EXERCISES IN MATH

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Second Semester 1998/1999

1. THE FUNDAMENTAL THEOREM OF ALGEBRA AND RATIONAL ZEROS

1. If $f(x)$ is a polynomial of lowest degree and real coefficients having zeros $1, 1-i, i$,and $1+i$, then the degree of $f(x)$ is:
   - (a) 7
   - (b) 8
   - (c) 6
   - (d) 4
   - (e) 5

2. Given that 1 is a zero of $f(x) = x^3 - 6x^2 + 8x - 3$, then the other zeros are:
   - (a) $\frac{5 \pm \sqrt{13}}{2}$
   - (b) $2i, 3i$
   - (c) $\frac{5 \pm \sqrt{13}i}{2}$
   - (d) ± 3
   - (e) ± 1

3. Which one of the following is TRUE ?
   - (a) Any polynomial of degree n has exactly n distinct zeros.
   - (b) Every polynomial has at least one nonreal complex zero.
   - (c) The polynomial $x+k$ is a factor of the polynomial $f(x)$ if and only if $f(-k) = 0$.
   - (d) If $f(x)$ is a polynomial and if $a+bi$ is a zero of $f(x)$, where a and b are real numbers, then $a-bi$ is also a zero of $f(x)$.
   - (e) If the polynomial $f(x)$ is divided by $x+k$, then the remainder is $f(k)$.

4. If the polynomial $P(x) = 2x^4 + 14x^3 - 54x^2 + 58x - 20$ has 1 as a zero of multiplicity 3, then the remaining zero(s) is(are):
   - (a) $10, 1$
   - (b) 1
   - (c) -10
   - (d) $-10, 1$
   - (e) $-10, 10$

5. If $4, -i, 2+i$ are the zeros of a polynomial $P(x)$ with real coefficients, then an expression of $P(x)$ is:
   - (a) $(x-4)\left(x^2-1\right)\left(x^2+4x+5\right)$
   - (b) $(x-4)\left(x^2+1\right)\left(x^2-4x-5\right)$
   - (c) $(x-4)\left(x^2+1\right)\left(x^2+4x+5\right)$
   - (d) $(x-4)\left(x^2+1\right)\left(x^2-4x+5\right)$
   - (e) $(x+4)\left(x^2+1\right)\left(x^2-4x+5\right)$

6. If $1+i$ is a zero of the polynomial $P(x) = x^4 - 3x^3 + 2x^2 + 2x - 4$, then the other zeros are:
   - (a) $-1+i, 2, -1$
   - (b) $-1-i, 2, -1$
   - (c) $1-i, -2, 1$
   - (d) $-1+i, -2, 1$
   - (e) $1-i, 2, -1$

7. The number of rational zeros of $f(x) = x^5 - 5x^3 + 5$ is:
   - (a) 1
   - (b) 0
   - (c) 5
   - (d) 4
   - (e) 3

8. The polynomial $f(x) = 2x^3 + 4x^2 - x - 2$ has
   - (a) three rational zeros.
   - (b) one rational and two irrational zeros.
   - (c) two rational and one irrational zeros.
   - (d) three nonreal complex zeros.
   - (e) one rational and two nonreal complex zeros.

9. A polynomial of lowest degree and real coefficients having $-2i, i$, and 0 of multiplicity 2 as roots is:
   - (a) $x^5 + 3x^3 + 4x$
   - (b) $x^6 + 5x^4 + 4x^2$
   - (c) $x^4 + ix^3 + 2x^2$
   - (d) $x^2(x+2i)^2(x-i)^2$
   - (e) $x^2\left(x^2-4\right)\left(x^2-1\right)$

10. If $f(x)$ is a polynomial of degree 4 with real coefficients and $f(k) = 0$, where k is a complex number, then
    - (a) $(x+k)$ is a factor of $f(x)$.
    - (b) $f(x)$ has 4 distinct zeros.
    - (c) k and $-k$ are zeros of $f(x)$.
    - (d) If $\left(1+\sqrt{5}\right)$ is a zero, then $\left(1-\sqrt{5}\right)$ must be a zero.
    - (e) If $(1+i)$ is a zero of multiplicity 2, then $(1-i)$ is also a zero of multiplicity 2.

11. The rational zeros of $f(x) = x^3 - 2x^2 - 4x + 8$ are:
    - (a) $1, 2, 3$
    - (b) $-2, -2, 2$
    - (c) $2, 2, \frac{3}{5}$
    - (d) $\frac{1}{2}, 4, 2$
    - (e) $2, 2, -2$

12. The number of possible rational zeros of $f(x) = 4x^5 + 4x^4 - 37x^3 - 37x^2 + 9x + 9$ is:
    (a) 9
    (b) 6
    (c) 3
    (d) 12
    (e) 18

13. The number -2 is a zero of $f(x) = x^4 + 2x^3 - 7x^2 - 20x - 12$ of multiplicity
    (a) 1
    (b) 2
    (c) 3
    (d) 4
    (e) 5

14. The number of rational zeros of $f(x) = 2x^4 - 3x^3 + 3x^2 - 3x + 1$ is
    (a) 4
    (b) 0
    (c) 2
    (d) 1
    (e) 3

15. The set of all distinct zeros of $f(x) = x\left(x^2 - 6x + 9\right)\left(x^2 + 4\right)^2$ is
    (a) $\{3, 2, -2\}$
    (b) $\{0, 3, 2i, -2i\}$
    (c) $\{0, -3, 2, -2\}$
    (d) $\{0, 3, 2, -2\}$
    (e) $\{0, 3, -3, 2i, -2i\}$

16. The sum of all zeros of $f(x) = 2x^3 - x^2 - 2x + 1$ is
    (a) $\frac{5}{2}$
    (b) $\frac{3}{2}$
    (c) -2
    (d) $\frac{1}{2}$
    (e) 3

17. If 1 is a zero of multiplicity 2 of $f(x) = x^4 + ax^3 + 2x^2 - 2x + b$, then
    (a) $a = -2, b = 1$
    (b) $a = 1, b = 1$
    (c) $a = 2, b = -1$
    (d) $a = 1, b = -2$
    (e) $a = -1, b = 1$

18. If the polynomial $f(x)$ with only real coefficients has $1 - 2i$ as a zero, then which one of the following is a factor of $f(x)$?
    (a) $x^2 - 2x + 5$
    (b) $x^2 - 2x - 5$
    (c) $x^2 + 2x - 5$
    (d) $x^2 - 2x + 4$
    (e) $x^2 - 2x - 4$

19. The polynomial $f(x)$ of lowest degree with only real coefficients having -2 and i as zeros, and $f(0) = 4$ is
    (a) $2x^3 + 6x^2 + 4$
    (b) $2x^3 + 10x^2 + 8x + 4$
    (c) $\frac{1}{2}x^3 + 2x^2 + \frac{3}{2}x + 4$
    (d) $2i(x + 2)(x - i)$
    (e) $2x^3 + 4x^2 + 2x + 4$

20. Let P be a polynomial of degree 5 with real coefficients. Which one of the following is TRUE?
    (a) P has 2 or 4 nonreal zeros.
    (b) It is possible for P to have the zeros $1, 2, i,$ and $1 + i$.
    (c) It is not possible for P to have the zeros $1, 2, 3,$ $i + 2$, and $-i + 2$.
    (d) It is possible for $1 + i$ to be a zero of multiplicity 3 of P.
    (e) P has at least one real zero.

21. For which integer n is $x + k$ a factor of $x^n + k^n$?
    (a) 2
    (b) 3
    (c) 4
    (d) 5
    (e) 7

22. The number of possible rational zeros of $f(x) = x^3 + \frac{1}{2}x^2 - \frac{11}{2}x - 5$ is
    (a) 6
    (b) 12
    (c) 2
    (d) 4
    (e) 0

23. The values of A such that $f(x) = x^3 + 2x^2 + Ax + 3$ has a rational zero are
    (a) $2, 3, 4, 6$
    (b) $-2, -3, -4, -6$
    (c) $-2, -6, -16, 4$
    (d) $-16, 4, 3, -3$
    (e) $2, 6, -16, 4$

24. Which one of the following is TRUE?
    (a) If $1 - i$ is a zero of $f(x) = x^2 - 2x + 1 + 2i$,

then $1+i$ is also a zero.
(b) $x^3+\sqrt{2}x^2+6x+\sqrt{13}=0$ has one root $a+bi$, then a second root is $a-bi$.
(c) x^3-x+1 has two rational zeros.
(d) x^6+x^2+1 has at least one real zero.
(e) x^6+x^2 has no real zeros.

25. The equation of the polynomial with rational coefficients, one of whose roots is $\sqrt{13}-\sqrt{3}$ is
(a) $x^2-13x+3$
(b) $x^2+13x-3$
(c) x^4-32x^2+100
(d) $\left(x-\sqrt{13}+3\right)^2$
(e) $x-\sqrt{13}+\sqrt{3}$

26. The sum of the values of k so that when x^3-3x^2+x-1 is divided by $x-k$ the remainder is 2, is
(a) 3
(b) $3+2i$
(c) $2i$
(d) -3
(e) $3-2i$

27. Given that $x-i$ is a factor of the polynomial $f(x)=x^4+x^3+2x^2+x+1$, then one of the following is also a factor.
(a) ix^2-x-1
(b) x^2+x+i
(c) x^2+x+1
(d) x^2+ix-1
(e) x^2-x-1

28. The number of noninteger rational zeros of the function $f(x)=(x+1)\left(4x^3+4x^2+3x+1\right)$ is equal to
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

29. The sum of all rational zeros of $f(x)=x^3-\frac{4}{3}x^2-\frac{13}{3}x-2$ is
(a) $-\frac{2}{3}$
(b) $-\frac{4}{3}$
(c) $\frac{2}{3}$
(d) $\frac{4}{3}$
(e) 3

30. The lowest degree of a polynomial with real coefficients having the following zeros $1-\sqrt{3}, 2+i, 3,$ and $2i$ is equal to
(a) 6
(b) 7
(c) 5
(d) 8
(e) 4

31. The number of distinct zeros of $f(x)=-x^4\left(x^2+5\right)^2\left(x^4-2x^2-3\right)^3$ is
(a) 20
(b) 9
(c) 7
(d) 8
(e) 10

32. The number of solutions of the equation $x^4-4x^3+6x^2-4x+1=0$ is
(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

33. Given that i is a zero of the polynomial $f(x)=8x^5-12x^4+14x^3-13x^2+6x-1$, then the rational zeros of $f(x)$ are
(a) $\frac{1}{2}$
(b) $\frac{1}{2},\frac{1}{2}$
(c) $\frac{1}{2},\frac{1}{2},\frac{1}{2}$
(d) $\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}$
(e) $\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}$

34. The number of rational zeros of the polynomial $P(x)=x\left(6x^3+9x^2+5x+1\right)$ is
(a) 1
(b) 4
(c) 2
(d) 3
(e) 0

35. The sum of all integer zeros of $f(x)=x^3-\frac{4}{3}x^2-\frac{5}{3}x+\frac{2}{3}$ is
(a) 0
(b) 3
(c) 1
(d) -3
(e) -1

2. GRAPHS OF POLYNOMIAL FUNCTIONS

1. The graph of the polynomial $y = (x-1)^2 (x-2)^2 (x+1)$, where $x \geq 0$ is
   (a) above or on the x-axis.
   (b) increasing.
   (c) above the x-axis.
   (d) decreasing.
   (e) below the x-axis.

2. The set of all x for which the graph of the function $f(x) = x^3 - 4x^2 + 5x - 2$ is completely below the x-axis, is
   (a) $(-\infty, 1) \cup (2, \infty)$
   (b) $(-1, 2)$
   (c) $(-\infty, 1) \cup (1, 2)$
   (d) $(1, \infty)$
   (e) $(-\infty, -2) \cup (-1, \infty)$

3. The polynomial $f(x) = x(x+1)^2 (x-2)^3$
   (a) is below the x-axis if $0 < x < 2$ only.
   (b) has 4 turning points.
   (c) is above the x-axis if $x > 0$.
   (d) is decreasing if $-1 < x < 0$.
   (e) has 6 x-intercepts.

4. The function $f(x) = x^2 (x-3)^3 (x+1)$ has
   (a) 2 turning points below the x-axis and 1 turning point above the x-axis.
   (b) 2 turning points above the x-axis and 1 turning point below the x-axis.
   (c) 3 turning points above the x-axis.
   (d) 2 turning points above the x-axis and 1 turning point on the x-axis.
   (e) 2 turning points below the x-axis and 1 turning point on the x-axis.

5. The number of turning points of the graph of $f(x) = x^5 - x^3$ is
   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) 5

6. The graph of $f(x) = 2x^3 + 5x^2 - x - 6$ lies above the x-axis in the interval(s)
   (a) $(-\infty, -2) \cup \left(-\frac{3}{2}, 1\right)$
   (b) $\left(-2, -\frac{3}{2}\right) \cup (1, \infty)$
   (c) $(-\infty, 1)$
   (d) $(-2, \infty)$
   (e) $(-2, 1) \cup (1, \infty)$

7. The number of turning points of the graph of $f(x) = x^4 - x^3 - x^2 - 2x - 6$ is
   (a) 1
   (b) 2
   (c) 4
   (d) 5
   (e) 0

8. The graph of $f(x) = 3x^4 + x^3 - 2x^2$ has
   (a) all turning points below the x-axis.
   (b) one turning point below the x-axis and two above it.
   (c) one turning point below the x-axis and one above it.
   (d) two turning points below the x-axis and one above it.
   (e) two turning points below the x-axis and one on it.

9. The number of turning points of $y = x\left(x^2 - 1\right)^2$ is
   (a) 2
   (b) 1
   (c) 4
   (d) 3
   (e) 5

10. If $P(x) = x^2 (x-2)(x+3)^3$, which one of the following is TRUE ?
    (a) The graph of $P(x)$ is above the x-axis for $-\infty < x < -3$ only.
    (b) The graph of $P(x)$ is above the x-axis for $-\infty < x < -3$ and $2 < x < \infty$.
    (c) The graph of $P(x)$ is above the x-axis for $-\infty < x < -3$ and $0 < x < 2$.
    (d) The graph of $P(x)$ is below the x-axis for $-2 < x < 0$ and $0 < x < 3$.
    (e) The graph of $P(x)$ is below the x-axis for $-3 < x < 0$ only.

11. If $P(x) = x^2 (x+1)(x-2)$, which one of the following is TRUE ?
    (a) $P(x)$ is negative for $-\infty < x < -1$.
    (b) $P(x)$ is negative for $0 \leq x \leq 2$.
    (c) $P(x)$ has two turning points.
    (d) $P(x)$ has three turning points.
    (e) $P(x)$ ha four turning points.

3. GRAPHS OF RATIONAL FUNCTIONS

1. The graph of the function $f(x) = \frac{x^2+2x}{2x-1}$ has
   (a) no vertical asymptotes.
   (b) one vertical asymptote and one oblique asymptote.
   (c) two vertical asymptotes.
   (d) one oblique asymptote only.
   (e) one horizontal asymptote and one vertical asymptote.

2. The rational function $f(x) = \frac{x^2-3x+2}{x^2-4}$ has
   (a) two vertical asymptotes, one horizontal asymptote and no oblique asymptotes.
   (b) two vertical asymptotes, one horizontal asymptote and one oblique asymptotes.
   (c) one vertical asymptote, one horizontal asymptote and no oblique asymptotes.
   (d) one vertical asymptote, one horizontal asymptote and one oblique asymptote.
   (e) no vertical asymptotes, one horizontal asymptote and no oblique asymptotes.

3. The rational function $f(x) = \frac{x^3+x+1}{x^2+1}$ has
   (a) no asymptotes.
   (b) no vertical asymptotes, no horizontal asymptotes and one oblique asymptote.
   (c) two vertical asymptotes, no horizontal asymptotes and one oblique asymptote.
   (d) no vertical asymptotes, one horizontal asymptote and one oblique asymptote.
   (e) one vertical asymptote, no horizontal asymptotes and one oblique asymptote.

4. The rational function $f(x) = \frac{x^2+3x+2}{x^2+2x-3}$ has
   (a) one vertical asymptote and one horizontal asymptote.
   (b) two horizontal asymptotes and one oblique asymptotes.
   (c) one horizontal asymptote and one oblique asymptote.
   (d) two vertical asymptotes and one horizontal asymptote.
   (e) one vertical asymptote and one oblique asymptote.

5. The rational function $f(x) = \frac{x^3+2x^2}{x^2+2x+1}$ has
   (a) two vertical asymptotes and one oblique asymptote given by $y = x$.
   (b) one vertical asymptote and one horizontal asymptote given by $y = 1$.
   (c) no vertical or horizontal asymptotes.
   (d) one vertical asymptote and one oblique asymptote given by $y = x$.
   (e) one vertical asymptote and one oblique asymptote given by $y = x + 4$.

6. The rational function $y = \frac{ax^2+2x+1}{bx-4}$ has a horizontal asymptote if
   (a) $a = 0, b = 0$
   (b) $a \neq 0, b \neq 0$
   (c) $a \neq 0, b = 0$
   (d) $a = 0, b \neq 0$
   (e) a, b are any real numbers.

7. The graph of the function $f(x) = \frac{2-3x+x^2}{x^2+4x-5}$ has
   (a) one vertical asymptote and one horizontal asymptote.
   (b) two horizontal asymptotes and two vertical asymptotes.
   (c) one horizontal asymptote and two vertical asymptotes.
   (d) one oblique asymptote and one horizontal asymptote.
   (e) two vertical asymptotes only

8. The rational function $f(x) = \frac{x-x^2}{x^2-2x+2}$ has
   (a) no asymptotes.
   (b) two vertical asymptotes and one oblique asymptote.
   (c) two vertical asymptotes and one horizontal asymptote.
   (d) one horizontal asymptote given by $y = 1$ only.
   (e) one horizontal asymptote given by $y = -1$ only.

9. If $f(x) = \frac{x^2-1}{x^2}$. Which one of the following is TRUE ?
   (a) $f(x)$ has two vertical asymptotes.
   (b) $f(x)$ has $x = 0$ as a missing point.
   (c) $f(x)$ has an oblique asymptote.
   (d) $f(x)$ has only $x = 1$ as x-intercept.
   (e) $f(x)$ has $y = 1$ as a horizontal asymptote.

10. The asymptotes of the graph of $f(x) = \frac{x^2-4x+3}{x^2-x}$ are
   (a) $x = 0, y = 1$
   (b) $x = 0, x = 1, y = 1$
   (c) $x = 0, x = 1$
   (d) $x = 1, y = 1$
   (e) $x = 0, x = 1, y = x$

11. If $x = 2$ is a vertical asymptote and $y = 3$ is a hori-

zontal asymptote of $f(x) = \frac{bx+1}{2x+a}$, then

(a) $a = -4, b = 6$
(b) $a = -2, b = -3$
(c) $a = -1, b = 3$
(d) $a = -4, b = 2$
(e) $a = -2, b = 6$

12. The oblique asymptote of the graph of $f(x) = \frac{x^2-3}{x+2}$ is

(a) $y = x$
(b) $y = x - 1$
(c) $y = 1$
(d) $y = x - 2$
(e) $y = x - 3$

13. The rational function $f(x) = \frac{x^2-1}{(x+1)(x^2+4)}$ has

(a) two vertical asymptotes and one horizontal asymptote.
(b) two vertical asymptotes and one oblique asymptote.
(c) three vertical asymptotes and one horizontal asymptote.
(d) no vertical asymptote and one horizontal asymptote.
(e) one horizontal asymptote and one vertical asymptote.

14. The graph of $y = \frac{(2a-1)x+1}{ax-6}$ has the line $x = 2$ as vertical asymptote, then it has a horizontal asymptote given by

(a) $y = \frac{1}{3}$
(b) $y = \frac{3}{2}$
(c) $y = \frac{2}{3}$
(d) $y = \frac{5}{3}$
(e) $y = 2$

15. The equation of the oblique asymptote of the graph of the equation $f(x) = \frac{1-2x-6x^2}{7+3x}$ is

(a) $y = -2x + \frac{16}{3}$
(b) $y = -2x + 4$
(c) $y = -2x - 4$
(d) $y = -2x - \frac{16}{3}$
(e) $y = x - 2$

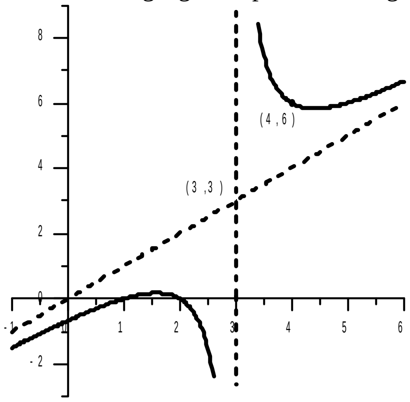
16. The function $f(x) = \frac{(x+3)(x^2-4)}{x+3}$ represents

(a) a parabola.
(b) no graph.
(c) a parabola with one missing point.
(d) a graph with the vertical asymptote $x = -3$.
(e) a graph with the horizontal asymptote $y = 1$.

