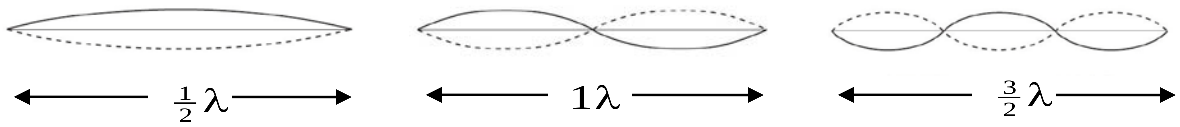
RESONANCE IN STRINGS

INTRODUCTION

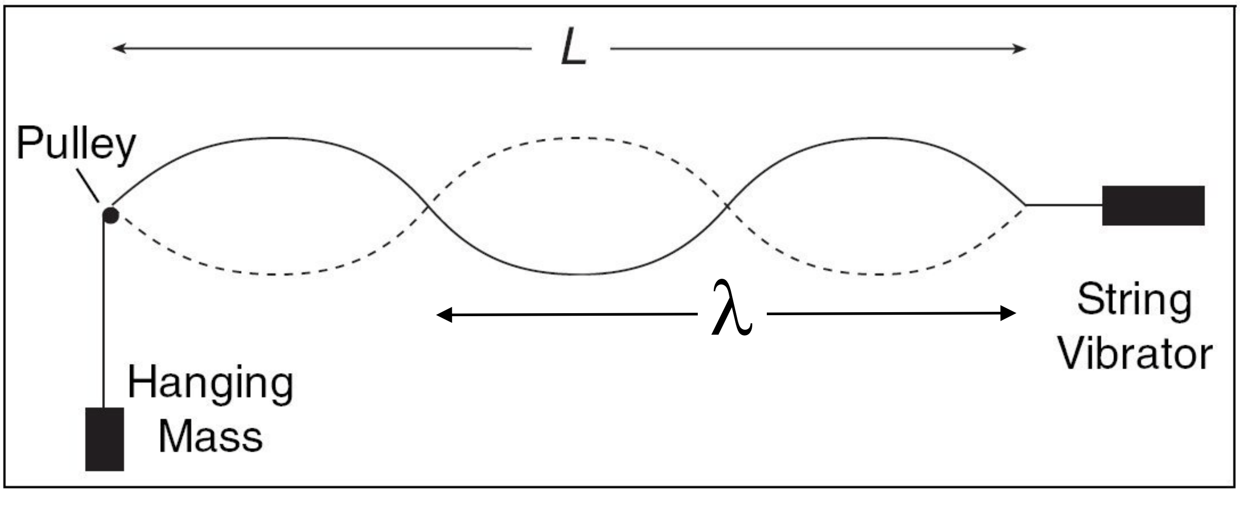
A sine wave generator drives a string vibrator to create a standing wave pattern in a stretched string. The driving frequency and the length, density, and tension of the string are varied to investigate their effect on the speed of the wave in the vibrating string.

THEORY

A stretched string has many natural modes of vibration (three examples are shown below). If the string is fixed at both ends then there must be a node (place of no amplitude) at each end and at least one anti-node (place of maximum amplitude). It may vibrate as a single segment, in which case the length (L) of the string is equal to 1/2 the wavelength (λ) of the wave. It may also vibrate in two segments with a node at each end and one node in the middle; then the wavelength is equal to the length of the string. It may also vibrate with a larger integer number of segments. In every case, the length of the string equals some integer number of half wavelengths.



If you drive a stretched string at an arbitrary frequency, you will probably not see any particular mode: Many modes will be mixed together. But, if the driving frequency, the tension and the length are adjusted correctly, one vibrational mode will occur at much greater amplitude than the other modes.
In this experiment, standing waves are set up in a stretched string by the vibrations of an electrically-driven String Vibrator. The arrangement of the apparatus is shown below. The tension in the string equals the weight of the masses suspended over the pulley. You can alter the tension by changing the masses. You can adjust the amplitude and frequency of the wave by adjusting the output of the Sine Wave Generator, which powers the string vibrator.



