Dimensional Analysis

The word *dimension* has a special meaning in physics. It denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or fathoms, it is still a distance. We say its dimension is *length.*

The symbols we use in this book to specify the dimensions of length, mass, and time are L, M, and T, respectively [the *dimensions* of a quantity will be symbolized by a capitalized, non-italic letter, such as L. The *symbol* for the quantity itself will be italicized, such as *L* for the length of an object, or *t* for time]. We shall often use brackets [1 to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this hook is *v*, and in our notation the dimensions of speed are written *[v]* = L/T. As another example, the dimensions of area *A* are *[A]* L^2. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1.1. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text.

In many situations, you may have to derive or check a specific equation. A useful and powerful procedure called *dimensional anal sis* can be used to assist in the derivation or to check your final expression. Dimensional analysis makes use of the fact that

Table 1.1 Units of Area, Volume, Speed, and Acceleration

	Area	**Volume**	**Speed**	**Acceleration**
System	**(L^2)**	**(L^3)**	**(L/T)**	**(L/T2)**
SI	**m^2**	**m^3**	**m/s**	**m/s^2**
SI	**ft^2**	**ft^3**	**ft/s**	**ft/s^2**

dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to help determine whether an expression has the correct form. The relationship can be correct only if the dimensions on both sides of the equation are the same.

To illustrate this procedure, suppose you wish to derive an equation for the position *x* of a car at a time *t* if the car starts from rest and moves with constant acceleration a. In Chapter 2, we shall find that the correct expression is $x = \frac{1}{2}at^2$. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2 (Table 1.1), and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$

$$L = \frac{L}{\not{T}^2} \bullet \not{T}^2 = L$$

The dimensions of time cancel as shown, leaving the dimension of length on the right- hand side.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \alpha a^n t^m$$

where *n* and *m* are exponents that must be determined and the symbol α **indicates a** proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = \mathbf{L} = L^1 J^0$$

Because the dimensions of acceleration are **L/T^2 and the dimension** of time is T, we have

$$(L/T^2)^n T^m = L^1 J^0$$

$$(L^n T^{m-2n}) = L^1 J^0$$

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that $n = 1$. From the exponents of T, we see that $m - 2n = 0$, which, once we substitute for *n*, gives us $m = 2$. Returning to our original expression $x \alpha a^n t^m$, we conclude that $x \alpha a t^2$. This result differs by a factor of $\frac{1}{2}$ from the correct expression, which is $x = \frac{1}{2} a t^2$.

Quick Quiz 1 .2 True or False: Dimensional analysis can give you the numerical value of constant of proportionality that may appear in an algebraic expression.

1.4 Symbols for Quantities

Some quantities have a small number of symbols that represent them. For example, the symbol for time is almost always t. Others quantities might have various symbols depending on the usage. Length may be described with symbols such as x, y, and *z* (for position), *r* (for radius), a, *b*, and *c* (for the legs of a right triangle), *l* (for the length of an object), *d* (for a distance), *h* (for height), etc.

Example 1.2 Analysis of an Equation

Show that the expression $v = at$ is dimensionally correct, where v represents speed, a acceleration, and t an instant of time

Solution For the speed term, we have from Table 1.6

$$[v] = \frac{L}{T}$$

The same table gives us L T2 for the dimensions of acceleration, and so the dimensions of at are

$$[at] = \frac{L}{T^{2}} T = \frac{L}{T}$$

Therefore, the expression is dimensionally correct. (If the expression were given as $v = at^2$ it would be dimensionally *incorrect.* Try it and see!)

Example 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration.

Solution Let us take a to be

$$a=kr^{n}v^{m}$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r and v, we see that the dimensional equation must be

$$\frac{L}{T^{2}} = L^{n}\left(\frac{L}{T}\right)^{m} = \frac{L^{n+m}}{T^{m}}$$

This dimensional equation is balanced under the conditions

$$n + m = 1 \text{ and } m = 2$$

Therefore n=-1, and we can write the acceleration expression as

$$a = kr^{-1} v^{2} = k\frac{v^{2}}{r}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if , for example, v were in km/h and you wanted a in rn/s^2.