17. The graph of the function $f(x) = \frac{x^3-2x^2-13x-10}{x^2-3x-10}$

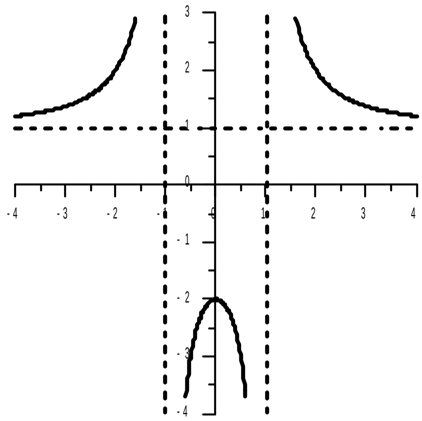
(a) has missing points at $x = -2$ and $x = 5$.
(b) has two vertical asymptotes given by $x = -2$ and $x = 5$.
(c) is the line $f(x) = x + 1$.
(d) has three x-intercepts.
(e) has only one vertical asymptote given by $x = -2$.

18. The following figure represents the graph of



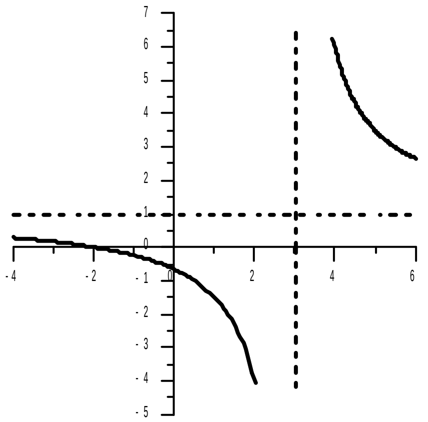
(a) $y = \frac{x^2-3x}{x-3}$
(b) $y = \frac{2x^2-6x+2}{x-3}$
(c) $y = \frac{x^2-3x+2}{x-3}$
(d) $y = \frac{x^2-3x+2}{(x-3)^2}$
(e) $y = x - 3$

19. The following figure represents the graph of



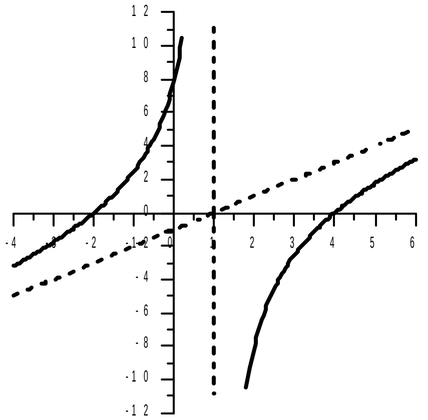
(a) $y = \frac{2x^2}{x^2-1}$
(b) $y = \frac{1}{(x-1)^2}$
(c) $y = \frac{x^2+2}{x^2-1}$
(d) $y = \frac{2x^2}{(x-1)^2}$
(e) $y = \frac{x^2-2}{x^2+1}$

20. The following figure represents the graph of



(a) $y = \frac{x+2}{x}$
(b) $y = \frac{x}{x-3}$
(c) $y = \frac{-x-3}{x-3}$
(d) $y = \frac{x+2}{-x+3}$
(e) $y = \frac{x+2}{x-3}$

21. The function that corresponds to the following figure is



(a) $y = \frac{(x+2)(x+4)}{x+1}$
(b) $y = \frac{(x-2)(x+4)}{(x+1)^2}$
(c) $y = \frac{(x+2)(x-4)}{(x-1)^2}$
(d) $y = \frac{x^2+2x-4}{x-1}$
(e) $y = \frac{(x+2)(x-4)}{x-1}$

22. The oblique asymptote of the graph of $f(x) = \frac{3(x^2+1)}{5x-1}$ is

(a) $y = x$
(b) $y = \frac{3}{5}x$
(c) $y = \frac{5}{3}x$
(d) $y = \frac{3}{5}x + \frac{3}{25}$
(e) $y = \frac{5}{3}x + \frac{25}{3}$

4. EXPONENTIAL AND LOGARITHMIC FUNCTIONS

1. If $\log_c 2 = \frac{2}{3}$, then $\log_8 c =$

(a) $\frac{2}{9}$
(b) $\frac{1}{2}$
(c) $\frac{8}{27}$
(d) $\frac{27}{8}$
(e) 2

2. The value of $\log_8 \frac{\sqrt[3]{16}}{4}$ is

(a) $\frac{10}{3}$
(b) $\frac{2}{3}$
(c) $\frac{1}{6}$
(d) $\frac{2}{9}$
(e) $-\frac{2}{9}$

3. Let $\ln 2 = x$ and $\ln 3 = y$. If $2^{t+1} = 3^{2t-1}$, then $t =$

(a) $\frac{x+y}{2y+x}$
(b) $\frac{-x-y}{2x+y}$
(c) $\frac{x-y}{2x+y}$
(d) $\frac{x+y}{2y-x}$
(e) $\frac{-x+y}{x-2y}$

4. $[\log_9 35 - \log_9 7] \cdot [\log_5 9] =$

(a) $(\log_9 28) \cdot (\log_5 9)$
(b) $\log_9 5$
(c) 1
(d) 0
(e) $\log_5 9$

5. $\ln \ln e^{e^{x+3}} - e^{\ln x} =$

(a) $e^{x+3} - e^x$
(b) 3
(c) $\ln \ln \left[e^{e^x+3} - e^{\ln x}\right]$
(d) $3x$
(e) $\ln \ln e^{x+3-\ln x}$

6. If $\ln 2 = x$ and $\ln 10 = y$, then $\ln 16000 + \ln 5 =$

(a) $12xy$
(b) $4x + 3y$
(c) $4y + 3x$
(d) $x^4 + y^3$
(e) $x^3 + y^4$

7. The expression $(e^x + e^{-x})^4 - (e^x - e^{-x})^4$ is equal to

(a) $8e^{2x}\left(1-e^{-4x}\right)$
(b) $2\left(e^{4x}+4e^{2x}+4e^{-2x}+e^{-4x}\right)$
(c) $8\left(e^{2x}+e^{x}-e^{-x}+e^{-2x}\right)$
(d) $8e^{-2x}\left(e^{4x}+1\right)$
(e) $8e^{-2x}$

8. $\log_8 e^3 x =$
(a) $3+\ln x$
(b) $\frac{1+\ln x}{\ln 2}$
(c) $\frac{e^3+\ln x}{\ln 8}$
(d) $\frac{\ln x}{\ln 2}$
(e) $\frac{3+\ln x}{3\ln 2}$

9. If $\log_3\left[\log_{\frac{1}{2}} x\right]=1$, then $x=$
(a) $\sqrt{3}$
(b) $\frac{1}{9}$
(c) $\frac{1}{8}$
(d) $\frac{3}{2}$
(e) 8

10. $\left[\sqrt{2}\right]^{\frac{\log 9}{\log 2}} =$
(a) 81
(b) $\frac{1}{3}$
(c) 9
(d) $\frac{9}{2}$
(e) 3

11. $3\log_2 x-\log_{\sqrt{2}} y+\log_4 z^2 =$
(a) $\log_2 \frac{x^3 z}{y^2}$
(b) $\log_2 \frac{y^2}{x^3 z}$
(c) $\log_2 \frac{x^3 z^2}{y^2}$
(d) $\log_2 \frac{x^3 z}{\sqrt{y}}$
(e) $\log_2 \frac{x^3\sqrt{z}}{y^2}$

12. If $\log 0.04 = x$, then $\log 80 =$
(a) $\frac{3x}{2}+4$
(b) $\frac{3x}{2}+2$
(c) $\left(\frac{x}{2}+1\right)^3+1$
(d) $\frac{3}{10}\left(\frac{x}{2}+1\right)$
(e) $\frac{3x}{2}-2$

13. If $\ln 2 = 0.7$ and $\ln 3 = 1.1$, then $\log_{36}\left(\frac{e^3}{12}\right) =$
(a) $\frac{e^3}{9}$
(b) $\frac{1}{3}$
(c) $\frac{5}{36}$
(d) $\frac{27}{36}$
(e) $\frac{10e^3-25}{36}$

14. If $y=\ln(x-3)+1$, then $x=$
(a) $3+(y-1)^e$
(b) $2+y^e$
(c) $3+e^{y-1}$
(d) $-3+e^{1-y}$
(e) $3-e+e^y$

15. The value of $(\log_5 16)\left(\log_2\sqrt{5}\right)-\left(\sqrt{e}\right)^{-6\ln 2} =$
(a) $\frac{15}{8}$
(b) $\frac{17}{8}$
(c) $\frac{17}{9}$
(d) $\frac{19}{9}$
(e) 14

16. If $t=\frac{10^x-10^{-x}}{10^x+10^{-x}}$, then $x=$
(a) $\frac{1}{2}\log\frac{t+1}{t-1}$
(b) $2\log_{0.1}\frac{t+1}{1-t}$
(c) $2\log\frac{t-1}{t+1}$
(d) $\frac{1}{2}\log\frac{1+t}{1-t}$
(e) $2\log_{0.1}\frac{t-1}{t+1}$

17. If $(\log_c x)\cdot(\log_5 c)=3$, then $x=$
(a) 25
(b) 200
(c) 101
(d) 125
(e) 100

18. If $\log x=2$, $\log y=3$, and $\log z=5$, then $\log\frac{x^3 y}{\sqrt{z}}-\log_x z=$
(a) 9
(b) $\frac{13}{2}$
(c) $\frac{7}{2}$
(d) 4
(e) 0

19. If $\ln 2=0.7$ and $\ln 3=1.1$, then $\log_{\frac{2}{3}}\frac{4e^2}{27} =$
(a) $\frac{3}{4}$
(b) $-\frac{1}{4}$
(c) $\frac{1}{2}$
(d) $\frac{1}{3}$
(e) $\frac{1}{4}$

20. The equation of the form $y=a^x$ whose graph contains the point $(3,8)$ is
(a) $y=3^x$
(b) $y=2^x$
(c) $y=4^x$
(d) $y=8^x$

(e) $y = \left(\frac{1}{2}\right)^x$

21. $\log_a \frac{1}{x} =$
    (a) $\log_{\frac{1}{a}} \frac{1}{x}$
    (b) x
    (c) a
    (d) 1
    (e) $\log_{\frac{1}{a}} x$

22. If $a > 0$, $a \neq 1$, and $y = \frac{\log(\ln a)}{\log a}$, then $a^y =$
    (a) $\ln a$
    (b) $\frac{1}{\ln a}$
    (c) a
    (d) e
    (e) $\log a$

23. If $f(x) = a^x$ and $f(-2) = \frac{1}{3}$, then $f(6) =$
    (a) 9
    (b) 27
    (c) 12
    (d) $\frac{1}{27}$
    (e) 18

24. If $\log_x(\log_2 8) = 2$, then $x =$
    (a) 9
    (b) $\frac{1}{3}$
    (c) $\sqrt{2}$
    (d) 8
    (e) $\sqrt{3}$

25. $2\log 5 + \frac{1}{2}\log 16 =$
    (a) 5
    (b) 2
    (c) 1
    (d) 10
    (e) 3

26. If $f(t) = 3^{2-t}$ is written in the form $f(t) = ka^t$, then k and a are respectively
    (a) $\frac{1}{3}, 3$
    (b) $9, 3$
    (c) $3, \frac{1}{3}$
    (d) $9, \frac{1}{3}$
    (e) $9, -3$

27. If $\log 2 = a$ and $\log 3 = b$, then $\log_4 60 =$
    (a) $2 + a + b$
    (b) $\frac{1+a+b}{2a}$
    (c) $\frac{1+a+b}{a^2}$
    (d) $\frac{a+b}{a^2}$
    (e) $\frac{a+b}{2a}$

28. The value of $\ln(\ln e) + e^{-2\ln\sqrt{5}}$ is
    (a) $-2\sqrt{5}$
    (b) -5
    (c) $\frac{4}{5}$
    (d) $\frac{1}{25}$
    (e) $\frac{1}{5}$

29. If $\log_3 a = \frac{1}{3}$, then $\log_a\left(\frac{1}{9}\right) =$
    (a) 9
    (b) -6
    (c) $-\frac{2}{3}$
    (d) $\frac{1}{6}$
    (e) -2

30. If $\ln 2 = x$ and $\ln 6 = y$, then $\log_9 4 =$
    (a) $\frac{y}{x+y}$
    (b) $\frac{x}{x+y}$
    (c) $\frac{y}{y-x}$
    (d) $\frac{xy}{x-y}$
    (e) $\frac{x}{y-x}$

31. If $\log_2(x-1) = \frac{1}{2}$, then the value of $\log_2\left(2x^2 - 4x + 2\right) =$
    (a) 1
    (b) 3
    (c) $\frac{5}{4}$
    (d) 2
    (e) $\frac{1}{4}$

32. The solution set of the equation $\log_{\frac{1}{x}} x^2 = 6$ is
    (a) $\emptyset$
    (b) $\{-1\}$
    (c) $\{1\}$
    (d) $\{\log_2 6\}$
    (e) $\left\{e^6\right\}$

33. Which one of the following is FALSE ?
    (a) $\ln e^x = x$ for any real number x.
    (b) $e^{\ln x} = x$ for any real number x.
    (c) $\ln \frac{1}{10} < \ln \frac{1}{3}$
    (d) $\log_{\frac{1}{3}} 4 > \log_{\frac{1}{3}} 5$
    (e) $g(x) = \left(\frac{1}{3}\right)^{-x}$ is an increasing function.

34. The domain of the function $y = \log\left(1 - x^2\right)$ is
    (a) $(-1, 1)$
    (b) $(1, \infty)$

(c) $(-\infty, 1)$
(d) $[-1, 1]$
(e) $(-\infty, -1) \cup (1, \infty)$

35. If $x > 0$, which of the following is TRUE ?
(a) $\log(1+x) = \frac{x}{1+x}$
(b) $\log(1+x) < \frac{x}{1+x}$
(c) $\log(1+x) > x$
(d) $\log(1+x) < x$
(e) non of the above.

36. Which one of the following is FALSE ?
(a) $\log_{\frac{1}{2}} 8 = -3$
(b) $\log_a xy = \log_a x + \log_a y$, $x > 0, y > 0, a > 0$, and $a \neq 1$
(c) $y = \log_a x$ if and only if $x = a^y$, $x > 0, a > 0$, and $a \neq 1$
(d) $a^{\log_a x} = x$, $x > 0, a > 0$, and $a \neq 1$
(e) $\frac{\log_a x}{\log_a y} = \log_a (x - y)$

37. If $f(x) = 3^{x+1}$, $g(x) = \left(\frac{1}{3}\right)^{x+5}$, then
(a) $g(x)$ is an increasing function
(b) the range of $f(x)$ is $[0, \infty)$
(c) $f(x)$ is a decreasing function
(d) the domain of $g(x)$ is $(0, \infty)$
(e) the graph of $f(x)$ and $g(x)$ intersect at $\left(-3, \frac{1}{9}\right)$

38. The graph of $y = \log_3(2x+1) - 2$ has
(a) x-intercept $= 2$; y-intercept $= -1$
(b) x-intercept $= 4$; y-intercept $= -1$
(c) x-intercept $= 3$; y-intercept $= -2$
(d) x-intercept $= 4$; y-intercept $= -2$
(e) x-intercept $= 3$; y-intercept $= 1$

39. If $f(x) = \log(2x-1) - 3$, then $f^{-1}(-2) =$
(a) $\frac{11}{2}$
(b) 10
(c) 4
(d) $-\frac{1}{2}$
(e) $\frac{9}{2}$

40. Suppose the number of rabbits in a colony is $y = y_0 3^{\frac{t}{7}}$, where t is time in months and y_0 is the rabbit population at $t = 0$. The rabbits are doubled when $t =$
(a) $\frac{\ln 2}{7}$
(b) $7\log_2 3$
(c) $\frac{\ln 3}{7}$
(d) $7\log_3 2$
(e) $7\ln 2$

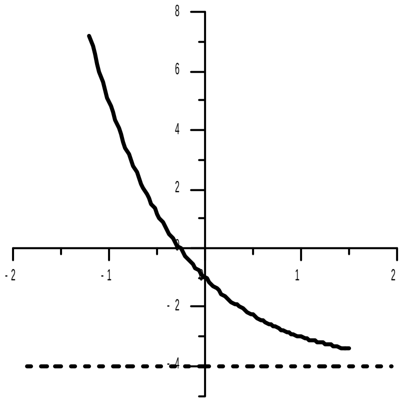
41. If the amount of a certain radioactive material present after t days is $P(t) = 800e^{-(\ln 2)t}$ grams, then the time needed for the material to decay to 200 grams is
(a) 3
(b) 2
(c) 4
(d) $\frac{3}{2}$
(e) 1

42. The number of mice in a colony is $P = P_0 e^{\frac{t}{4}}$, where t is time in months. The number of mice in the colony is tripled when $t =$
(a) $\frac{\ln 3}{4}$
(b) $\ln 12$
(c) $\ln 81$
(d) $4\log_3 e$
(e) $\frac{\log_3 e}{4}$

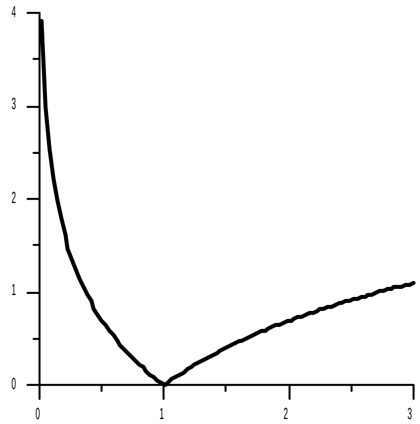
43. The number of mice in a colony is $P = P_0 10^{\frac{t}{2}}$. If the number of mice at time $t = 2$ is 10^6, then the number of mice in the colony at $t = 6$ is
(a) 10^8
(b) 10^5
(c) 10^6
(d) 10^{15}
(e) 10^7

44. In the formula $P(t) = P_0 e^{kt}$, if $P(25) = \frac{1}{2}P_0$, then $P(75) =$
(a) $P_0 \ln 8$
(b) $3P_0$
(c) $\frac{P_0}{8}$
(d) $\frac{P_0}{3}$
(e) $P_0 \ln 2$

45. The following figure represents the graph of
(a) $y = \log_{\frac{1}{4}}(x-1)$
(b) $y = \log_{\frac{1}{4}}(x+1)$
(c) $y = 2^{-x+1} - 6$
(d) $y = 3^{-x+1} - 4$
(e) $y = -3^{-x+1} + 2$

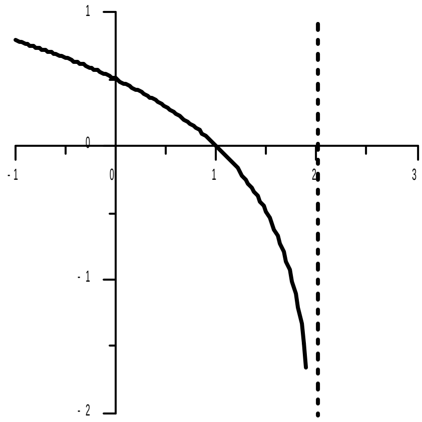


46. The following figure represents the graph of



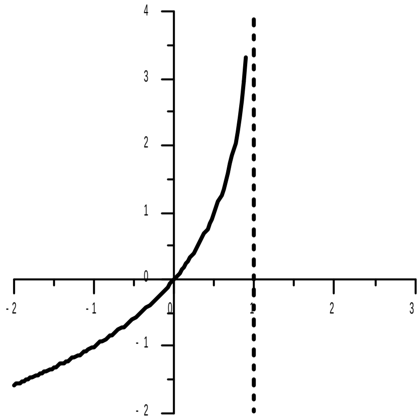
(a) $y = x \ln x$
(b) $y = \frac{\ln x}{x}$
(c) $y = |\ln x|$
(d) $y = \ln |x|$
(e) $y = \frac{x}{\ln x}$

47. The following figure represents the graph of



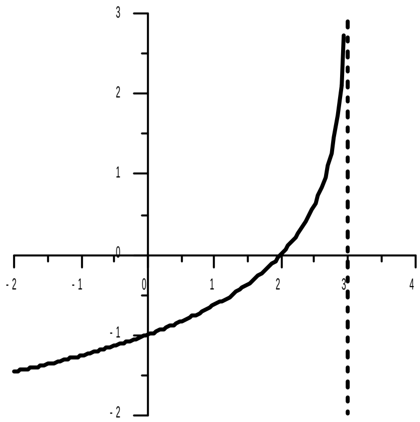
(a) $y = \log_4 (x - 2)$
(b) $y = \log_4 (2 - x)$
(c) $y = \log_4 |2 - x|$
(d) $y = \log_{\frac{1}{4}} (x - 2)$
(e) $y = \left|\log_{\frac{1}{4}} (x - 2)\right|$

48. The following figure represents the graph of



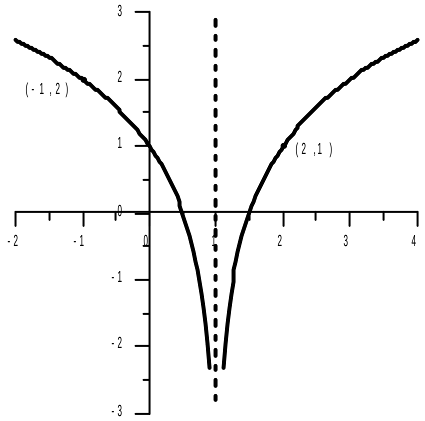
(a) $y = \log_2 (x - 1)$
(b) $y = \log_{\frac{1}{2}} (1 - x)$
(c) $y = \log_2 (1 - x)$
(d) $y = \log_{\frac{1}{2}} (x - 1)$
(e) $y = \ln (1 - x)$

49. The following figure represents the graph of



(a) $y = \log_3 (2 + x)$
(b) $y = \log_{\frac{1}{3}} (2 - x)$
(c) $y = \log_3 (3 - x)$
(d) $y = \log_{\frac{1}{3}} (3 - x)$
(e) $y = \log_{\frac{1}{3}} (3 + x)$

50. The following figure represents the graph of



(a) $y = 1 + \log_2 |x - 1|$
(b) $y = 1 + \log_2 |x - 2|$
(c) $y = 1 + \log_2 \left|x - \frac{3}{2}\right|$
(d) $y = \log_2 |x - 1|$
(e) $y = -1 + \log_2 |x - 1|$

51. If $(343)^{3-x} = (49)^x$, then $x =$

(a) $\frac{5}{9}$
(b) $\frac{9}{5}$
(c) 9
(d) 2
(e) $\frac{1}{2}$

52. The equation $2\log_2 x - \log_2 (x-1) = 2$ has

(a) two positive real solutions.
(b) one negative real solution.
(c) one positive real solution.
(d) one positive and one negative real solutions.
(e) no real solutions.