L is the length of the vibrating part of the string and λ is the wavelength of the wave. For the string shown above vibrating in 3 segments, $\lambda = \frac{2}{3} L$.

For any wave the product of its wavelength λ with the frequency f, λf, gives the speed of the wave, v.

$v = \lambda f$ (1)

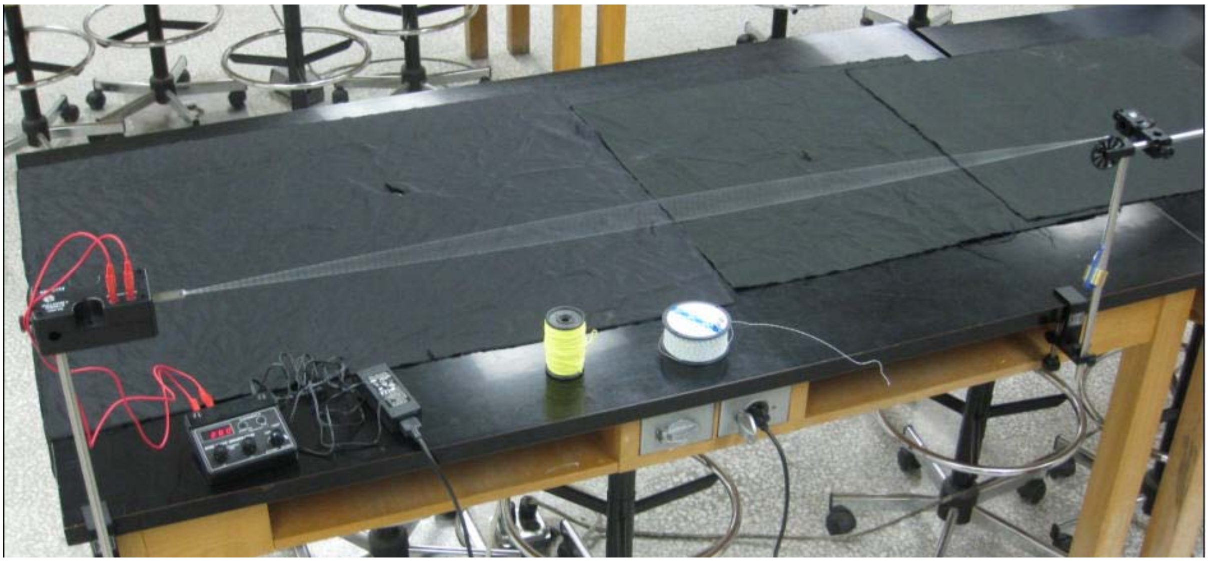
However, the speed of a wave in a given medium depends only on the physical properties of the medium and in the case of a uniform string it is given by

$$v = \sqrt{\frac{T}{\mu}} \qquad (2)$$

where μ is the linear density (the mass per unit length) of the string and T is the tension in the string. The Tension (T) is applied by the hanging a mass (m), and is equal to the weight (mg) of the hanging mass, i.e. T = mg.

SETUP

1. You are provided with a long piece of braided string with length L_1 = 4.00 m, another piece of braided string about 1.5 m long and a piece of yellow cord about 1.5 m long. For what follows here use the braided strings, yellow cord is used later in the *further investigations*.
2. Measure the mass of the long braided string (L_1=4.00 m) using the triple beam balance and calculate the linear density, μ (mass/length).
3. As shown in the picture, use the two clamps to position the Sine Wave Generator and pulley about 120 cm apart. Attach the braided string (~ 1.5 m long) to the vibrating blade, run it over the pulley, and hang about 150 g of mass from it.



4. Measure from the knot where the string attaches to the string vibrator to the top of the pulley. This is distance L. (Note that L is not the total length of the string, only the part that is vibrating.)

