Exercises

1) For which k is $\begin{cases} x + kz = 1 \\ y + z = 2 \\ 2x + y = 5 \end{cases}$

a) inconsistent
b) independent
c) dependent

$$\left[\begin{array}{ccc|c} 1 & 0 & k & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 5 \end{array}\right] \xrightarrow{-2R_1+R_3} \left[\begin{array}{ccc|c} 1 & 0 & k & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & -2k & 3 \end{array}\right] \xrightarrow{-R_2+R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & k & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -2k-1 & 1 \end{array}\right]$$

a) To inconsistent, it is enough that the last equation is a contradiction

$$(-2k-1)z = 1$$

$$-2k-1 = 0 \Rightarrow \boxed{k = -\tfrac{1}{2}}$$

b) To be independent, $(-2k-1) \neq 0 \iff \boxed{k \neq -\tfrac{1}{2}}$

c) We cannot make this system dependent

2) For which k & m has $\begin{cases} x + 2y + 2z = -1 \\ y + 2z = 3 \\ x - 3y + kz = m \end{cases}$

a) No sol^n
b) 1 sol^n
c) ∞ nbr sol^n.

First try putting it in row-echelon form.

3) If $\begin{cases} y = -2x - 2z + 1 \\ x = -2y - z + 2 \\ z = x - y \end{cases}$ becomes $\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right]$

$\Rightarrow$ Find a, b, c.

4) Solve $\left[\begin{array}{ccc|c} 3 & 3 & 4 & -3 \\ 4 & 4 & 2 & 1 \end{array}\right]$.