53. The solution set of $\left(\frac{3}{2}\right)^{|2x-1|} = \frac{27}{8}$ is

(a) $\{1, -3\}$
(b) $\{-1\}$
(c) $\{-1, 2\}$
(d) $\{2,3\}$
(e) $\{-1, -2\}$

54. The solution set of $2\log(x-3) = \log(x+5) + \log 4$ consists of

(a) one positive and one negative integers.
(b) two irrational numbers.
(c) $\emptyset$
(d) only one positive integer.
(e) one irrational and one rational numbers.

55. The solution set of the inequality $x^2 e^x - 2xe^x > 0$ is

(a) $(0, \infty)$
(b) $(0, 2)$
(c) $(-\infty, 0) \cup (0, \infty)$
(d) $(-\infty, 0) \cup (0, 2)$
(e) $(-\infty, 0) \cup (2, \infty)$

56. The solution set of $\left(\frac{2}{3}\right)^{x-2} = \left(\frac{27}{8}\right)^{-2(x+3)}$ consists of

(a) only one positive integer.
(b) only one irrational number.
(c) two real numbers.
(d) only one negative integer.
(e) $\emptyset$

57. The sum of all real solutions of the equation $(\log x)^2 + 2 - \log x^3 = 0$ is

(a) 110
(b) 3
(c) 100
(d) $e^2 + e$
(e) $\frac{1}{10}$

58. The solution set of the equation $\log(3x-1) = 1 - \log x$ consists of

(a) one positive integer and one negative rational number.
(b) only one irrational number.
(c) two irrational numbers.
(d) only one integer.
(e) one negative integer and one positive rational number.

59. If x_1 and x_2 are the solutions of the equation $e^x - 5 + 6e^{-x} = 0$, then $x_1 + x_2 =$

(a) 0
(b) $\ln 6$
(c) 5
(d) $\ln 9$
(e) -2

60. The equation $2^{2x} + 6 \cdot 2^x + 4 = 0$ has

(a) two solutions only.
(b) one positive real solution only.
(c) no solution.
(d) infinitely many solutions.
(e) one negative real solution only.

61. If $\log_2 6x - \log_2 3x = 2\log_2 k$, and $x > 0$, then $k =$

(a) $\pm\sqrt{2}$
(b) $\sqrt{3}$
(c) $\log_2 3$

(d) $\frac{3}{2}$
(e) $\sqrt{2}$

62. The solution set of the equation $2^{2x+1} - 7 \cdot 2^x - 4 = 0$ is
    (a) $\{2\}$
    (b) $\{-1\}$
    (c) $\{-2, 2\}$
    (d) $\{0\}$
    (e) $\{1\}$

63. The solution set of the equation $2 \log \sqrt{x+3} + \log(2-x) = \log(-2x)$ consists of
    (a) two positive real numbers.
    (b) two negative real numbers.
    (c) one positive and one negative real numbers.
    (d) only positive real number only.
    (e) one negative real number only.

64. The solution set of the equation $\ln x = -(\ln x)^2$ consists of
    (a) one natural and one irrational numbers.
    (b) two irrational numbers.
    (c) two rational numbers.
    (d) $\emptyset$
    (e) only one real number.

65. The solution set of the inequality $\log_3 x + 2 \log_9 x > 2$ is
    (a) $(0, \infty)$
    (b) $(3, \infty)$
    (c) $(0, 1)$
    (d) $(1, 3)$
    (e) $(0, 1) \cup (1, 3)$

66. The solution set of $\log_{3x+1} 4 = -2$ is
    (a) $\left\{-\frac{1}{2}, -\frac{1}{6}\right\}$
    (b) $\left\{-\frac{1}{2}\right\}$
    (c) $\left\{-\frac{1}{6}\right\}$
    (d) $\left\{\frac{1}{2}, \frac{1}{6}\right\}$
    (e) $\emptyset$

67. The graphs of the two functions $f(x) = e^{x^2}$ and $g(x) = (e^x)^2$ intersect at $x =$
    (a) $0, -2$
    (b) 0
    (c) $0, 1$
    (d) $0, 2$
    (e) $1, 2$

68. The solution set of $\ln(x-2) - \log_{e^{-1}}(x+2) - \ln\left(e^{\ln 12}\right) = 0$ is
    (a) $\{4, -4\}$
    (b) $\{2\}$
    (c) $\{0\}$
    (d) $\emptyset$
    (e) $\{4\}$

69. The solution set of $\log_2 \sqrt{x-2} + \log_4(x-4) = \frac{1}{2}(3 + \log_2 3)$ is
    (a) $\{3, 8\}$
    (b) $\{\sqrt{24}, 8\}$
    (c) $\{-2, 8\}$
    (d) $\emptyset$
    (e) $\{8\}$

70. The solution set of $(125)^{x(x-5)} = \left(\frac{1}{25}\right)^3$ is
    (a) $\{2, 3\}$
    (b) $\emptyset$
    (c) $\left\{\frac{5 \pm \sqrt{33}}{2}\right\}$
    (d) $\left\{\frac{5 \pm \sqrt{17}}{2}\right\}$
    (e) $\left\{\frac{5 \pm \sqrt{37}}{2}\right\}$

71. The solution set of the inequality $\log_{\frac{1}{2}} x^2 > -4$ is
    (a) $(-\infty, -4) \cup (4, \infty)$
    (b) $[-16, 16]$
    (c) $(-4, 0) \cup (0, 4)$
    (d) $(-\infty, -16) \cup (16, \infty)$
    (e) $(-\infty, \infty)$

72. The solution set of $(e^x - 3)(e^x + 1) = -3$ is
    (a) $\{\ln 2\}$
    (b) $\{0, \ln 2\}$
    (c) $\{0\}$
    (d) $\{0, e^2\}$
    (e) $\{0, 2\}$

73. If $\log_5 x < \log x$, then x belongs to the interval
    (a) $[0, 1]$
    (b) $(0, 1)$
    (c) $(1, \infty)$
    (d) $(-\infty, 1)$
    (e) $(0, 1]$

74. The solution set of $(\ln x)^2 - \ln x^3 + 2 = 0$ is
    (a) $\{-1, -2\}$
    (b) $\{1, 2\}$
    (c) $\{e^2\}$

(d) $\{10, 100\}$
(e) $\{e, e^2\}$

75. The solution set of $\log_x 64 < 3$ is
    (a) $(0, 1)$
    (b) $(4, \infty)$
    (c) $(0, 1) \cup (4, \infty)$
    (d) $(1, 4)$
    (e) $(0, \infty)$

76. The solution set of $\log_2 x < -1$ is
    (a) $(0, \infty)$
    (b) $\left(0, \frac{1}{2}\right)$
    (c) $\left(\frac{1}{2}, \infty\right)$
    (d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$
    (e) $(2, \infty)$

5. ANGLES AND THE RADIAN MEASURE

1. The radian measure of the central angle which cuts off an arc of length 6 inches on a circle of radius 5 inches is
   (a) 1.6
   (b) $\frac{5}{6}$
   (c) 1.2
   (d) 0.625
   (e) 6

2. The length of the arc on a circle of radius 6 cm cut by a central angle of $120°$ is
   (a) 4π cm
   (b) $\frac{\pi}{9}$ cm
   (c) 720 cm
   (d) 20 cm
   (e) $\frac{9}{\pi}$ cm

3. The angle $31°15'$ in radian measure is
   (a) $\frac{22\pi}{120}$
   (b) $\frac{144}{25\pi}$
   (c) $\frac{25\pi}{144}$
   (d) $\frac{5625}{\pi}$
   (e) $\frac{25\pi}{36}$

4. The length of the arc of a circle of diameter 12 cm cut by a central angle of $40°$ is
   (a) $\frac{2\pi}{3}$
   (b) $\frac{8\pi}{3}$
   (c) $\frac{4\pi}{3}$
   (d) $\frac{27}{\pi}$
   (e) 240

5. Converting $36°$ into radians and $\frac{7\pi}{45}$ radians into degrees, we get respectively
   (a) $\frac{\pi}{5}, 28°$
   (b) $\frac{28\pi}{5}, 14°$
   (c) $\frac{5}{\pi}, 28°$
   (d) $\frac{28\pi}{5}, 56°$
   (e) $\frac{5}{28\pi}, 28°$

6. The degree measure of a central angle which cuts off an arc of length 10 cm in a circle of radius 6 cm is
   (a) $\left(\frac{5}{3\pi}\right)^\circ$
   (b) $\left(\frac{\pi}{300}\right)^\circ$
   (c) $\left(\frac{108}{\pi}\right)^\circ$
   (d) $\left(\frac{\pi}{108}\right)^\circ$
   (e) $\left(\frac{300}{\pi}\right)^\circ$

7. The length of the smaller arc of a circle of radius 1 cm whose angle is $90°$ is
   (a) $\frac{\pi}{2}$ cm
   (b) $\frac{\pi}{4}$ cm
   (c) π cm
   (d) $\frac{3\pi}{4}$ cm
   (e) $\frac{\pi}{9}$ cm

8. What is the smallest positive angle coterminal with $-743°$?
   (a) $257°$
   (b) $167°$
   (c) $77°$
   (d) $357°$
   (e) $337°$

9. Which one of the following is NOT coterminal with $50°$?
   (a) $410°$
   (b) $230°$
   (c) $-310°$
   (d) $770°$
   (e) $-670°$

10. The degree measure of a central angle that cuts off an arc length 7π cm on a circle of radius 3 cm, is
    (a) $3780°$
    (b) $210°$
    (c) $420°$
    (d) $120°$

(e) $480°$

11. Suppose that a point P is on the circle of radius 2 cm and center O, and the ray OP is rotating with angular velocity $\frac{\pi}{6}$ radians per second. The linear velocity of P is
    (a) 3π cm/second
    (b) $\frac{3}{\pi}$ cm/second
    (c) $\frac{12}{\pi}$ cm/second
    (d) $\frac{\pi}{12}$ cm/second
    (e) $\frac{\pi}{3}$ cm/second

6. TRIGONOMETRIC FUNCTIONS OF ANGLES AND FUNDAMENTAL PROPERTIES

1. If $\cot^2 x = 4$ and x terminates in quadrant IV, then
    (a) $\cos x = 2$
    (b) $\sec x = \frac{\sqrt{3}}{2}$
    (c) $\csc x = -\sqrt{5}$
    (d) $\tan x = \frac{1}{4}$
    (e) $\sin x = 1$

2. $\tan 945° - \sin\left(-\frac{79\pi}{6}\right)$ is equal to
    (a) $\frac{3}{2}$
    (b) $\frac{1}{2}$
    (c) $\frac{2-\sqrt{3}}{2}$
    (d) $\frac{\sqrt{3}}{2}$
    (e) $-\frac{\sqrt{3}}{2}$

3. If $\tan x = -\frac{4}{3}$, and $\sec x = -\frac{5}{3}$, then $\csc x =$
    (a) $\frac{5}{4}$
    (b) $-\frac{5}{4}$
    (c) $\frac{4}{3}$
    (d) $\frac{20}{9}$
    (e) $-\frac{20}{9}$

4. $\frac{\cos 60° + \sin 270° + \sec 240°}{(\tan 135°)(\sec 150°) - (\csc 90°)(\cot 30°)} =$
    (a) $\frac{5\sqrt{3}}{2}$
    (b) $\frac{\sqrt{3}}{5}$
    (c) $-\frac{5\sqrt{3}}{2}$
    (d) $\frac{2\sqrt{3}}{5}$
    (e) $-\frac{2\sqrt{3}}{5}$

5. $\frac{\tan 225° + \csc 150° - \tan 60°}{\sec 300° + \cot 210°} =$
    (a) 1
    (b) $\frac{3+\sqrt{3}}{2-\sqrt{3}}$
    (c) $\frac{3-\sqrt{3}}{2+\sqrt{3}}$
    (d) $\frac{3+\sqrt{3}}{2+\sqrt{3}}$
    (e) $\frac{3-\sqrt{3}}{2-\sqrt{3}}$

6. $\cot \frac{41\pi}{3} =$
    (a) $-\sqrt{3}$
    (b) $-\frac{\sqrt{3}}{3}$
    (c) $\sqrt{3}$
    (d) $\frac{\sqrt{3}}{3}$
    (e) $-\sqrt{2}$

7. If the point $(-3, 4)$ lies on the terminal side of an angle θ in standard position, then $\sin(-\theta) + \sec\theta$ is equal to
    (a) $\frac{37}{15}$
    (b) $-\frac{13}{15}$
    (c) $\frac{13}{15}$
    (d) $-\frac{37}{20}$
    (e) $-\frac{37}{15}$

8. If $\csc\theta = -2$, and θ is in quadrant III, then $\cot(-\theta) + \cos(-\theta)$ is equal to
    (a) $\frac{3\sqrt{3}}{2}$
    (b) $-\frac{3\sqrt{3}}{2}$
    (c) $\frac{\sqrt{3}}{2}$
    (d) $-\frac{\sqrt{3}}{2}$
    (e) $-\frac{\sqrt{3}}{6}$

9. If $\tan\alpha = m$, α terminates in quadrant II, then $\csc\alpha =$
    (a) $\frac{\sqrt{m^2+1}}{m}$
    (b) $-\sqrt{m^2+1}$
    (c) $\frac{m}{\sqrt{m^2+1}}$
    (d) $-\frac{\sqrt{m^2+1}}{m}$
    (e) $-\frac{m}{\sqrt{m^2-1}}$

10. $(\sin 510°)(\csc 330°) + [\cos(-330°)](\sec 210°) =$
    (a) 0
    (b) -1
    (c) -2
    (d) 1
    (e) 2

11. $\cot 225° + \sec 150° - \tan 60° =$
    (a) $-1 - \sqrt{3}$

(b) $\frac{3-\sqrt{3}}{3}$
(c) $1+\sqrt{3}$
(d) $-1+\sqrt{3}$
(e) $\frac{3-5\sqrt{3}}{3}$

12. If $\cos t = \frac{2}{3}$ and t terminates in quadrant IV, then $\tan t =$
(a) $\frac{\sqrt{5}}{2}$
(b) $\frac{3}{2}$
(c) $-\frac{\sqrt{5}}{3}$
(d) $-\frac{\sqrt{5}}{2}$
(e) $\frac{\sqrt{5}}{3}$

13. If $\sec\theta = -\frac{3}{2}$, $\tan\theta = \frac{\sqrt{5}}{2}$, then $\csc\theta =$
(a) $\frac{3\sqrt{5}}{5}$
(b) $-\frac{3\sqrt{5}}{5}$
(c) $-\frac{2}{3}$
(d) $-\frac{\sqrt{5}}{3}$
(e) $\frac{\sqrt{5}}{3}$

14. Which one of the following is undefined?
(a) $\cos(\ln 1)$
(b) $\ln(\cos\pi)$
(c) $\sin(\ln e)$
(d) $\log(\ln e^2)$
(e) $\log(\cos 0)$

15. $\cos\left(-17\frac{\pi}{3}\right) =$
(a) $\frac{\sqrt{3}}{2}$
(b) $-\frac{1}{2}$
(c) $-\frac{\sqrt{3}}{2}$
(d) $\frac{\sqrt{2}}{2}$
(e) $\frac{1}{2}$

16. If $\csc(-x) = 3$, and x terminates in quadrant IV, then $\sec x$ is equal to
(a) $-\frac{3}{2}$
(b) $\frac{1}{2\sqrt{2}}$
(c) $\frac{3}{2\sqrt{2}}$
(d) $2\sqrt{2}$
(e) $-\frac{3}{2\sqrt{2}}$

17. If $\left(-1,-\frac{4}{3}\right)$ is on the terminal side of angle θ in standard position, then $\sec\theta - \csc\theta$ is equal to
(a) $-\frac{1}{4}$
(b) $\frac{1}{5}$
(c) 0
(d) $-\frac{5}{12}$
(e) $\frac{1}{7}$

18. Suppose that $90^\circ < \theta < 180^\circ$. The signs of $\sin 2\theta$, $\tan\frac{\theta}{2}$, $\cos\theta$ are
(a) negative, positive, positive
(b) positive, positive, positive
(c) positive, negative, positive
(d) negative, positive, negative
(e) negative, negative, negative

19. Given $\cos x = -\frac{3}{5}$. The largest possible value of $\frac{\sec x - \tan x}{\sin x}$ is equal to
(a) $\frac{5}{12}$
(b) $\frac{1}{2}$
(c) $\frac{3}{4}$
(d) $\frac{15}{4}$
(e) $\frac{2}{11}$

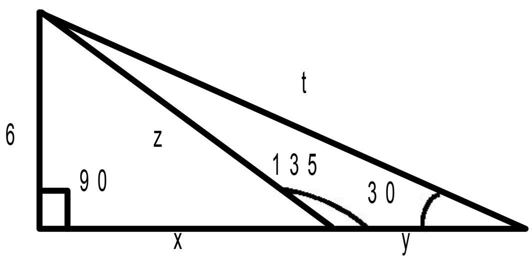
20. Which one of the following is TRUE ?
(a) $\tan 3 > \sin 3$
(b) $\sin 3 = \tan 3$
(c) $\sin 3 > \tan 3$
(d) $\sin 3 < 0$
(e) $\tan 3 > 0$

21. Which one of the following is NOT possible?
(a) $\tan x = 10$
(b) $\cot x = -3$
(c) $\sin x = \frac{\pi}{4}$
(d) $\cos x = \frac{\sqrt{2}}{100}$
(e) $\csc x = \frac{1}{2}$

22. Which one of the following is undefined?
(a) $\cos 40^\circ$
(b) $\sec 60^\circ$
(c) $\tan 360^\circ$
(d) $\cot 180^\circ$
(e) $\sin 180^\circ$

23. If $\csc\theta > 0$, $\cot\theta < 0$, then θ terminates in quadrant
(a) I
(b) II
(c) III
(d) IV
(e) I or II

24. In the figure, the expression $3x + y - z - t =$



(a) $6\left(\sqrt{3}+\sqrt{2}\right)$
(b) $6\left(\sqrt{3}-\sqrt{2}\right)+30$
(c) $6+\sqrt{3}-\sqrt{2}$
(d) $6\left(\sqrt{3}-\sqrt{2}\right)$
(e) $6+\sqrt{2}-\sqrt{3}$

7. TRIGONOMETRIC FUNCTIONS OF REAL NUMBERS

1. If $\left(-\frac{2}{3},\frac{\sqrt{5}}{3}\right)$ is the point on the unit circle corresponding to the arc length S, then the point on the unit circle corresponding to the arc length $\pi-S$ is
(a) $\left(\frac{\sqrt{5}}{3},-\frac{2}{3}\right)$
(b) $\left(-\frac{\sqrt{5}}{3},-\frac{2}{3}\right)$
(c) $\left(-\frac{2}{3},-\frac{\sqrt{5}}{3}\right)$
(d) $\left(\frac{2}{3},-\frac{\sqrt{5}}{3}\right)$
(e) $\left(\frac{2}{3},\frac{\sqrt{5}}{3}\right)$

2. If P and Q are two points on the unit circle corresponding to the arc lengths $\frac{23\pi}{4}$ and $\frac{274\pi}{40}$, respectively, then the shortest distance along the unit circle from P to Q is
(a) $\frac{29\pi}{20}$
(b) $\frac{11\pi}{20}$
(c) $\frac{3\pi}{4}$
(d) $\frac{11\pi}{10}$
(e) $\frac{9\pi}{10}$

3. If the point $\left(\frac{\sqrt{3}}{6},y\right)$, where $y<0$ corresponds to an arc length S on the unit circle, then the point on the unit circle corresponds to the arc length $\pi-S$ is
(a) $\left(\frac{\sqrt{3}}{6},-\frac{\sqrt{33}}{6}\right)$
(b) $\left(\frac{\sqrt{3}}{6},\frac{\sqrt{33}}{6}\right)$
(c) $\left(y,-\frac{\sqrt{3}}{6}\right)$
(d) $\left(y,\frac{\sqrt{33}}{6}\right)$
(e) $\left(-\frac{\sqrt{3}}{6},-\frac{\sqrt{33}}{6}\right)$

4. The coordinates of the point corresponding to an arc length of $-\frac{5\pi}{6}$ on the unit circle are
(a) $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
(b) $\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
(c) $\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
(d) $\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
(e) $\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$

5. The point corresponding to an arc length of $-\frac{3\pi}{2}$ on the unit circle is
(a) $(-1,0)$
(b) $\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
(c) $(0,-1)$
(d) $\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
(e) $(0,1)$

6. The terminal side of angle $\theta=480^\circ$ intersects the unit circle at
(a) $\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
(b) $\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
(c) $\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
(d) $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
(e) $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

7. The line segment from the origin to the point $(-7,24)$ intersects the unit circle at
(a) $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
(b) $\left(-\frac{3}{5},\frac{4}{5}\right)$
(c) $\left(-\frac{7}{25},\frac{24}{25}\right)$
(d) $\left(-\frac{24}{25},-\frac{7}{25}\right)$
(e) $\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$

8. On the unit circle, if the arc length $\frac{100\pi}{3}$ terminates at (a,b) and the arc length $\frac{55\pi}{3}$ terminates at (c,d), then $ac+bd$ is equal to
(a) 0
(b) $\frac{1}{2}$
(c) -1
(d) $-\frac{1}{2}$

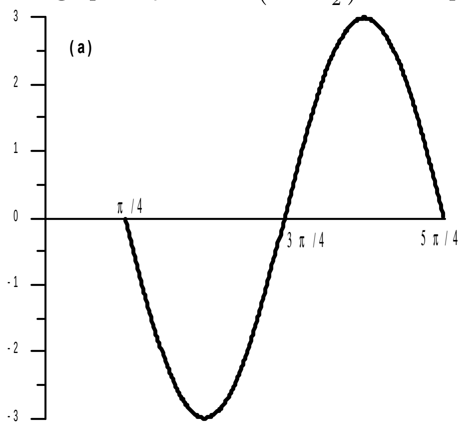
(e) $-\frac{\sqrt{3}}{2}$

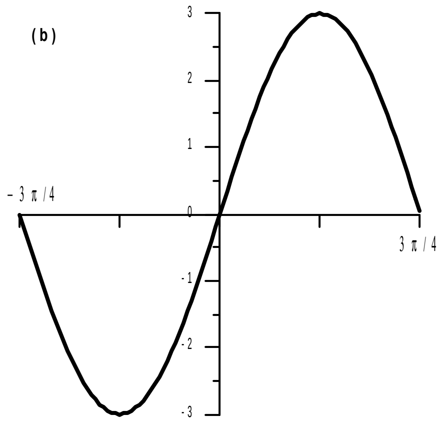
9. On the unit circle, if the point $\left(\frac{3}{5}, y\right)$ corresponds to arc length t where $y < 0$, then $\cos(3\pi - t) + \sin(t - 3\pi)$ is equal to

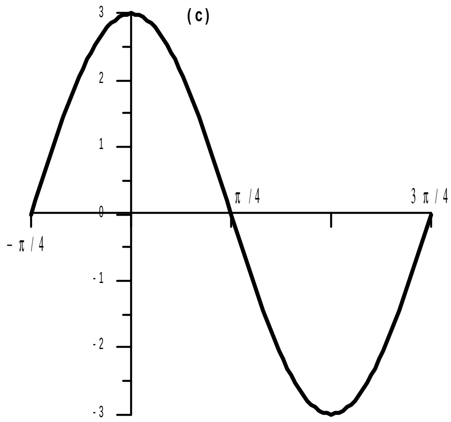
(a) $-\frac{7}{5}$
(b) $-\frac{1}{5}$
(c) $\frac{1}{5}$
(d) $\frac{7}{5}$
(e) -1

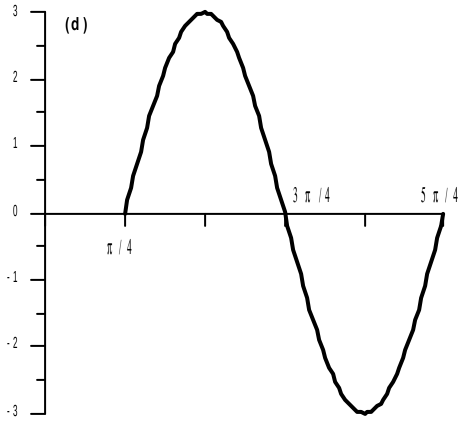
8. GRAPHS OF TRIGONOMETRIC FUNCTIONS

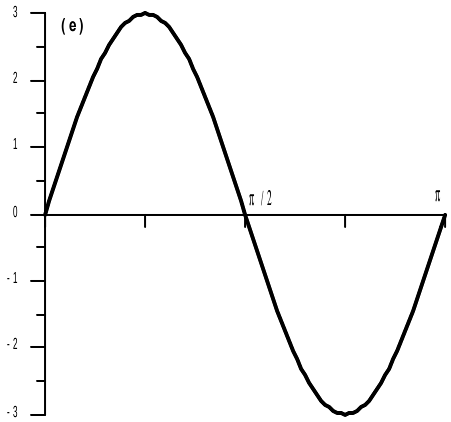
1. The graph of $y = 3\sin\left(2x - \frac{\pi}{2}\right)$ for one period is



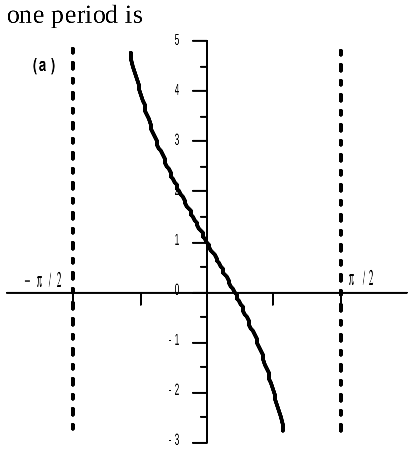


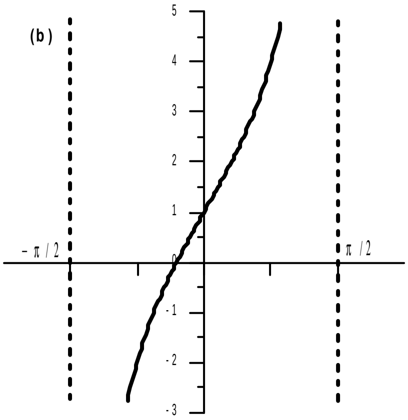


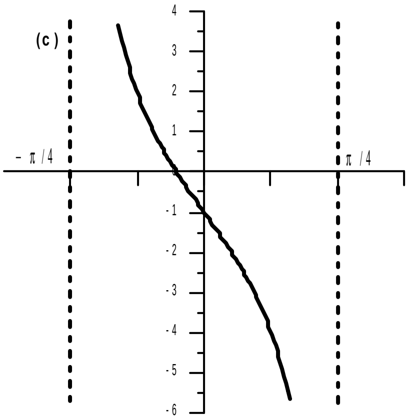


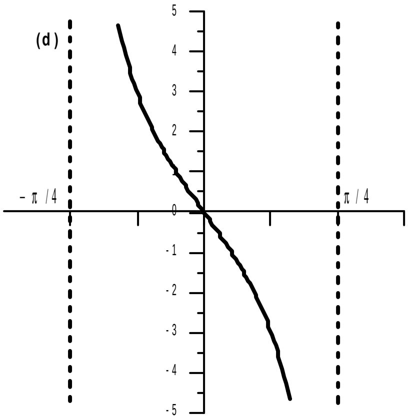


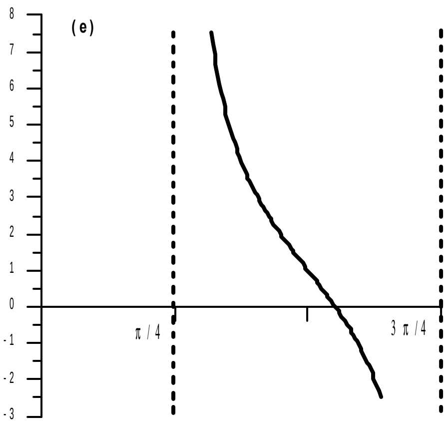
2. The graph of the function $y = 1 - 3\tan(2x - \pi)$ for one period is



(b)


(c)


(d)


(e)


3. The function $y = -\frac{3}{4}\cos\left(\frac{\pi}{4} - 2x\right)$ has Amplitude A, Period P, and Phase Shift S:

   (a) $A = -\frac{3}{4}, P = \pi, S = \frac{\pi}{8}$ to the right
   (b) $A = \frac{3}{4}, P = \frac{\pi}{4}, S = \frac{\pi}{8}$ to the right
   (c) $A = \frac{3}{4}, P = \pi, S = \frac{\pi}{8}$ to the left
   (d) $A = \frac{3}{4}, P = \pi, S = \frac{\pi}{8}$ to the right
   (e) $A = \frac{3}{4}, P = \pi, S = \frac{\pi}{4}$ to the right

4. The function $y = \sin^2(2\pi x)$ has

   (a) a period of $\frac{1}{2}$
   (b) a period of 1
   (c) a period of $\sqrt{2\pi}$
   (d) a period of π^2
   (e) no period

5. The range of the function $y = 3\sec x - 1$ is

   (a) $(-\infty, \infty) \setminus \{n\pi | n$ is an even integer$\}$
   (b) $(-\infty, \infty) \setminus \left\{\frac{n\pi}{2} | n\right.$ is an odd integer$\left.\right\}$
   (c) $[-4, 2]$
   (d) $(-\infty, -4) \cup (2, \infty)$
   (e) $(-\infty, -4] \cup [2, \infty)$

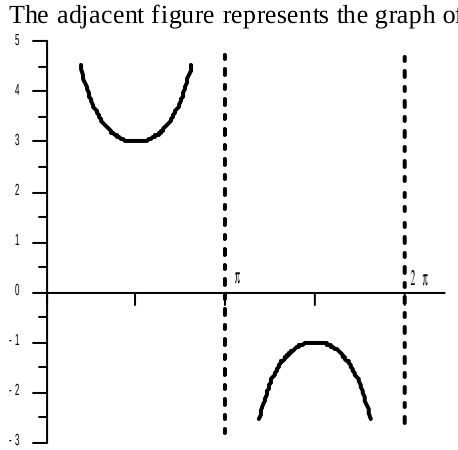
6. The function $y = x + \sin x$ is

   (a) not periodic
   (b) has period 2π
   (c) has period $1 + \pi$
   (d) has period $1 - \pi$
   (e) has period π

7. If n is any integer, the vertical asymptotes of $y = 3\cot\left(x - \frac{\pi}{2}\right)$ are at

   (a) $x = (2n + 1)\frac{\pi}{2}$
   (b) $x = (2n + 1)\pi$
   (c) $x = n + \frac{\pi}{2}$
   (d) $n\pi$
   (e) $2n\pi$

8. The adjacent figure represents the graph of



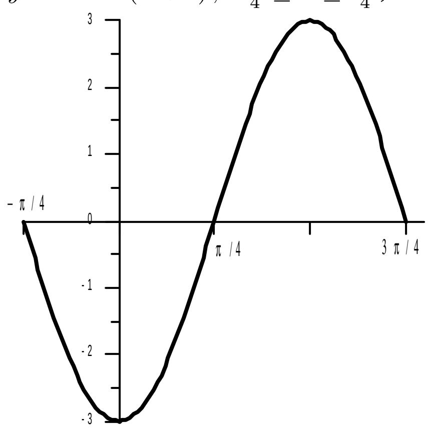
   (a) $y = 3\csc x$
   (b) $y = -\sec x$

(c) $y = 2\csc x + 1$
(d) $y = 2\sec x + 1$
(e) $y = \csc x + 2$

9. The function $y = -6\sin(\pi - 4x)$ has
   (a) Amplitude $= 6$, Period $= \frac{\pi}{2}$, Phase Shift $= \frac{\pi}{2}$ to the right.
   (b) Amplitude $= 6$, Period $= \frac{\pi}{4}$, Phase Shift $= \frac{\pi}{4}$ to the right.
   (c) Amplitude $= 6$, Period $= \frac{\pi}{2}$, Phase Shift $= \frac{\pi}{4}$ to the right.
   (d) Amplitude $= 6$, Period $= \frac{\pi}{2}$, Phase Shift $= \frac{\pi}{4}$ to the left.
   (e) Amplitude $= -6$, Period $= -\frac{\pi}{2}$, Phase Shift $= \frac{\pi}{4}$ to the left.

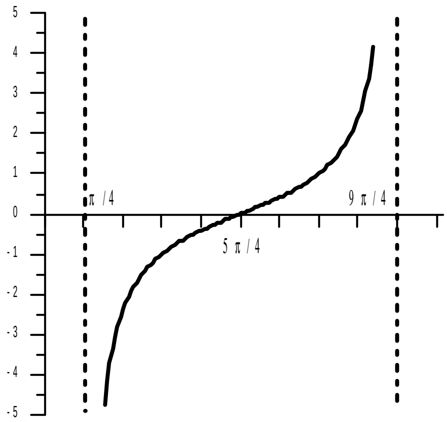
10. The function $f(x) = 1 + \sec\frac{\pi}{3}$ has
   (a) Period $= 3\pi$, Range $= (-\infty, 0) \cup (2, \infty)$
   (b) Period $= \frac{2\pi}{3}$, Range $= (-\infty, -1] \cup [1, \infty)$
   (c) Period $= 6\pi$, Range $= [0, 2]$
   (d) Period $= 6\pi$, Range $= (-\infty, 0] \cup [2, \infty)$
   (e) Period $= \frac{2\pi}{3}$, Range $= [0, 2]$

11. If the adjacent figure represents the graph of $y = a\sin b(x + c)\,, -\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, then

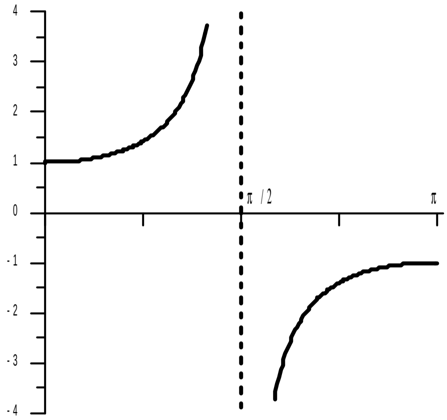


   (a) $a = -3, b = 1, c = -\frac{\pi}{4}$
   (b) $a = -3, b = 2, c = -\frac{\pi}{4}$
   (c) $a = 3, b = 2, c = \frac{\pi}{4}$
   (d) $a = -3, b = 2, c = \frac{\pi}{2}$
   (e) $a = -3, b = 2, c = \frac{\pi}{4}$

12. The adjacent figure represents the graph of
   (a) $y = -\cot\left(\frac{1}{2}x - \frac{\pi}{8}\right)$
   (b) $y = \cot\left(\frac{1}{2}x - \frac{\pi}{8}\right)$
   (c) $y = -\cot\left(x - \frac{\pi}{4}\right)$
   (d) $y = -\cot\left(x + \frac{\pi}{4}\right)$
   (e) $y = \cot\left(x + \frac{\pi}{4}\right)$



13. The adjacent figure represents



   (a) $\csc x$
   (b) $\sec x$
   (c) $\tan x$
   (d) $\cot x$
   (e) none of these

14. The function $f(x) = \frac{1}{2} - \cos\left(2x - \frac{\pi}{2}\right)$ has
   (a) Period $= \pi$, Phase Shift $= \pi$ to the right
   (b) Period $= \pi$, Phase Shift $= \frac{\pi}{4}$ to the right
   (c) Period $= 2\pi$, Phase Shift $= \frac{\pi}{4}$ to the left
   (d) Period $= \pi$, Amplitude $= \pi$
   (e) Amplitude $= \frac{1}{2}$, Phase Shift $= \frac{\pi}{4}$ to the left

15. The range of the function $y = 2\csc x - 1$ is
   (a) $(-\infty, -1] \cup [3, \infty)$
   (b) $[-3, 1]$
   (c) $(-\infty, -3) \cup (1, \infty)$
   (d) $(-\infty, -1) \cup (3, \infty)$
   (e) $(-\infty, -3] \cup [1, \infty)$

16. The period of $\tan^2 x$ is
   (a) $\frac{1}{2}$
   (b) 1
   (c) $\frac{\pi}{2}$

(d) π
(e) undefined

17. The domain of $y = \tan x$ is
(a) $(-\infty, \infty) \setminus [\pi k]$
(b) $(-\infty, \infty)$
(c) $[-2\pi, 2\pi]$
(d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(e) $(-\infty, \infty) \setminus \left[\frac{\pi}{2} + \pi k\right]$

18. The asymptotes of $f(x) = 3 - 2\sec\left(2x - \frac{\pi}{4}\right)$ are given by $x = \frac{A+Bn}{C}\pi$, where n is an integer, then
(a) $A = 1, B = 4, C = 8$
(b) $A = 1, B = 2, C = 2$
(c) $A = 3, B = 4, C = 8$
(d) $A = 1, B = 2, C = 4$
(e) $A = 9, B = 2, C = 8$

19. Which one of the following has Period 2π and Phase Shift $\frac{\pi}{6}$ to the right?
(a) $y = \sec\left(x + \frac{\pi}{6}\right)$
(b) $y = \sec(6x - \pi)$
(c) $y = \sec(x - 3\pi)$
(d) $y = \frac{1}{2}\tan\frac{1}{2}(x + 3\pi)$
(e) $y = \csc\left(x - \frac{\pi}{6}\right)$

20. The x-intercepts of the graph of $y = x\cos x$ are
(a) $x = 0$ only
(b) $x = 0, \frac{\pi}{2} + n\pi$
(c) $x = \frac{\pi}{2} + n\pi$
(d) $x = n\pi$
(e) $x = \frac{n\pi}{2}$

21. How many zeros does the function $f(x) = \sec\frac{\pi}{2}x - x$ have in the interval $[0, 4]$? (Hint: Use graphs).
(a) 1
(b) 2
(c) 3
(d) 4
(e) 0

22. The function $y = 2 + \sin\left(x - \frac{\pi}{3}\right)$ is
(a) increasing on $[0, \pi]$
(b) decreasing on $[0, \pi]$
(c) decreasing on $\left[0, \frac{\pi}{3}\right]$
(d) increasing on $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$
(e) decreasing on $\left[\frac{\pi}{3}, \frac{5\pi}{6}\right]$

23. The range of the $f(x) = -\frac{5}{4} + \frac{3}{2}\csc\left(2x - \frac{\pi}{6}\right)$ is
(a) $\left(-\infty, -\frac{11}{4}\right] \cup \left[\frac{1}{4}, \infty\right)$
(b) $\left(-\infty, -\frac{1}{4}\right] \cup \left[\frac{11}{4}, \infty\right)$
(c) $\left[-\frac{11}{4}, \frac{1}{4}\right]$
(d) $(-\infty, -1] \cup [1, \infty)$
(e) $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{1}{4}, \infty\right)$

24. The number of vertical asymptotes of $y = 2\cot\left(3x - \frac{\pi}{2}\right)$ in the interval $\left(-\frac{\pi}{4}, \pi\right)$ is
(a) 4
(b) 3
(c) 2
(d) 1
(e) 5

25. The set of all x-intercepts of $y = \cot 2x, 0 \le x < 2\pi$ is
(a) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$
(b) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$
(c) $\{0, \pi\}$
(d) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$
(e) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

26. If $f(x) = a\cos\left(bx - \frac{\pi}{3}\right), (a > 0)$ has Amplitude $= 2$ and Period $= \frac{\pi}{2}$, then $f\left(\frac{\pi}{2}\right)$ is equal to
(a) $-\sqrt{2}$
(b) -1
(c) $-\sqrt{3}$
(d) 1
(e) $\sqrt{3}$

27. A cosine function of Amplitude $= 3$ and Period $= 2$ is
(a) $y = 3\cos x$
(b) $y = -3\cos\pi x$
(c) $y = 2\cos\frac{2\pi}{3}x$
(d) $y = -2\cos 3x$
(e) $y = 3\cos 2\pi x$

28. Let n be any integer, the vertical asymptotes of $y = \cot\left(2x + \frac{\pi}{3}\right)$ are
(a) $x = n\pi$
(b) $x = (2n + 1)\frac{\pi}{2}$
(c) $x = n\frac{\pi}{2} - \frac{\pi}{6}$
(d) $x = n\pi + \frac{\pi}{6}$
(e) $x = \frac{n\pi}{3}$

29. The graph of $y = \sin\left(2x - \frac{\pi}{2}\right), 0 \le x \le \frac{5\pi}{4}$, is decreasing on
(a) $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

(b) $(0,\pi)$
(c) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)\cup\left(\pi,\frac{5\pi}{4}\right)$
(d) $\left(\frac{\pi}{2},\pi\right)$
(e) $\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{5\pi}{4}\right)$

30. Let $f(x) = a\cos bx$ with Period $= 10$. Suppose $f(5) = 2$, then $f(25) =$
(a) 4
(b) 2
(c) 22
(d) 10
(e) 0

9. VERIFYING IDENTITIES

1. If angle θ terminates in quadrant IV, then $\cos\theta$ expressed in terms of $\tan\theta$ is equal to
(a) $-\frac{\sqrt{\tan^2\theta+1}}{\tan^2\theta+1}$
(b) $\frac{\sqrt{\tan^2\theta+1}}{\tan^2\theta+1}$
(c) $\frac{1}{\sqrt{1-\tan^2\theta}}$
(d) $-\frac{1}{\sqrt{\tan^2\theta-1}}$
(e) $\frac{1}{\sqrt{\tan^2\theta-1}}$

2. When simplified, the expression $\frac{\csc x+\sec x}{\tan x+\cot x}$ is equal to
(a) $\cos x+\sin x$
(b) $\frac{1}{\sin x-\cos x}$
(c) $\sin x-\cos x$
(d) $\frac{1}{\sin x+\cos x}$
(e) $\cos x-\sin x$

3. $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\frac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}=$
(a) 0
(b) $\sin 2\theta$
(c) $-\cos 2\theta$
(d) 1
(e) 2

4. If $\sin x\cos x=\frac{3}{4}$, then $\frac{\tan^2 x-\sec^2 x}{\sec^2 x+\csc^2 x}=$
(a) $\frac{3}{4}$
(b) $-\frac{4}{3}$
(c) $\frac{4}{3}$
(d) $-\frac{9}{16}$
(e) $\frac{9}{16}$

5. $\frac{\tan x}{\sec x-\tan x}-\frac{\sec x}{\sec x+\tan x}=$
(a) -1
(b) 1
(c) $2\sec x\tan x-1$
(d) $2\sec x\tan x+1$
(e) $1-2\sec x\tan x$

6. $\frac{\cot x}{1+\sin x}+\frac{\cot x}{1-\sin x}=$
(a) $2\sec x\csc x$
(b) $2\cot x$
(c) $2\csc x$
(d) $2\tan x\sec^2 x$
(e) $2\cot x\cos^2 x$

7. Which one of the following is TRUE for all values of x for which the expressions are defined?
(a) $\left(\cot^2 x\right)\left(\sin^2 x-\tan^2 x\right)=-\sin^2 x$
(b) $\cot^2 x\left(1+\tan^2 x\right)=\sin^2 x$
(c) $\sin x=\sqrt{1-\cos^2 x}$
(d) $\cot^2 x-\csc^2 x=1$
(e) $2\sin x=\sin 2x$

8. $\frac{\tan^2 x-\sin^2 x}{\csc^2 x+\sec^2 x}=$
(a) $-\sin^6 x$
(b) $\sin^6 x$
(c) $-\cos^6 x$
(d) $\cos^6 x$
(e) 1

9. $\frac{-\sin x\csc x+\cos^2 x}{\cot^2 x+1}=$
(a) $\cos^4 x$
(b) $-\sin^4 x$
(c) $\sin^4 x$
(d) $-\cos^4 x$
(e) -1

10. $\frac{1}{\cos\theta}-\frac{\cos\theta}{1+\sin\theta}=$
(a) $\tan\theta$
(b) $\csc\theta$
(c) $\cot\theta$
(d) $\sin\theta$
(e) $\sec\theta$

11. Which one of the following is an identity?
(a) $\sqrt{\sin^2 x}=\sin x$
(b) $\sqrt{1-\cos^2 x}=\sin x$
(c) $1-\cot^2 x=\csc^2 x$
(d) $\frac{1+\sin x}{\cos x}=\frac{\cos x}{1-\sin x}$
(e) $\sec x=\sqrt{1+\tan^2 x}$

12. $\frac{\tan^2 x+\csc^2 x+1}{\tan^2 x}=$

(a) $\sec^4 x$
(b) $\sin^4 x$
(c) $\csc^4 x$
(d) $\cos^4 x$
(e) $\cot^4 x$

13. $\frac{\csc\theta+\cot\theta}{\tan\theta+\sin\theta} =$
(a) $\cot\theta + \csc\theta$
(b) $\cot\theta$
(c) $\frac{\cos\theta}{2(1-\cos\theta)}$
(d) $\cot\theta\csc\theta$
(e) $\frac{1}{2}(\cot\theta + \csc\theta)$

14. $\sec^4 x - 2\sec^2 x\tan^2 x + \tan^4 x =$
(a) -1
(b) $(\sec x - \tan x)^4$
(c) $(\sec x + \tan x)^2$
(d) $\sec^2 x + \tan^2 x$
(e) 1

15. $\frac{\sin x}{1+\cos x} =$
(a) $\frac{1-\cos x}{\sin x}$
(b) $\frac{1+\cos x}{\sin x}$
(c) $\frac{\cos x}{1+\sin x}$
(d) $\frac{\tan x}{1+\cot x}$
(e) $\frac{\cot x}{1+\tan x}$

16. $\csc^4\theta - \cot^4\theta =$
(a) $-1 + 2\cot^2\theta$
(b) $1 - 2\csc^2\theta$
(c) 1
(d) $\frac{1+\cos\theta}{1-\cos\theta}$
(e) $1 + 2\cot^2\theta$

17. $(1-\csc^2 x)(\sec^2 x - 1) =$
(a) $\tan^4 x$
(b) 1
(c) -1
(d) $-\cot^4 x$
(e) $\sin^2 x\cos^2 x$

18. $\frac{\tan^2 x}{1-\cos x} - \frac{\tan^2 x}{1+\cos x} =$
(a) $2\tan^2 x$
(b) $2\csc x$
(c) $2\sec x$
(d) $2\sec x\tan x$
(e) $2\csc x\cot x$

19. Replacing x with $3\sin\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$, then $\sqrt{9-x^2} =$
(a) $3\sin\theta$
(b) $3\cos\theta$
(c) $3\sin^2\theta$
(d) $3+\cos\theta$
(e) $3+\sin\theta$

10. TRIGONOMETRIC FUNCTIONS OF THE SUM OR DIFFERENCE OF TWO ANGLES

1. If $\cos\theta = \frac{3}{5}$ and θ is in quadrant IV, then $\tan(-\theta)$ is equal to
(a) $\frac{4}{3}$
(b) $\frac{3}{4}$
(c) $-\frac{3}{4}$
(d) $\frac{4}{5}$
(e) $-\frac{4}{3}$

2. $\cos\left(\frac{19\pi}{12}\right) =$
(a) $\frac{\sqrt{2}+\sqrt{6}}{4}$
(b) $\frac{\sqrt{2}-\sqrt{6}}{4}$
(c) $\frac{\sqrt{3}-1}{2}$
(d) $-\frac{\sqrt{2}+\sqrt{6}}{4}$
(e) $\frac{\sqrt{6}-\sqrt{2}}{4}$

3. Which one of the following is TRUE for all real values of x and y for which the expressions are defined?
(a) $\sec(\pi - x) = -\sec x$
(b) $\cos(x+y) + \cos(x-y) = -2\cos x\cos y$
(c) $\cos\left(\frac{\pi}{2} + x - y\right) = \sin(x-y)$
(d) $\csc\left(\frac{\pi}{2} - x\right) = -\sec x$
(e) $\cos(\pi + x - y) = \sin x\sin y - \cos x\cos y$

4. Which one of the following is TRUE for all real values of x for which the expressions are defined?
(a) $\sqrt{\cos^2 x} = \cos x$
(b) $\cos(-x+\pi) = -\cos x$
(c) $\sin(-x+2\pi) = \sin x$
(d) $\csc x = \sqrt{1+\cot^2 x}$
(e) $\tan^2 x + \sec^2 x + 1 = 0$

5. $\sin 255° =$
(a) $\frac{\sqrt{2}+\sqrt{6}}{4}$
(b) $\frac{\sqrt{2}-\sqrt{6}}{4}$

(c) $-\frac{\sqrt{2}+\sqrt{6}}{4}$
(d) $-\frac{\sqrt{2}+\sqrt{3}}{4}$
(e) $\frac{\sqrt{6}-\sqrt{2}}{4}$

6. $\cos 255^\circ =$
(a) $\frac{\sqrt{6}-\sqrt{2}}{4}$
(b) $\frac{-\sqrt{2}-\sqrt{6}}{4}$
(c) $\frac{\sqrt{2}-\sqrt{6}}{4}$
(d) $\frac{1-\sqrt{3}}{2}$
(e) $\frac{\sqrt{2}+\sqrt{6}}{4}$

7. If $\sin S = -\frac{4}{5}$, and $\cos t = -\frac{12}{13}$ such that S and t are in quadrant III, then $\tan(S-t) =$
(a) $\frac{63}{16}$
(b) $-\frac{63}{16}$
(c) $\frac{33}{56}$
(d) $-\frac{33}{56}$
(e) $\frac{11}{12}$

8. If $\cos S = -\frac{3}{5}$, and $\sin t = \frac{5}{13}$ such that S and t are in quadrant II, then $\cos(S-t) =$
(a) $\frac{33}{65}$
(b) $-\frac{16}{65}$
(c) $\frac{16}{65}$
(d) $-\frac{56}{65}$
(e) $\frac{56}{65}$

9. If $\cos\theta = 0.8$ and θ is in quadrant IV, then $\cos\left(\theta + \frac{\pi}{6}\right) =$
(a) $0.8 + \frac{\sqrt{3}}{2}$
(b) $\frac{4\sqrt{3}+1}{10}$
(c) $\frac{4\sqrt{3}-3}{10}$
(d) $\frac{4\sqrt{3}+3}{10}$
(e) $0.8 - \frac{\sqrt{3}}{2}$

10. Using the reduction identity, the expression $\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x =$
(a) $\cos\left(x - \frac{\pi}{6}\right)$
(b) $\sin\left(x + \frac{\pi}{3}\right)$
(c) $\sin\left(x - \frac{\pi}{3}\right)$
(d) $\sin\left(x + \frac{\pi}{6}\right)$
(e) $\sin\left(x - \frac{\pi}{6}\right)$

11. When $y = \cos x - \sqrt{3}\sin x$ is expressed in the form $y = a\sin(x+\alpha)$ where $0 \le \alpha \le 2\pi$, then
(a) $a = -2, \alpha = \frac{5\pi}{6}$
(b) $a = -2, \alpha = \frac{4\pi}{3}$
(c) $a = 2, \alpha = \frac{5\pi}{6}$
(d) $a = 2, \alpha = \frac{7\pi}{6}$
(e) $a = 2, \alpha = \frac{4\pi}{3}$

12. For all positive integers n, the value of $\cos(n+2)\pi - \sin(2n+3)\frac{\pi}{2}$ is
(a) 2
(b) 0
(c) $2(-1)^n$
(d) $-2(-1)^n$
(e) -2

13. The value of the expression $\sin 27^\circ \cos 57^\circ - \sin 63^\circ \cos 33^\circ =$
(a) $-\frac{\sqrt{3}}{2}$
(b) 2
(c) $\frac{1}{2}$
(d) $-\frac{1}{2}$
(e) $\frac{\sqrt{3}}{2}$

14. The Amplitude and the Period of the function $y = 3\sin\frac{x}{2} + 4\cos\frac{x}{2}$ are
(a) $7, \pi$
(b) $7, 4\pi$
(c) $5, \pi$
(d) $7, 2\pi$
(e) $5, 4\pi$

15. If $\alpha = -\frac{2\pi}{5}$, $\beta = 288^\circ$, and $\theta = 72^\circ$, then which one of the following is FALSE?
(a) $\tan\alpha = -\tan\beta$
(b) $\sin\alpha = -\sin\theta$
(c) $\sin^2\alpha + \cos^2\theta = 1$
(d) $\sec\alpha = \sec\theta$
(e) α and β are coterminal

16. If the terminal side of angle θ passes through $\left(-1, \sqrt{8}\right)$, then $\sin\left(\frac{5\pi}{6} - \theta\right) =$
(a) $\frac{\sqrt{3}+2\sqrt{2}}{6}$
(b) $\frac{3-4\sqrt{2}}{6}$
(c) $\frac{2\sqrt{2}-\sqrt{3}}{6}$
(d) $\frac{2\sqrt{6}-1}{6}$
(e) $\frac{2\sqrt{6}-\sqrt{3}}{6}$

17. The value of $\cos 20^\circ - \sin 70^\circ + \csc\frac{19\pi}{6} + \tan\left(-\frac{9\pi}{4}\right)$ is
(a) $-\frac{3}{2}$
(b) $-\frac{\sqrt{3}}{2} - 1$
(c) $\frac{\sqrt{3}}{2} + 1$
(d) $-2 - \sqrt{3}$

(e) -3

18. $\cos\left(\frac{\pi}{2}-x\right)=$

(a) $\cos\left(\frac{\pi}{2}+x\right)$
(b) $\sin\left(\frac{\pi}{2}+x\right)$
(c) $\cos(\pi-x)$
(d) $\sin\left(\frac{\pi}{2}-x\right)$
(e) $\sin x$

19. Which one of the following is TRUE for $f(x)=\sin x+\cos x$?

(a) $f(x)$ is even
(b) $f(x)$ is odd
(c) The maximum value of $f(x)$ is $\sqrt{2}$
(d) The Amplitude of $f(x)$ is 2
(e) The Period of $f(x)$ is 4π

20. $\cos 12^\circ\cos 42^\circ+\cos 78^\circ\cos 48^\circ=$

(a) $-\frac{\sqrt{3}}{2}$
(b) $-\frac{1}{2}$
(c) $\frac{1}{2}$
(d) 0
(e) $\frac{\sqrt{3}}{2}$

21. $\sin 195^\circ=$

(a) $\frac{\sqrt{2}-\sqrt{3}}{2}$
(b) $\frac{\sqrt{2}+\sqrt{6}}{4}$
(c) $\frac{\sqrt{2}-\sqrt{3}}{4}$
(d) $\frac{\sqrt{2}-\sqrt{6}}{4}$
(e) $\frac{\sqrt{6}-\sqrt{2}}{4}$

22. The minimum value of $f(x)=5\sin x+12\cos x$ is

(a) 5
(b) 17
(c) -13
(d) -7
(e) -17

23. When using the reduction identity, the Phase Shift of $y=-3\sin 2x+3\cos 2x$ is

(a) $\frac{7\pi}{8}$ to the right
(b) $\frac{3\pi}{8}$ to the right
(c) $\frac{7\pi}{8}$ to the left
(d) $\frac{3\pi}{4}$ to the left
(e) $\frac{3\pi}{8}$ to the left

24. $\frac{\tan 75^\circ-\cot 75^\circ}{1+\cot 15^\circ\cot 75^\circ}=$

(a) 1
(b) $\sqrt{3}$
(c) $\frac{\sqrt{3}}{3}$
(d) -1
(e) $\frac{\sqrt{3}}{2}$

25. $\sin\frac{29\pi}{7}\cos\frac{2\pi}{7}+\cos\frac{\pi}{7}\sin\frac{16\pi}{7}=$

(a) $-\sin\frac{\pi}{7}$
(b) $\sin\frac{9\pi}{7}$
(c) $\sin\frac{3\pi}{7}$
(d) $-\cos\frac{\pi}{7}$
(e) $-\sin\frac{3\pi}{7}$

26. If the terminal side of an angle θ lies on the line $y=3x, x<0$, then $\cos\left(\theta+\frac{\pi}{4}\right)=$

(a) $\frac{2\sqrt{5}}{5}$
(b) $\sqrt{5}$
(c) $\frac{\sqrt{10}}{10}$
(d) $\frac{4\sqrt{5}}{5}$
(e) $\frac{\sqrt{5}}{5}$

27. Which one of the following is FALSE ?

(a) $\cot 4>\cot 6$
(b) $\tan 6>\tan 4$
(c) The angle 701° is coterminal with the angle 341°
(d) $f(x)=\csc x\tan x$ is an even function
(e) $f(x)=|\sin 2x|$ has Period of $\frac{\pi}{2}$

28. If $\tan(x+y)=33$, and $\tan x=3$, then $\tan y=$

(a) $\frac{3}{10}$
(b) $-\frac{15}{49}$
(c) $-\frac{10}{3}$
(d) $\frac{49}{15}$
(e) $\frac{5}{17}$

29. If $\sin x=\frac{3}{5}$, and $\cos y=-\frac{5}{13}$ where x is in quadrant II and y is in quadrant III, then $\sin(x+y)=$

(a) $\frac{63}{65}$
(b) $-\frac{21}{65}$
(c) $-\frac{16}{65}$
(d) $-\frac{56}{65}$
(e) $\frac{33}{65}$

30. If $2\sin x-2\sqrt{3}\cos x=k\sin(x+t)$, then

(a) $k=-4, t=300^\circ$
(b) $k=4, t=300^\circ$
(c) $k=2, t=120^\circ$
(d) $k=-2, t=330^\circ$
(e) $k=4, t=240^\circ$

31. Which one of the following is equivalent to

$y = 3\sin x - 3\cos x$?

(a) $y = 3\sin x$
(b) $y = 3\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$
(c) $y = \sqrt{2}\sin 3x$
(d) $y = \sqrt{2}\cos 3x$
(e) $y = 3\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$

32. $\frac{\tan\frac{7\pi}{9} + \tan\frac{2\pi}{9}}{1 - \tan\frac{7\pi}{9}\tan\frac{2\pi}{9}} =$

(a) -1
(b) 1
(c) $\sqrt{3}$
(d) 0
(e) $\sqrt{7}$

33. Which one of the following is NOT an identity?

(a) $\csc(-\theta) = -\csc\theta$
(b) $\sec(90^\circ - \theta) = \csc\theta$
(c) $\cot(90^\circ - \theta) = \tan\theta$
(d) $\tan(-\theta) = \tan\theta$
(e) $\csc(\theta - 90^\circ) = -\sec\theta$

34. If $\cot(51^\circ - 3x) = \tan\left(25^\circ - \frac{x}{2}\right)$, then a value of x is

(a) -4°
(b) -18°
(c) -6°
(d) -28°
(e) -14°

35. The range of $f(x) = \sin 3x + \cos 3x$ is

(a) $\left[-\frac{1}{3}, \frac{1}{3}\right]$
(b) $[-3, 3]$
(c) $[-2, 2]$
(d) $\left[\sqrt{2}, 2\right]$
(e) $\left[-\sqrt{2}, \sqrt{2}\right]$

36. If A and B are the acute angles such that $\tan A = \frac{1}{4}$ and $\tan B = \frac{3}{5}$, then

(a) $A + B = 135^\circ$
(b) $A + B = 45^\circ$
(c) $90^\circ < A + B < 120^\circ$
(d) $\cot(A + B) > \tan(A + B)$
(e) $\sin(A + B) = \frac{3}{20}$

37. If $\tan x = \frac{3}{4}$ and $\tan y = \frac{1}{5}$, then $\tan(x + y) =$

(a) $\frac{19}{17}$
(b) $-\frac{19}{17}$
(c) $\frac{17}{19}$
(d) $-\frac{17}{19}$
(e) 1

38. If $0^\circ < \theta < 90^\circ$ and $\cos\theta = \sin 11^\circ$, then $\theta =$

(a) 101°
(b) 97°
(c) 79°
(d) 22°
(e) 11°

39. $\sin\frac{\pi}{10}\cos\frac{2\pi}{5} - \cos\frac{\pi}{10}\sin\left(-\frac{2\pi}{5}\right) =$

(a) 1
(b) 0
(c) $\frac{1}{2}$
(d) $-\frac{\sqrt{3}}{2}$
(e) $\frac{\sqrt{2}}{2}$

40. Let n be any positive integer, which one of the following is FALSE ?

(a) $\sin\left[(4n - 3)\frac{\pi}{2}\right] = (-1)^{n-1}$
(b) $\tan\left[\left(\frac{4n+1}{4}\right)\pi\right] = 1$
(c) $\sec(n\pi) = (-1)^n$
(d) $\cos[(3n - 2)\pi] = (-1)^n$
(e) $\sin\left[(2n - 1)\frac{\pi}{2}\right] = (-1)^{n-1}$

41. If $P\left(-\frac{3}{5}, \frac{4}{5}\right)$ is the point on the unit circle corresponding to an arc length S, then the point on the unit circle corresponding to arc length $S + 441\frac{\pi}{2}$ is

(a) $\left(\frac{4}{5}, \frac{3}{5}\right)$
(b) $\left(\frac{4}{5}, -\frac{3}{5}\right)$
(c) $\left(-\frac{3}{5}, -\frac{4}{5}\right)$
(d) $\left(\frac{3}{5}, -\frac{4}{5}\right)$
(e) $\left(-\frac{4}{5}, -\frac{3}{5}\right)$

42. If $P\left(x, \frac{5}{13}\right), x > 0$ is the point on the unit circle corresponding to an arc length S, then $\cos(3\pi - S) =$

(a) $-\frac{12}{13}$
(b) $\frac{12}{13}$
(c) $\frac{5}{13}$
(d) $-\frac{5}{13}$
(e) $-\frac{\sqrt{194}}{13}$

43. If $P\left(\frac{\sqrt{3}}{6}, \frac{\sqrt{33}}{6}\right)$ is the point on the unit circle corresponding to arc length S and $Q(b, c)$ is the point on the unit circle corresponding to arc length $S + \frac{\pi}{6}$, then $b =$

(a) $\frac{3+\sqrt{33}}{12}$
(b) $\frac{3\sqrt{11}+\sqrt{3}}{12}$
(c) $\frac{3-\sqrt{33}}{12}$

(d) $-\frac{\sqrt{3}}{6}$
(e) $\frac{\sqrt{33}-3}{12}$

44. If the point $\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$ on the unit circle corresponding to arc length $3\pi - S$, then $\cos(S+\pi) =$
(a) $-\frac{2}{3}$
(b) $\frac{\sqrt{5}}{3}$
(c) $\frac{\sqrt{3}}{3}$
(d) $-\frac{\sqrt{5}}{3}$
(e) $\frac{2}{3}$

11. DOUBLE ANGLE AND HALF-ANGLE IDENTITIES

1. $\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} =$
(a) $\sin\theta - \cos\theta$
(b) $\sec\theta$
(c) $-\sec\theta$
(d) $\csc\theta$
(e) $-\csc\theta$

2. $\sqrt{\frac{1+\cos 200°}{2}} =$
(a) $\cos 80°$
(b) $-\sin 100°$
(c) $\sin 100°$
(d) $-\cos 80°$
(e) $\cos 100°$

3. $\frac{2\sin 2\theta - \sin 4\theta}{2\sin 2\theta + \sin 4\theta} =$
(a) $\cot^2\theta$
(b) $\sec^2\theta$
(c) $\sin^2\theta$
(d) $\csc^2\theta$
(e) $\tan^2\theta$

4. $\frac{2}{1+\cos\theta} - \tan^2\frac{\theta}{2} =$
(a) $\cot^2\frac{\theta}{2}$
(b) 1
(c) -1
(d) $\sin^2\theta$
(e) $\tan^2\frac{\theta}{2}$

5. $-\frac{\tan\frac{x}{4}}{2-2\tan^2\frac{x}{4}} =$
(a) $-\frac{1}{2}\tan\frac{x}{8}$
(b) $\frac{1}{2}\tan\frac{x}{2}$
(c) $-\tan\frac{x}{2}$
(d) $\frac{1}{4}\tan\left(-\frac{x}{2}\right)$
(e) $-\tan\frac{x}{2}$

6. $\sqrt{\frac{1-\cos 400°}{2}} =$
(a) $\cos 400°$
(b) $\sin 200°$
(c) $\sin 20°$
(d) $-\cos 200°$
(e) $\cos 20°$

7. If $\sin t = -\frac{1}{4}$, then $\cos 2t =$
(a) $\frac{\sqrt{15}}{4}$
(b) $-\frac{\sqrt{15}}{4}$
(c) $\frac{\sqrt{15}}{2}$
(d) $\frac{7}{8}$
(e) $-\frac{7}{8}$

8. If $\cos^2 2\theta = \frac{1}{2}(1 + 2k\cos t\theta)$ for all θ, then
(a) $k = 1, t = \pm 1$
(b) $k = \frac{1}{2}, t = \frac{1}{2}$
(c) $k = \frac{1}{2}, t = \pm 4$
(d) $k = \frac{1}{2}, t = 2$
(e) $k = 4, t = \pm 4$

9. Which one of the following is an identity?
(a) $\sin x + \cos x = 1$
(b) $\csc^2 x + 1 = \cot^2 x$
(c) $\sec^2\frac{x}{2} - \tan^2\frac{x}{2} = 1$
(d) $\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}}$
(e) $\sin(x-y) = \sin x - \sin y$

10. If $\tan x = -\sqrt{8}, \frac{3\pi}{2} < x < 2\pi$, then $\cos\frac{x}{2} =$
(a) $\frac{\sqrt{6}}{3}$
(b) $-\frac{\sqrt{6}}{3}$
(c) $\frac{\sqrt{3}}{3}$
(d) $-\frac{\sqrt{3}}{3}$
(e) $-\frac{1}{\sqrt{2}}$

11. If $\left(-\frac{3}{5}, -\frac{4}{5}\right)$ is the point on the unit circle corresponding to arc length t, then the point corresponding to arc length $2t$ is
(a) $\left(-\frac{6}{5}, -\frac{8}{5}\right)$
(b) $\left(\frac{24}{25}, -\frac{7}{25}\right)$
(c) $\left(\frac{7}{25}, \frac{24}{25}\right)$
(d) $\left(-\frac{7}{25}, \frac{24}{25}\right)$
(e) $\left(\frac{24}{25}, \frac{7}{25}\right)$

12. If $\cos x = \frac{1}{3}$, then $\cos 2x =$

(a) $-\frac{8}{9}$
(b) $-\frac{4\sqrt{2}}{9}$
(c) $\frac{7}{9}$
(d) $\frac{4\sqrt{2}}{9}$
(e) $-\frac{7}{9}$

13. $\frac{4\tan 2x}{1+\tan^2 2x} =$
(a) $2\cos 4x$
(b) $2\cos\left(\frac{\pi}{2} - 4x\right)$
(c) $2\cos 2x$
(d) $2\sin 2x$
(e) $\sin 4x$

14. If $\sin\theta = \frac{1}{4}, \frac{\pi}{2} < \theta < \pi$, then $\sin 2\theta =$
(a) $-\frac{\sqrt{15}}{8}$
(b) $\frac{\sqrt{15}}{4}$
(c) $\frac{\sqrt{15}}{8}$
(d) $-\frac{\sqrt{15}}{4}$
(e) $\frac{1}{2}$

15. Given $\cos\theta = -\frac{3}{5}$ and θ terminates in quadrant II, then $\tan\frac{\theta}{2} =$
(a) $-\frac{1}{2}$
(b) $\frac{1}{2}$
(c) 2
(d) -2
(e) 0

16. If $\tan\theta = \frac{4}{3}$, which one of the following must be TRUE ?
(a) $\sin 2\theta = \frac{24}{25}$
(b) $\sin\theta > \cos\theta$
(c) $\cot(\theta - \pi) = -\frac{3}{4}$
(d) $|\sin\theta - \cos\theta| = 1$
(e) $\cot 2\theta = \frac{25}{24}$

17. If $3 - 3\cos 4\theta = k\sin^2 t\theta$, then
(a) $k = -2, |t| = 2$
(b) $k = 3, |t| = \pi$
(c) $k = 3, |t| = 4$
(d) $k = 6, |t| = 8$
(e) $k = 6, |t| = 2$

18. On the unit circle, if the arc length $\frac{\pi}{12}$ terminates at the point (a, b), then $ab =$
(a) 1
(b) $\frac{1}{4}$
(c) $\frac{\sqrt{3}}{4}$
(d) $\sqrt{3}$
(e) $\frac{1}{2}$

19. If $5\sin^2 5\theta - 5\cos^2 5\theta = 2k\cos t\theta$, then
(a) $k = \frac{3}{2}, t = -10$
(b) $k = -\frac{5}{2}, t = 5$
(c) $k = -\frac{5}{2}, t = 10$
(d) $k = \frac{2}{5}, t = -5$
(e) $k = -5, t = 5$

20. $\sqrt{\frac{1-\sin 440^\circ}{2}} =$
(a) $\cos 5^\circ$
(b) $\sin 5^\circ$
(c) $\cos 175^\circ$
(d) $\sin 95^\circ$
(e) $\sin 220^\circ$

21. $\frac{2\sin\theta - \sin 2\theta\cos\theta}{2\sin\theta(1-\sin^2\theta)} =$
(a) $1 + \tan^2\theta$
(b) $\cos 2\theta$
(c) $\tan 2\theta$
(d) $\sec^2\theta - 1$
(e) $\sec\theta$

22. $\cos\frac{17\pi}{3} + (\sin 75^\circ + \cos 75^\circ)^2 =$
(a) $\frac{3}{2}$
(b) $\frac{9}{16}$
(c) 0
(d) $1 + \sqrt{3}$
(e) 2

23. $\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 + k\sin tx$, then
(a) $k = -\frac{1}{2}, t = 2$
(b) $k = -1, t = 2$
(c) $k = -\frac{1}{2}, t = 1$
(d) $k = -\frac{1}{2}, t = \frac{1}{2}$
(e) $k = 1, t = -2$

24. If $\cos 2\theta = \frac{7}{25}$, whre $\frac{3\pi}{2} < 2\theta < 2\pi$, then $\cos\theta =$
(a) $\frac{4}{5}$
(b) $-\frac{7}{50}$
(c) $-\frac{4}{5}$
(d) $\frac{3}{5}$
(e) $-\frac{3}{5}$

25. If $\tan x = -\frac{1}{2}, \frac{\pi}{2} < x < \pi$, then $\cos\left(\frac{\pi}{2} - 2x\right) =$
(a) $\frac{4}{\sqrt{5}}$
(b) $\frac{3}{5}$

(c) $-\frac{4}{5}$
(d) $\frac{4}{3}$
(e) $-\frac{2}{3}$

26. The value of $\left(\sin\frac{\pi}{12}+\cos\frac{\pi}{12}\right)^2$ is equal to
(a) 1
(b) $1+\sqrt{3}$
(c) $\frac{3}{2}$
(d) $1+\frac{\sqrt{3}}{2}$
(e) $\frac{5}{2}$

27. If $\tan\theta=-\frac{3}{4}$, and $90^\circ<\theta<180^\circ$, then $\cos\frac{\theta}{2}=$
(a) $\frac{\sqrt{10}}{10}$
(b) $\frac{\sqrt{5}}{5}$
(c) $-\frac{3}{5}$
(d) $\frac{4}{5}$
(e) $\frac{3\sqrt{10}}{10}$

28. Which one of the following is an identity?
(a) $\cos^2 x-\sin^2 x=\sin 2x$
(b) $\cos(-x)\sec x=-1$
(c) $\cos^4\frac{x}{2}-\sin^4\frac{x}{2}=\cos x$
(d) $\tan^2 x-\sec^2 x=1$
(e) $\frac{1}{2}\sin 2x=\sin x$

29. $\sin\frac{x}{2}=-\sqrt{\frac{1-\cos x}{2}}$ is an identity if
(a) $0\le x\le 2\pi$
(b) $0\le x\le \pi$
(c) $\pi\le x\le 2\pi$
(d) $2\pi\le x\le 4\pi$
(e) $\pi\le x\le 3\pi$

12. TRIGONOMETRIC EQUATIONS

1. The solution set of the equation $\cos^2\frac{x}{2}-\sin^2\frac{x}{2}=-\frac{\sqrt{3}}{2}, 0\le x<2\pi$ is
(a) $\left\{\frac{2\pi}{3},\frac{4\pi}{3}\right\}$
(b) $\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}\right\}$
(c) $\left\{\frac{5\pi}{6},\frac{11\pi}{6}\right\}$
(d) $\left\{\frac{5\pi}{12},\frac{7\pi}{12}\right\}$
(e) $\left\{\frac{5\pi}{6},\frac{7\pi}{6}\right\}$

2. The solution set of the equation $3\sin x=\sqrt{2}+\sin x, 0\le x<\pi$ is
(a) $\left\{\frac{\pi}{4},\frac{\pi}{3}\right\}$
(b) $\left\{\frac{\pi}{6},\frac{\pi}{3}\right\}$
(c) $\left\{\frac{\pi}{6},\frac{3\pi}{4}\right\}$
(d) $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$
(e) $\left\{\frac{\pi}{3},\frac{2\pi}{3}\right\}$

3. The solution set of the equation $2\sin x\cos x=\sqrt{3}\cos x$ in $[0^\circ,360^\circ)$ is
(a) $\{30^\circ,90^\circ,150^\circ,270^\circ\}$
(b) $\{60^\circ,90^\circ,120^\circ,270^\circ\}$
(c) $\{60^\circ,120^\circ\}$
(d) $\{30^\circ,150^\circ\}$
(e) $\{45^\circ,90^\circ,135^\circ,270^\circ\}$

4. The solution set of $2\cos^2 x-2\sin^2 x=\sqrt{3}$ in $[0,\pi)$ is
(a) $\left\{\frac{\pi}{6},\frac{11\pi}{6}\right\}$
(b) $\left\{\frac{\pi}{4}\right\}$
(c) $\left\{\frac{11\pi}{12}\right\}$
(d) $\left\{\frac{\pi}{3}\right\}$
(e) $\left\{\frac{\pi}{12},\frac{11\pi}{12}\right\}$

5. The sum of the solutions of the equation $\sec\theta+1=\tan^2\theta$ where $0\le\theta<2\pi$ is
(a) 2π
(b) $\frac{4\pi}{3}$
(c) 3π
(d) $\frac{7\pi}{6}$
(e) 4π

6. The number of solutions of $3\cot^3\theta=\cot\theta$ in $\left[\frac{\pi}{3},\frac{3\pi}{2}\right]$ is
(a) 4
(b) 3
(c) 6
(d) 5
(e) 9

7. The solution set of $3\sec^2\frac{x}{2}=4$ in $[0,2\pi)$ is
(a) $\left\{\frac{\pi}{12},\frac{5\pi}{12}\right\}$
(b) $\left\{\frac{\pi}{3},\frac{5\pi}{3}\right\}$
(c) $\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}\right\}$
(d) $\left\{\frac{\pi}{12},\frac{5\pi}{12},\frac{7\pi}{12},\frac{11\pi}{12}\right\}$
(e) $\left\{\frac{\pi}{6},\frac{5\pi}{6}\right\}$

8. The solution set of $\tan t\sin t=\sqrt{3}\sin t$ in $[30^\circ,210^\circ]$ is
(a) $\{30^\circ,120^\circ,180^\circ\}$
(b) $\{30^\circ,60^\circ,120^\circ\}$
(c) $\{90^\circ,120^\circ,180^\circ\}$

(d) $\{30°, 180°\}$
(e) $\{60°, 180°\}$

9. The number of solutions of $\csc \frac{x}{2} = \sqrt{2}$ in $(\pi, 6\pi)$ is
(a) 0
(b) 3
(c) 1
(d) 4
(e) 2

10. The solution set of the equation $\sin 3x - \sin x = \cos 2x$ in $(0°, 360°)$ is
(a) $\{30°, 45°, 135°, 150°, 225°, 315°\}$
(b) $\{30°, 135°, 225°\}$
(c) $\{45°, 150°, 315°\}$
(d) $\{30°, 45°\}$
(e) $\{225°, 315°\}$

13. INVERSE TRIGONOMETRIC FUNCTIONS

1. $\sec(\arccos x) + \sin(\arccos x) =$
(a) $\frac{1}{x} - \sqrt{1-x^2}$
(b) $-\frac{1}{x} + \sqrt{1-x^2}$
(c) $\frac{1}{x} + \sqrt{1-x^2}$
(d) $x + x\sqrt{x^2-1}$
(e) $x - \sqrt{x^2-1}$

2. The function $y = \arcsin(2x) + \frac{\pi}{2}$ has
(a) domain $= [-2, 2]$, and range $= [0, \pi]$
(b) domain $= [-1, 1]$, and range $= [-\pi, 0]$
(c) domain $= \left[-\frac{1}{2}, \frac{1}{2}\right]$, and range $= [0, \pi]$
(d) domain $= \left[-\frac{1}{2}, \frac{1}{2}\right]$, and range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(e) domain $= [-2, 2]$, and range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

3. $\cos\left(\arctan\left(-\frac{1}{4}\right)\right) =$
(a) $\frac{1}{4}$
(b) $\frac{4}{\sqrt{17}}$
(c) $-\frac{4}{\sqrt{17}}$
(d) $\frac{1}{\sqrt{17}}$
(e) $-\frac{1}{4}$

4. The function $y = \arcsin(3x+1) - \pi$ has
(a) domain $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and range $= [-1, 1]$
(b) domain $= \left[-\frac{2}{3}, 0\right]$, and range $= \left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$
(c) domain $= \left[0, \frac{2}{3}\right]$, and range $= [0, 2\pi]$
(d) domain $= [-1, 1]$, and range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(e) domain $= [-2\pi, 0]$, and range $= \left[-\frac{2}{3}, 0\right]$

5. $\tan\left[\arcsin\left(\frac{3}{5}\right) + \arccos\left(-\frac{4}{5}\right)\right] =$
(a) $\frac{3}{2}$
(b) $\frac{24}{25}$
(c) 0
(d) $-\frac{24}{7}$
(e) $\frac{24}{7}$

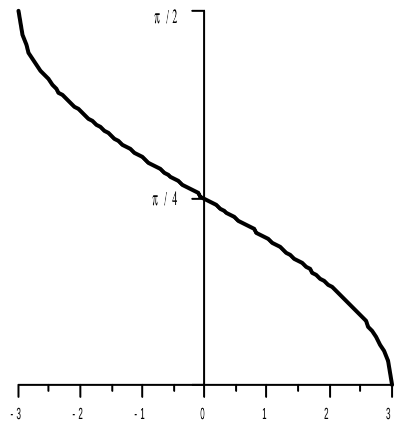
6. $\sin\left[\frac{\pi}{2} - \arcsin\left(\frac{4}{5}\right) + \arctan\left(-\frac{3}{4}\right)\right] =$
(a) $\frac{24}{25}$
(b) $\frac{\sqrt{3}}{5}$
(c) 1
(d) $-\frac{12}{25}$
(e) 0

7. Which one of the following is TRUE for all values for which the variables are defined?
(a) $2\arccos(x) = \arccos(2x)$
(b) If $y = \arcsin(x+2)$, then $x = -2 + \sin y$
(c) $\arctan x = \frac{\arcsin x}{\arccos x}$
(d) $y = \arcsin x$ is an even function
(e) $\text{arccot}\, x = \frac{1}{\arctan x}$

8. The function $y = \cos(\arcsin 2x)$ has
(a) domain $= \left[0, \frac{1}{2}\right]$, and range $= [-1, 1]$
(b) domain $= \left[-\frac{1}{2}, \frac{1}{2}\right]$, and range $= [-1, 1]$
(c) domain $= \left[-\frac{1}{2}, \frac{1}{2}\right]$, and range $= [0, 1]$
(d) domain $= \left[0, \frac{1}{2}\right]$, and range $= [0, 1]$
(e) domain $= \left[-\frac{1}{2}, \frac{1}{2}\right]$, and range $= \left[-\frac{1}{2}, \frac{1}{2}\right]$

9. $\text{arccsc}\left(\frac{1}{\sin\frac{3\pi}{4}}\right) =$
(a) undefined
(b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{4}$
(d) $\frac{4}{3\pi}$
(e) $\frac{5\pi}{4}$

10. The adjacent figure represents the graph of
(a) $y = 3\cos^{-1}\frac{x}{2}$
(b) $y = 2\sin^{-1} 3x$
(c) $y = \frac{1}{2}\cos^{-1} 3x$
(d) $y = 3\sin^{-1}\frac{x}{2}$
(e) $y = \frac{1}{2}\cos^{-1}\frac{x}{3}$



11. $\cos\left[\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right] =$
   (a) $|x|$
   (b) x
   (c) $-x$
   (d) $\frac{1}{x}$
   (e) $\frac{1}{|x|}$

12. $\tan\left[\frac{\pi}{4} - \sec^{-1}\left(-\frac{5}{4}\right)\right] =$
   (a) 7
   (b) $\frac{1}{7}$
   (c) -7
   (d) 1
   (e) $\frac{1}{4}$

13. $\cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\sin\frac{7\pi}{6}\right) =$
   (a) $\frac{2\pi}{3}$
   (b) $\frac{11\pi}{6}$
   (c) $\frac{5\pi}{6}$
   (d) 2π
   (e) $\frac{\pi}{2}$

14. VECTORS

1. If $\overrightarrow{v} = \langle -2, -6\sqrt{3}\rangle$ and $\overrightarrow{u} = -\sqrt{3}i - j$, then the vector $\frac{\overrightarrow{v}}{2} - \sqrt{3}\,\overrightarrow{u}$ has
   (a) magnitude $= 4$, direction angle $= 60^\circ$
   (b) magnitude $= 4$, direction angle $= 300^\circ$
   (c) magnitude $= 4$, direction angle $= 330^\circ$
   (d) magnitude $= 16$, direction angle $= 60^\circ$
   (e) magnitude $= 16$, direction angle $= 300^\circ$

2. If $\theta, 0 \leq \theta \leq \pi$, is the angle between the vectors $\overrightarrow{u} = -i + 2j$ and $\overrightarrow{v} = 2i - j$, then $\sin\theta =$
   (a) $\frac{4}{5}$
   (b) $-\frac{3}{5}$
   (c) $\frac{1}{5}$
   (d) $-\frac{4}{5}$
   (e) $\frac{3}{5}$

3. A unit vector which is opposite in direction to $\overrightarrow{v} = \langle -2, 3\rangle$ is
   (a) $\left\langle -\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\right\rangle$
   (b) $\left\langle \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}\right\rangle$
   (c) $\left\langle \frac{2}{5}, -\frac{3}{5}\right\rangle$
   (d) $\left\langle -\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right\rangle$
   (e) $\left\langle \frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\right\rangle$

4. If $\overrightarrow{u} = \langle 1, 2\rangle$ and $\overrightarrow{v} = \langle -3, \sqrt{3} - 1\rangle$, then the magnitude and direction angle of $\overrightarrow{u} + \overrightarrow{v} + i - j$, respectively, are
   (a) $2, 150^\circ$
   (b) $2, 120^\circ$
   (c) $4, 120^\circ$
   (d) $4, 150^\circ$
   (e) $2, 60^\circ$

5. Given the vectors $\overrightarrow{u} = \langle 3, -4\rangle$, $\overrightarrow{v} = \langle 4, 3\rangle$, and $\overrightarrow{w} = \langle a, b\rangle$. If $\overrightarrow{w}$ is a unit vector opposite in direction to the vector $\overrightarrow{u} + \overrightarrow{v}$, then
   (a) $a = -\frac{3}{5}, b = \frac{4}{5}$
   (b) $a = -\frac{7}{10}, b = \frac{1}{10}$
   (c) $a = \frac{7\sqrt{2}}{10}, b = -\frac{\sqrt{2}}{10}$
   (d) $a = -\frac{4}{5}, b = -\frac{3}{5}$
   (e) $a = -\frac{7\sqrt{2}}{10}, b = \frac{\sqrt{2}}{10}$

6. The smallest angle between the vectors $\overrightarrow{u} = -i - 2j$ and $\overrightarrow{v} = -i + 3j$ is
   (a) 45°
   (b) 90°
   (c) 120°
   (d) 150°
   (e) 135°

7. If $\overrightarrow{u} = \langle 2\cos 80^\circ, 2\sin 80^\circ\rangle$, $\overrightarrow{v} = \langle 3\cos 20^\circ, 3\sin 20^\circ\rangle$, then $\overrightarrow{u} \cdot \overrightarrow{v} =$
   (a) 3
   (b) 2
   (c) 0
   (d) 12
   (e) $3\sqrt{3}$

8. If t is the angle between the unit vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ and $\cos t = \frac{1}{3}$, then $\left|\overrightarrow{u} - \overrightarrow{v}\right|$

(a) 2
(b) $\frac{4}{3}$
(c) $\frac{2\sqrt{3}}{3}$
(d) $\frac{2\sqrt{6}}{3}$
(e) $\frac{8}{3}$

9. For the vectors $\overrightarrow{u} = \langle 4, 3\rangle$ and $\overrightarrow{v} = \langle 2, 1\rangle$, let α be the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$, where $0^\circ \le \alpha \le 180^\circ$, then $\left(\overrightarrow{u} - \overrightarrow{v}\right) \cdot \left(\overrightarrow{u} + \overrightarrow{v}\right) + 2\left|\overrightarrow{u}\right|\left|\overrightarrow{v}\right| \cos\alpha =$
(a) -2
(b) 0
(c) 20
(d) 22
(e) 42

10. If $\overrightarrow{u}$ and $\overrightarrow{v}$ are unit vectors such that $\overrightarrow{u} \cdot \overrightarrow{v} = -\frac{3}{4}$, then $\left|\overrightarrow{u} + \overrightarrow{v}\right| =$
(a) 2
(b) $\frac{1}{2}$
(c) $\frac{\sqrt{14}}{2}$
(d) $\frac{\sqrt{2}}{2}$
(e) $\frac{\sqrt{5}}{2}$

11. The two unit vectors which are perpendicular to $\overrightarrow{v} = i + 2j$ are
(a) $\left\langle -2\frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right\rangle$ and $\left\langle 2\frac{\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}\right\rangle$
(b) $\left\langle 2\frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right\rangle$ and $\left\langle -2\frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right\rangle$
(c) $\left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right\rangle$ and $\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$
(d) $-\frac{2}{5}i + \frac{1}{5}j$ and $\frac{2}{5}i - \frac{1}{5}j$
(e) $2i - j$ and $-2j + j$

12. If $\overrightarrow{u} = \langle 2, 1\rangle$ and $\overrightarrow{v} = \langle 1, -2\rangle$, then $\overrightarrow{u} + \overrightarrow{v}$ is perpendicular to
(a) $\overrightarrow{v}$
(b) $\overrightarrow{u}$
(c) $\overrightarrow{u} + \overrightarrow{v}$
(d) $\overrightarrow{u} - \overrightarrow{v}$
(e) $\overrightarrow{u} - 2\overrightarrow{v}$

13. If θ is the angle between $\overrightarrow{u} = i + 3j$ and $\overrightarrow{v} = -i + 3j$, and $0^\circ \le \theta \le \pi$, then $\tan\theta =$
(a) $-\frac{3}{5}$
(b) $\frac{3}{5}$
(c) $\frac{4}{5}$
(d) $-\frac{3}{4}$
(e) $\frac{3}{4}$

14. If $\overrightarrow{u}$ and $\overrightarrow{v}$ are unit vectors and the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$ is 60°, then the magnitude of the vectors $2\overrightarrow{u} - 3\overrightarrow{v}$ is
(a) $\left(\sqrt{2} - \sqrt{3}\right)^2$
(b) $\sqrt{13}$
(c) 5
(d) $\sqrt{7}$
(e) 1

15. For vectors $\overrightarrow{u}$ and $\overrightarrow{v}$, if $\left|\overrightarrow{u} + \overrightarrow{v}\right| = \sqrt{50}$ and $\left|\overrightarrow{u}\right| = \left|\overrightarrow{v}\right| = 4$, then $\overrightarrow{u} \cdot \overrightarrow{v} =$
(a) 2
(b) 3
(c) $\frac{\sqrt{26}}{4}$
(d) 9
(e) 0

16. Which one of the following is NOT a unit vector?
(a) $\left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2}\right\rangle$
(b) $\langle -1, 0\rangle$
(c) $\sin\frac{\pi}{5}, \cos\frac{\pi}{5}\rangle$
(d) $\left\langle \frac{1}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}\right\rangle$
(e) $\frac{1}{5}, \frac{4}{5}\rangle$

17. The vector of magnitude 2 and opposite to $\overrightarrow{u} = \langle -1, 1\rangle$ is
(a) $\sqrt{3}, -1\rangle$
(b) $\left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$
(c) $-\sqrt{2}, \sqrt{2}\rangle$
(d) $\sqrt{2}, -\sqrt{2}\rangle$
(e) $\langle 1, -1\rangle$

18. If $\left|\overrightarrow{u}\right| = 2$ with direction angle $\theta = 30^\circ$, then the vector $\overrightarrow{u} =$
(a) $\langle 1, 2\rangle$
(b) $\langle 3, 1\rangle$
(c) $\langle 2, 2\rangle$
(d) $1, \sqrt{3}\rangle$
(e) $\sqrt{3}, 1\rangle$

19. The magnitude and direction angle of $\overrightarrow{u} = 2\langle 6, -1\rangle - 3\langle 4, 5\rangle$ are
(a) $17, 180^\circ$
(b) $\sqrt{13}, 0^\circ$
(c) $\sqrt{7}, 90^\circ$
(d) $17, 0^\circ$
(e) $17, 270^\circ$

20. If $\overrightarrow{w}$ has magnitude 24 and direction angle 30°, then

the horizontal and vertical components of $\overrightarrow{w}$, respectively, are

(a) $12, 12\sqrt{3}$
(b) $12\sqrt{3}, 12$
(c) $12\sqrt{2}, 12$
(d) $12, 12\sqrt{2}$
(e) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

21. If θ is the direction angle of $\overrightarrow{u} = \langle -3, 4\rangle$, then $\tan 2\theta =$

(a) $-\frac{8}{3}$
(b) -6
(c) $\frac{12}{7}$
(d) $\frac{24}{7}$
(e) $\frac{24}{25}$

22. If $\overrightarrow{u}$ and $\overrightarrow{v}$ are two nonzero perpendicular vectors, then

(a) $|\overrightarrow{u} + \overrightarrow{v}| > |\overrightarrow{u} - \overrightarrow{v}|$
(b) $|\overrightarrow{u} - \overrightarrow{v}| > |\overrightarrow{u} + \overrightarrow{v}|$
(c) $|\overrightarrow{u} + \overrightarrow{v}| = 0$
(d) $|\overrightarrow{u} + \overrightarrow{v}| = |\overrightarrow{u} - \overrightarrow{v}|$
(e) $|\overrightarrow{u} - \overrightarrow{v}| = 0$

23. If $\overrightarrow{u} = 4i + j$, $\overrightarrow{v} = \langle 1, 3\rangle$, and $\overrightarrow{w} = \overrightarrow{u} - \overrightarrow{v} - 2j$, then

(a) $\langle -1, 1\rangle$ is a unit vector opposite to $\overrightarrow{w}$
(b) $\langle 1, -1\rangle$ is a unit vector opposite to $\overrightarrow{w}$
(c) $-\frac{3}{5}, \frac{4}{5}\rangle$ is a unit vector opposite to $\overrightarrow{w}$
(d) $\langle 3, 4\rangle$ is a unit vector in the direction of $\overrightarrow{w}$
(e) The direction angle of $\overrightarrow{u}$ is greater than the direction angle of $\overrightarrow{v}$

24. If $\overrightarrow{v} = \langle 2, -1\rangle$ and $\overrightarrow{v} = -3i + 2j$, then the direction angle of the vector $2\overrightarrow{u} + \overrightarrow{v} + i - 2\sqrt{3}j$ is

(a) $120°$
(b) $210°$
(c) $330°$
(d) $300°$
(e) $150°$

15. SYSTEMS OF EQUATIONS

1. If (α, β, γ) is the solution of the system
$2x - y + 3z = 9$
$3x + y + 2z = 11$
$x - y + z = 2$

(a) α, and γ are positive integers and β is a negative integer.
(b) α, and β are positive integers and γ is a negative integer.
(c) α, and γ are negative integers and β is a positive integer.
(d) α, and β are negative integers and γ is a positive integer.
(e) α, γ, and β are positive integers.

2. The value of k for which the system
$3x - 2y + 1 = 0$
$x + ky = 0$
has no solution is

(a) $-\frac{3}{2}$
(b) $\frac{3}{2}$
(c) $-\frac{2}{3}$
(d) $\frac{2}{3}$
(e) -2

3. The solution set of the following system is
$\frac{1}{x} - \frac{1}{y} = \frac{7}{10}$
$\frac{3}{x} + \frac{5}{y} = \frac{1}{2}$

(a) $\{(2, 5)\}$
(b) $\{(2, -5)\}$
(c) $\left\{\left(\frac{1}{2}, \frac{10}{13}\right)\right\}$
(d) $\left\{\left(-2, \frac{10}{12}\right)\right\}$
(e) $\left\{\left(\frac{9}{10}, 5\right)\right\}$

4. The solution set of the following system is
$\frac{2}{x+2} + \frac{1}{1-y} = 1$
$\frac{1}{x+2} + \frac{3}{y-1} = 11$

(a) $\left\{\left(\frac{3}{2}, -\frac{4}{3}\right)\right\}$
(b) $\{(0, 4)\}$
(c) $\left\{\left(-\frac{21}{8}, \frac{21}{5}\right)\right\}$
(d) $\{(2, 3)\}$
(e) $\left\{\left(-\frac{3}{2}, \frac{4}{3}\right)\right\}$

5. The solution set of the following system is
$4x + y + z = 6$
$2x - y = 0$

(a) $\{(x, 2x, 6x - 6)\,|x \in \Re\}$
(b) $\{(x, 2x, 6 - 6x)\,|x \in \Re\}$
(c) $\{(x, x, 6x)\,|x \in \Re\}$
(d) $\{(x, x, 6x - 6)\,|x \in \Re\}$
(e) $\{(2x, x, 6x)\,|x \in \Re\}$

6. The solution set of the following system is
$x + y - z = 6$
$2x - y + z = -9$
$x - 2y + 3z = 1$

(a) $\{(-1, -23, 16)\}$

(b) $\{(1, 23, 16)\}$
(c) $\{(-1, -23, 16)\}$
(d) $\{(0, 0, -6)\}$
(e) $\{(-1, 23, 16)\}$

7. If (x, y) is a solution of the system
$2x + y = 4$
$3x - 2y = -1$,
then (x, y) satisfies
(a) $x^2 + 2y^2 = 8$
(b) $2x^2 - y^2 = -2$
(c) $x^2 + y^2 = 3$
(d) $x + y = -3$
(e) $3x - 2y = 1$

8. Consider the following system
$x + 2y - z = 2$
$3x - y + mz = -7$
$5x + 3y = n$
where m and n are real numbers. Which one of the following is TRUE?
(a) The system has infinite number of solutions if $m = 2$ and $n = -3$.
(b) The system has no solution if $m = 2$ and $n = -3$.
(c) The system is independent if $m = 2$ and $n = \frac{13}{10}$.
(d) The system has a single solution if $m = 2$ and $n \neq -3$.
(e) The system is inconsistent if $m = 2$ and $n \neq \frac{13}{10}$.

9. Given the system
$-x + ky + 3 = 0$
$5x - 4y = 0$.
If the system is inconsistent, then $k =$
(a) $\frac{6}{7}$
(b) $\frac{4}{5}$
(c) $\frac{1}{3}$
(d) $\frac{1}{2}$
(e) $\frac{8}{9}$

10. The values of k_1 and k_2 for which the following system of equations has infinitely many solutions are
$2x + 5y + k_1 = 0$
$3x - k_2 y = 2$
(a) $k_1 = -5, k_2 = -2$
(b) $k_1 = -\frac{4}{3}, k_2 = -\frac{15}{2}$
(c) $k_1 = \frac{4}{3}, k_2 = \frac{15}{2}$
(d) $k_1 = -\frac{4}{3}, k_2 = \frac{15}{2}$
(e) $k_1 = \frac{4}{3}, k_2 = -\frac{15}{2}$

11. The solution set of the following system satisfies
$\frac{3}{\sqrt[3]{x}} + \frac{4}{\sqrt[3]{y}} = -1$
$\frac{2}{\sqrt[3]{x}} - \frac{2}{\sqrt[3]{y}} = -3$
(a) $x + y = 1$
(b) $x + y = -1$
(c) $x + y = 7$
(d) $x + y = -7$
(e) $x + y = -9$

12. If $(a, b - c, b + c)$ is a solution of the system
$x + 3y + 2z = 1$
$y + z = 0$
$z = 1$,
then the values of a, b, and c are
(a) $a = -2, b = 0, c = 1$
(b) $a = -1, b = \frac{1}{3}, c = 2$
(c) $a = 2, b = 0, c = 1$
(d) $a = 1, b = -\frac{1}{3}, c = \frac{1}{2}$
(e) $a = 2, b = 1, c = 0$

13. Which one of the following is TRUE about the system
$x + y + z = 1$
$x - y = 0$
$y + z = 2$
(a) The system is inconsistent.
(b) The system has a unique solution.
(c) The system has infinite number of solutions.
(d) The system has two solutions.
(e) The system has three solutions.

14. The solution of the following system is
$y = e^x - 5$
$y = -2e^x + 1$
(a) $(2, -3)$
(b) $(\ln 3, -1)$
(c) $(-\ln 2, 2)$
(d) $(\ln 2, 2)$
(e) $(\ln 2, -3)$

15. If the line $x + y = k$ and the curve $xy = 1$ have no points in common, then
(a) $k < -2$ or $k > 2$
(b) $k \leq 4$
(c) $k = \pm 2$
(d) $k = \pm 4$
(e) $-2 < k < 2$

16. If the graphs of the equations $x + y = k$ and $xy = 1$ have one point in common, then $k =$
(a) 2
(b) -2
(c) 2 or -2

(d) $\frac{1}{4}$
(e) 4

17. The set of values of k for which the following system of equations has only one solution is
$2x^2 + y^2 = 6$
$y - x = 3k$
(a) $\{2, -2\}$
(b) $\{1, 2\}$
(c) $\{1\}$
(d) $\{1, -1\}$
(e) $\emptyset$

18. For real x and y, the solution set of the following system is
$x^4 - y^4 = 0$
$x^2 + 2xy + 3y^2 = 6$
(a) $\{(1,1), (-1,-1), (\sqrt{3}, -\sqrt{3}), (-\sqrt{3}, \sqrt{3})\}$
(b) $\{(1,1), (-1,-1), (\sqrt{3}, \sqrt{3}), (-\sqrt{3}, -\sqrt{3})\}$
(c) $\{(1,1), (-1,-1)\}$
(d) $\{(1,1), (-1,-1), (2,2), (-2,-2)\}$
(e) $\{(1,1), (-1,-1), (-1,1), (1,-1)\}$

19. The following system has
$(x+1)^2 + 2(y-2)^2 = 10$
$2(x+1)^2 + 3(y-2)^2 = 16$
(a) three real solutions.
(b) one real solution.
(c) no real solutions.
(d) two real solutions.
(e) four real solutions.

20. If the straight line $x - y = -2$ intersects the ellipse $3x^2 + 2y^2 = 5$ at the points (a, b) and (c, d), then $a + b + c + d =$
(a) 2
(b) $-\frac{12}{5}$
(c) 0
(d) $-\frac{8}{5}$
(e) $\frac{4}{5}$

21. Consider the system
$y - x^2 + 9 = 0$
$y + x^2 - k = 0$
where x, y, and k are real numbers. The values of k for which the system has exactly two different solutions are
(a) $k > -9$
(b) $k < -9$
(c) $k < 9$
(d) $k > 9$
(e) $k > 0$

22. A value of b such that the straight line $3x - y = b$ touches the circle $x^2 + y^2 = 25$ at only one point is
(a) $\sqrt{10}$
(b) 25
(c) $5\sqrt{2}$
(d) $10\sqrt{5}$
(e) $5\sqrt{10}$

23. Consider the equations
$(x-1)^2 + 2(y-2)^2 = 2$
$(x-2)^2 - (y-2)^2 = 2$
where x and y are real numbers. The solution set of the system is
(a) $\{(\frac{1}{3}, 1+\sqrt{7}), (\frac{1}{3}, 1-\sqrt{7})\}$
(b) $\{(\frac{1}{3}, 1+\sqrt{7}), (-\frac{1}{3}, 1+\sqrt{7})\}$
(c) $\left\{\left(\frac{1}{3}, 2+\frac{\sqrt{7}}{3}\right), \left(\frac{1}{3}, 2-\frac{\sqrt{7}}{3}\right)\right\}$
(d) $\left\{\left(3, 2+\frac{\sqrt{7}}{3}\right), \left(3, 2-\frac{\sqrt{7}}{3}\right), \left(\frac{1}{3}, 2+\frac{\sqrt{7}}{3}\right), \left(\frac{1}{3}, 2-\frac{\sqrt{7}}{3}\right)\right\}$
(e) $\left\{\left(3, 2+\frac{\sqrt{7}}{3}\right), \left(3, 2-\frac{\sqrt{7}}{3}\right), \left(\frac{1}{3}, 1+\frac{\sqrt{7}}{3}\right), \left(\frac{1}{3}, 1-\frac{\sqrt{7}}{3}\right)\right\}$

24. Let C be the circle $x^2 + y^2 = 9$ and L be the line $x + 2y = 7$. Which one of the following is TRUE?
(a) L and C have two points in common.
(b) L and C have no points in common.
(c) L touches C only at $(\sqrt{45}, \sqrt{3} - 1)$.
(d) L touches C only at $(\sqrt{3} - 1, \sqrt{35})$.
(e) L touches C only at $\left(\sqrt{35}, \frac{7-\sqrt{35}}{2}\right)$.

16. MATRICES: NOTATION, DEFINITIONS, AND MULTIPLICATION

1. Given $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ x & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 0 \\ 6 & 0 \end{bmatrix}$. If $AB = 2A^2 - C$, then $x =$
(a) 0
(b) 2
(c) 4
(d) -4
(e) -2

2. Let $A = \begin{bmatrix} 2 & -3 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}$. If X is a 2×2 matrix such that $X = 2A - B$, then
(a) $X = -3B$

(b) $X = 2A$
(c) $X = -2B$
(d) $X = 2B$
(e) $X = -3B$

3. If $A = \begin{bmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix}$, then
   (a) $A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   (b) $A - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   (c) $AB = BA$
   (d) $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   (e) $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

4. If $A = \begin{bmatrix} 1 & 4 \\ 0 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 2 \\ 3 & 4 & 0 \end{bmatrix}$, then the element a_{32} of A is
   (a) 0
   (b) 2
   (c) 6
   (d) 3
   (e) -4

5. Let $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} a & \frac{1}{2} \\ 3 & b \end{bmatrix}$. If $AB = 2C$, then
   (a) $a = 1, b = -2$
   (b) $a = -1, b = 2$
   (c) $a = -1, b = -2$
   (d) $a = 2, b = 3$
   (e) $a = 1, b = 2$

6. If $A = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix}$, then the element in the second row and third column of $(A^2 - A)$ is
   (a) 0
   (b) 6
   (c) 2
   (d) 3
   (e) -6

7. If $C = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & 2 & 1 \\ 4 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$, then the element in the second row and third column of CD is
   (a) 2
   (b) 1
   (c) -3
   (d) 8
   (e) 3

8. If C is 4×3, A and B are 3×4, then the size of $C \cdot (2A + 3B)$ is
   (a) 4×4
   (b) 3×4
   (c) 3×3
   (d) 4×3
   (e) 4×6

9. If A and B are two matrices of size 4×3, then the size of $B^T \cdot (2A + 3B)$ is
   (a) 4×3
   (b) 3×4
   (c) 3×3
   (d) 4×4
   (e) undefined

10. If $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$, then $A^T - 2B + C^2 =$
    (a) $\begin{bmatrix} -3 & -3 \\ -8 & -9 \end{bmatrix}$
    (b) $\begin{bmatrix} 3 & 3 \\ 8 & 9 \end{bmatrix}$
    (c) $\begin{bmatrix} 8 & 9 \\ 3 & 3 \end{bmatrix}$
    (d) $\begin{bmatrix} -8 & -9 \\ -3 & -3 \end{bmatrix}$
    (e) $\begin{bmatrix} -9 & -3 \\ -8 & -3 \end{bmatrix}$

11. If $A = \begin{bmatrix} -1 & 0 \\ 3 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ -1 & 2 \end{bmatrix}$, then $(A + B) \cdot B^T =$
    (a) $\begin{bmatrix} 7 & 3 \\ -1 & 1 \end{bmatrix}$
    (b) $\begin{bmatrix} 3 & -1 & 1 \\ 9 & -4 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(d) $B^T \cdot (A+B)$
(e) $\begin{bmatrix} 3 & 1 & 1 \\ 9 & 4 & -2 \\ -2 & -1 & 1 \end{bmatrix}$

12. Let A and B be square matrices of the same order and A^T is the transpose of A. Which one of the following is not always true?
(a) $(A^T)^T = A$
(b) $(A+B)^T = A^T + B^T$
(c) $(A+B)^2 = A^2 + 2AB + B^2$
(d) $(AB)^T = B^T A^T$
(e) $c(A+B) = cA + cB$, where c is a real number.

13. If $A = \begin{bmatrix} 3 & 2 \\ x & 0 \\ -2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$
and $B^T A^T = 2\begin{bmatrix} \frac{3}{2} & \frac{1}{2} & \frac{x}{2} \\ y & -1 & -2 \end{bmatrix}$, then
(a) $x = -1, y = 6$
(b) $x = -\frac{1}{2}, y = 6$
(c) $x = -2, y = 6$
(d) $x = 2, y = 3$
(e) $x = -1, y = 3$

17. SOLUTION OF LINEAR SYSTEMS BY MATRIX METHODS

1. If the augmented matrix of a system of linear equations is $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 2 & 3 & -2 \\ 0 & 0 & 0 & 3 & -3 \end{array}\right]$,
then the system has
(a) solution set $\{(1,2,0,1)\}$
(b) solution set $\{(0,0,0,0)\}$
(c) no solution
(d) infinitely many solutions
(e) solution set $\{(1,-1,-2,-3)\}$

2. If the augmented matrix of a system of linear equations is $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$,
then the solution set is
(a) $\{(0,0,0,2)\}$
(b) $\{(1,2,3,4)\}$
(c) $\{(0,0,0,0)\}$
(d) $\emptyset$
(e) $\{(0,0,-1,2)\}$

3. If the augmented matrix of a system of linear equations is $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 4 & 2 & 2 \end{array}\right]$,
then
(a) the system has infinitely many solutions.
(b) the system has a unique solution.
(c) the matrix can not be the augmented natrix of a 4×4 system.
(d) the system has two solutions.
(e) the system has no solution.

4. If $AX = B$ is the matrix equation which represents the system
$3x + 2y = 1$
$2x + y = 6$,
then $X =$
(a) $\begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix}$
(b) $\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix}$
(c) $\begin{bmatrix} 1 \\ 6 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix}$
(e) $\begin{bmatrix} -1 & 2 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix}$

5. Which one of the following represents an inconsistent system?
(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$
(b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$
(c) $\left[\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 1 \end{array}\right]$
(d) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$
(e) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 1 & 1 & 1 & 0 \end{array}\right]$

6. Given the matrices $A = \begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & -5 \end{bmatrix}$, $C = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$. If $(A - B)X = C$, then $X =$

(a) $\begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -1 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$

7. If the augmented matrix of a system of linear equations is $\left[\begin{array}{ccc|c} -1 & 1 & 0 & -1 \\ 0 & 1 & -1 & 6 \\ 1 & 0 & 1 & -1 \end{array}\right]$.
The solution (x, y, z) of the system has

(a) two negative and one positive value.
(b) all the values positive.
(c) all the values negative.
(d) two positive and one negative value.
(e) $x = 3$ and two negative values.

8. If the system $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 3 & 2 & 4 & 5 \\ 2 & 1 & 1 & 6 \end{array}\right]$ is written as $\left[\begin{array}{ccc|c} 1 & m & n & 2 \\ 0 & 1 & k & 1 \\ 0 & 0 & 1 & -\frac{3}{2} \end{array}\right]$,
then $mnk =$

(a) 0
(b) 1
(c) -1
(d) 2
(e) -2

9. The solution set of $\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 2 & -1 & 1 & 1 \\ 4 & 1 & 3 & 5 \end{array}\right]$ is

(a) $\{(1, 1, -5)\}$
(b) $\{(2, 0, -5)\}$
(c) $\{(0, 1, -4)\}$
(d) $\{(x, 2x, x - 7) \mid x \in \Re\}$
(e) $\emptyset$

10. If (a, b, c) is a solution for $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 2 & -1 & 3 & 0 \\ 1 & 1 & 1 & 2 \end{array}\right]$,
then $3a + 4b + c =$

(a) 13
(b) 7
(c) 15
(d) 9
(e) 0

11. Given the system $\left[\begin{array}{ccc|c} 1 & -2 & 4 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 2 & 6 & A \end{array}\right]$
Which one of the following is FALSE ?

(a) The system is inconsistent for all $A \neq -2$.
(b) The system is consistent with infinitely many solutions for $A = -2$.
(c) The system has no unique solution for any real number A.
(d) The system can be made consistent or inconsistent for a suitable value of A.
(e) The system is consistent for any real A.

18. INVERSE OF A MATRIX

1. If $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the inverse of $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then

(a) $a = \frac{2}{3}, c = \frac{1}{3}$
(b) $a = \frac{1}{3}, c = -\frac{2}{3}$
(c) $a = -\frac{1}{3}, c = \frac{2}{3}$
(d) $a = \frac{1}{3}, c = \frac{2}{3}$
(e) $a = -\frac{1}{3}, c = \frac{1}{3}$

2. If the matrix $\begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & -1 \\ 3 & 5 & 7 \end{bmatrix}$ is the multiplication inverse of $\begin{bmatrix} 4 & -13t & -7t \\ x & 5t & yt \\ -1 & 2t & 2t \end{bmatrix}$,
then

(a) $x = -1, y = 2, t = \frac{1}{3}$
(b) $x = -1, y = -5, t = \frac{1}{3}$
(c) $x = -1, y = 5, t = -\frac{1}{5}$
(d) $x = 1, y = 2, t = \frac{1}{3}$
(e) $x = -1, y = 2, t = \frac{1}{5}$

3. If $a \neq 0$ and $A = \begin{bmatrix} a & 0 & 0 \\ 0 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix}$, then

(a) $AA^{-1} = I$
(b) $A^{-1}A = I$
(c) $A^{-1} = \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}$

(d) A^{-1} does not exist.
(e) $A^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4. If A and B are matrices of order $n \times n$, then
(a) if A^{-1} exists, then $ABA^{-1} = B$ where $B \neq I$.
(b) A^{-1} and B^{-1} are $(n+1) \times (n+1)$ matrices.
(c) if $AB = O$, then either $A = O$ or $B = O$, where O is an $n \times n$ zero matrix.
(d) if A^{-1} exists, then (AA^{-1}) is the $n \times n$ identity matrix.
(e) $(A + 2B)(A - 2B) = A^2 - 4B^2$.

5. If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$, and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then
(a) $A^{-1} = -A, B^{-1}$ does not exist.
(b) $A^{-1} = A, B^{-1}$ does not exist.
(c) $A^{-1} = A, B^{-1} = O$.
(d) A^{-1} does not exist, $B^{-1} = O$.
(e) $A^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ -\frac{3}{7} & -\frac{2}{7} \end{bmatrix}$, B^{-1} does not exist.

6. If $A^{-1} = \begin{bmatrix} 2 & -1 & 1 \\ -3 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$, then the element in the second row and third column of $(AB)^{-1}$ is equal to
(a) 1
(b) 4
(c) -2
(d) 3
(e) -6

7. If $A^{-1} = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & x \\ -6 & 1 & -4 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 1 \\ -3 & 2 & 0 \end{bmatrix}$, then $x =$
(a) -3
(b) -1
(c) 0
(d) 2
(e) 4

8. Given the system
$3x - 2y = 4$
$x + 3y = 5$
The element in the first row and first column of the iverse of the coefficient matrix is
(a) $\frac{3}{7}$
(b) $-\frac{3}{2}$
(c) 3
(d) $\frac{3}{11}$
(e) $\frac{2}{11}$

9. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} x & 3x & -2x \\ 2x & 0 & 2x \\ -x & 3x & 2x \end{bmatrix}$, which of the following is TRUE?
(a) $x < -\frac{1}{2}$
(b) $\frac{1}{2} < x < 1$
(c) $x > 1$
(d) $-\frac{1}{2} < x < \frac{1}{2}$
(e) $1 < x < 2$

10. If A and B are $n \times n$ matrices and A^{-1} and B^{-1} exist, then which one of the following is not always true?
(a) $(AB)^T = B^T A^T$
(b) A^{-1} is $n \times n$
(c) $AA^{-1} = I$
(d) $(AB)^{-1} = A^{-1}B^{-1}$
(e) $(A + B)^2 = A^2 + B^2 + AB + BA$

11. If $\begin{bmatrix} -2 & -3 \\ -3 & -4 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 4 & n \\ m & 2 \end{bmatrix}$, then m and n are
(a) $-1, -\frac{5}{3}$
(b) $-3, -\frac{13}{3}$
(c) $2, \frac{7}{3}$
(d) $-3, -3$
(e) $1, 1$

12. If the matrix equation $A^3 = I$ is true and A^{-1} exists, then $A^{-1} =$
(a) A^2
(b) A
(c) A^3
(d) I
(e) A^6

13. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, then the element in the second row and third column of A^{-1} is equal to
(a) 0
(b) 1
(c) -2
(d) 2

(e) -1

14. Given the matrix equation $AXC = B$. If A^{-1} and C^{-1} exist, then $X =$
   (a) $A^{-1}BC^{-1}$
   (b) $BA^{-1}C^{-1}$
   (c) $A^{-1}C^{-1}B$
   (d) $BC^{-1}A^{-1}$
   (e) $C^{-1}BA^{-1}$

19. DETERMINANTS

1. If A is a square matrix with inverse A^{-1} and transpose A^T, then which one of the following is always true?
   (a) $\left|A^T\right| = -\left|A\right|$
   (b) $\left|AA^{-1}\right| = 1$
   (c) $\left|AA^T\right| = 1$
   (d) $\left|A^{-1}\right| = \left|A\right|$
   (e) $\left|A^{-1}\right| = \left|A^T\right|$

2. If $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ 2 & 3 & 4 \end{vmatrix} = -3$,
   then $\begin{vmatrix} 2 & 3 & 4 \\ x-4 & y-6 & z-8 \\ -2 & -2 & -2 \end{vmatrix} =$
   (a) 0
   (b) 6
   (c) -3
   (d) -6
   (e) 3

3. The solution set of $\begin{vmatrix} x & x & 0 \\ 2 & 1+x & 2 \\ -1 & 0 & x \end{vmatrix} = 0$ is
   (a) $\{-1, 2\}$
   (b) $\{0, -1, 2\}$
   (c) $\{0, -1, -2\}$
   (d) $\{-1, -2\}$
   (e) $\left\{0, \frac{1+\sqrt{7}i}{2}\right\}$

4. The value of $\begin{vmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{vmatrix}$ is equal to
   (a) $b^2\left(a^2 - c^2\right)$
   (b) $ad^2 + b^2c^2$
   (c) $(ad - bc)(d - c)$
   (d) $(ad - bc)^2$
   (e) 0

5. If $A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$, $B = \begin{vmatrix} c & -2b & -3a \\ f & -2e & -3d \\ i & -2h & -3g \end{vmatrix}$ and $|A| = 2$, then $|B| =$
   (a) 12
   (b) -12
   (c) 6
   (d) -6
   (e) -2

6. The determinant $\begin{vmatrix} x & 2 & 4 \\ 1 & 3 & 0 \\ 1-2x & -1 & -8 \end{vmatrix} =$
   (a) 1
   (b) -32
   (c) 0
   (d) $-48x$
   (e) $x(1-2x)$

7. If a 3×3 matrix A with elements a_{ij} has $a_{11} = -1$, $a_{21} = 3$, $a_{31} = 4$, and the minors of a_{11}, a_{21}, and a_{31} are 5, -2, and 3 respectively, then $|A| =$
   (a) -4
   (b) 23
   (c) -11
   (d) 1
   (e) 13

8. The determinant $\begin{vmatrix} \sin\theta & -\cos\theta \\ -\sin 2\theta & \cos 2\theta \end{vmatrix} =$
   (a) $\cos 3\theta$
   (b) $-\sin\theta$
   (c) $-\cos\theta$
   (d) $\sin 3\theta$
   (e) $\sin\theta$

9. The cofactor of x in $\begin{vmatrix} -3 & 0 & -1 & 0 \\ 2 & 4 & 6 & 2 \\ 0 & x & -2 & 4 \\ 1 & 3 & 1 & 0 \end{vmatrix}$ is
   (a) $-4x$
   (b) 4
   (c) -4
   (d) $4x$
   (e) -8

10. If $A = \begin{bmatrix} 1 & 2 & 3 \\ a & b & c \\ x & y & z \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 & -6 \\ 3a & 3b & 6c \\ x & y & 2z \end{bmatrix}$,

and $C = \begin{bmatrix} 2 & 4 & 6 \\ 2x & 2y & 2z \\ 2a & 2b & 2c \end{bmatrix}$,
then

(a) $B = -6A, C = 2A$
(b) $|B| = -6|A|, |C| = -8|A|$
(c) $|B| = -6|A|, |C| = -2|A|$
(d) $|B| = 6|A|, |C| = 8|A|$
(e) $|B| = -8|A|, |C| = -2|A|$

11. If A is a 3×3 matrix, then

(a) $|2A| = 8|A|$
(b) $|2A| = 2|A|$
(c) $|2A| = 16|A|$
(d) $|2A| = 4|A|$
(e) $|2A| = 6|A|$

12. The cofactor of the element in the third row and second column of $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 1 & 2 & 6 \end{bmatrix}$ is

(a) 0
(b) 2
(c) -7
(d) -8
(e) -2

13. If $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$, then $|A| =$

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

14. If $B = \begin{bmatrix} 4 & 2 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$, then $\left|A^T B^T\right| =$

(a) 1
(b) -7
(c) -3
(d) 4
(e) 10

15. The solution set of $\begin{vmatrix} 1 & 0 & 0 \\ 0 & x & 1 \\ 0 & 1 & x \end{vmatrix} = 0$ is

(a) $\{1\}$
(b) $\{0,1\}$
(c) $\{-1,0\}$
(d) $\{-1\}$
(e) $\{-1,1\}$

16. Let A and B be 3×3 matrices. Which one of the following is FALSE ?

(a) $(AB)^{-1} = B^{-1}A^{-1}$
(b) $(|A|+1)^2 = |A|^2 + 2|A| + 1$
(c) $\left|A^T\right| = |A|$
(d) $\left|A^{-1}\right| = |A|$
(e) $|3A| = 27|A|$

17. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then M_{21} and C_{13} are

(a) $3, 3$
(b) $3, -3$
(c) $-3, 3$
(d) $6, -9$
(e) $-6, 9$

18. The sum of all values of x for which $\begin{vmatrix} -1 & 3 & 0 \\ 0 & 2 & x \\ 1 & -x & 1 \end{vmatrix} = 0$ is

(a) 2
(b) 1
(c) -1
(d) -2
(e) 3

19. If $\begin{vmatrix} 3 & x & u \\ 3 & y & v \\ 3 & z & w \end{vmatrix} = 1$, then $\begin{vmatrix} x & z & y \\ 2 & 2 & 2 \\ u & w & v \end{vmatrix}$ is equal to

(a) $\frac{2}{3}$
(b) $\frac{3}{2}$
(c) $-\frac{3}{2}$
(d) 1
(e) 0

20. If A is 5×5 and $|A| = 4$, then $2|A| + \left|2A^{-1}\right| =$

(a) 8
(b) $\frac{17}{2}$
(c) 128
(d) 136
(e) 16

21. If $A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then $\left|(AB)^T\right| =$

(a) 26
(b) 45
(c) 130
(d) 85

(e) -25

22. If $\begin{vmatrix} 1 & 0 & 1 \\ 0 & \sin\theta & \cos\theta \\ \sec\theta & -\cos\theta & \sin\theta \end{vmatrix} = 0, 0 \leq \theta \leq \pi$, then $\theta =$

(a) $\frac{3\pi}{4}$
(b) 0
(c) $\frac{\pi}{2}$
(d) $\frac{\pi}{3}$
(e) $\frac{\pi}{4}$

23. The determinant $\begin{vmatrix} 1 & 2 & 3 \\ a & b & c \\ 2+a & 4+b & 6+c \end{vmatrix}$ is

(a) equal to 0 only if $a = b = c = 0$
(b) equal to 0 only if $a = -b$ and $b = -c$
(c) never equal to 0
(d) always equal to 0
(e) equal to zero only if $a = -2, b = -4$, and $c = -6$

24. If A is a 4×4 matrix and $|A| = \frac{3}{2}$, then $\frac{1}{4}\,|-2A| =$

(a) $-\frac{3}{3}$
(b) $-\frac{3}{32}$
(c) $\frac{81}{4}$
(d) -3
(e) 6

25. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, I is the 2×2 identity matrix, then $|A - 3I| =$

(a) 2
(b) 3
(c) 0
(d) -4
(e) -1

26. The minor M_{23} of the element x in the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta & 1 \\ 1 & 1 & x \\ -\sin\theta & \cos\theta & 0 \end{bmatrix}$ is

(a) 1
(b) 0
(c) $\cos 3\theta$
(d) $\sin 3\theta$
(e) $-\cos\theta$

ANSWERS

Question \ Section	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	e	a	b	b	c	c	e	d	b	a	b	e	c	b	e	e	c	c	b
2	a	c	c	e	a	b	e	e	a	e	a	d	c	e	c	a	e	a	d
3	c	a	b	d	c	a	e	d	e	a	e	b	b	b	b	c	e	d	b
4	c	e	d	c	c	a	d	a	d	b	b	e	b	b	e	b	b	d	d
5	d	b	d	b	a	c	e	e	c	c	d	c	c	e	b	b	c	b	b
6	e	b	d	c	e	b	e	a	a	c	c	d	e	e	e	d	b	b	c
7	b	a	a	d	a	e	c	a	a	c	d	b	b	a	b	a	d	d	e
8	b	e	e	e	e	b	c	c	b	e	c	e	c	c	a	a	c	d	b
9	b	c	e	c	b	d	c	c	b	d	c	d	c	e	b	c	e	d	c
10	e	b	a	e	c	c		d	a	e	b	a	e	d	b	a	d	d	b
11	e	d	a	a	e	e		e	d	c	d		a	a	c	e	e	d	a
12	e		d	a		d		a	c	c	e		a	d	c	c		a	c
13	b		d	c		b		b	d	d	b		e	e	b	e		e	a
14	c		d	c		b		b	e	e	a			d	e			a	c
15	b		b	a		e		e	a	a	c			d	e				e
16	d		c	d		c		d	e	d	a			e	c				d
17	a		a	d		d		e	c	e	e			d	d				b
18	a		c	d		d		c	c	e	b			e	a				e
19	e		c	b		d		e	b	c	c			e	e				a
20	e		e	b		c		b		e	b			b	e				e
21	b		e	e		e		a		d	d			d	a				d
22	b		d	a		d		d		c	e			d	e				e
23	c			b		b		a		d	a			c	c				d
24	b			e		d		a		b	c			d	b				e
25	c			b				b		c	c								d
26	a			d				d		e	c								c
27	c			b				b		b	a								
28	b			e				c		a	c								
29	d			b				d		e	d								
30	a			e				b		b									
31	c			d						e									
32	d			a						d									
33	c			b						d									
34	c			a						a									
35	c			d						e									
36				e						b									
37				e						a									
38				d						c									
39				a						a									
40				d						a									
41				b						e									
42				c						a									
43				a						c									
44				c						a									

Section 4 (Continued)

45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
D	c	b	b	d	a	b	c	c	d	e	d	a	d	b	c
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76
E	a	e	a	b	c	d	e	e	d	c	a	b	e	c	b