5. Turn on the Sine Wave Generator and turn the Amplitude knob all the way down (counter-clockwise). Connect the Sine Wave Generator to the string vibrator using two banana patch cords. Polarity does not matter.

PROCEDURE

<u>Part I:</u>

1. Set the Amplitude knob about midway. Use the Coarse (1.0) and Fine (0.1) Frequency knobs of the Sine Wave Generator to adjust the vibrations so that the string vibrates in *one* segment. Adjust the driving amplitude and frequency to obtain a large-amplitude wave, but also check the end of the vibrating blade: <u>The point where the string attaches should be a node</u> (see the picture above). You may want to decrease the driving amplitude to be able to get the right resonance frequency with the node at the <u>end of the vibrating blade</u> than away from it. It is more important to have a good node at the blade than it is to have the largest amplitude possible. However, it is desirable to have a large amplitude while keeping a good node.

2. Record the frequency. How much uncertainty is there in this value? How much can you change the frequency before you see an effect?

3. Repeat steps 1 and 2 for a standing wave with *two* segments. The string should vibrate with a node at each end and one node in the center. Do not change the hanging mass.

4. How is the frequency of the two-segment wave related to the frequency of the one-segment wave? Calculate the ratio of the frequencies. Is the ratio what you would expect?

5. With the wave vibrating in two segments, the length of the string, L, is one wavelength ($L = \lambda$). Does it look like one wavelength? Since the string vibrates up and down so fast, it is hard to see that when one side is up, the other is down. Try touching the string at an anti-node. What happens? Try touching the string at the central node. Can you hold the string at the node and not significantly affect the vibration?

6. What was the wavelength when the string was vibrating in one segment? Use Equation 1 to calculate the speed of the one-segment wave. Calculate the speed of the two-segment wave. How do these two values compare? Are they *about* the same (find the percentage difference between the two values to justify your answer)? Why?

7. Calculate the tension in the string caused by the hanging mass (don't forget the mass hanger) and using your measured density of the string, calculate the speed of the wave using equation (2). How does this compare with the speed calculated (for the two segment wave) in step 6?

8. Adjust the frequency so that the string vibrates in *three* segments. What is the velocity now? Has it changed? Does the speed of the wave depend on the wavelength and the frequency?

9. Set the frequency to a value between the frequencies that produced waves of two and three segments. Adjust the frequency so that no particular standing waveform is

present. Unclamp the string vibrator on the table and slowly move it towards the pulley. (Do not let go of the string vibrator without clamping it to the table again.). Without changing the driving frequency or the hanging mass decrease the length of vibrating string until it vibrates in *two* segments. Measure the new wavelength and calculate the speed of the wave. Is it about the same as before? Does the speed of the wave depend on the length of the string?

Part II:

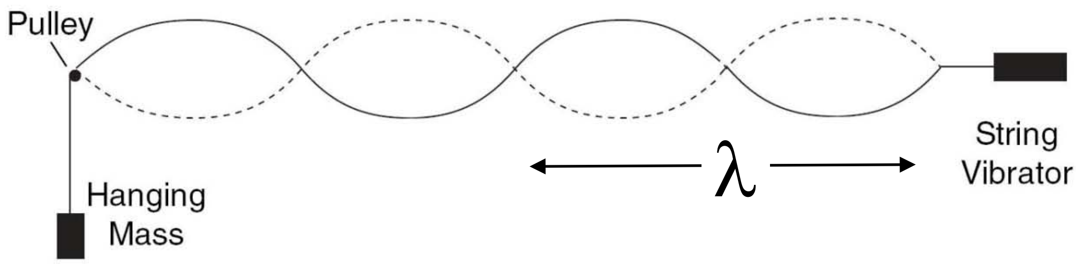
1. Clamp the String Vibrator back at its original position, about 120 cm from the pulley. You should re-measure the length, L. Hang about 50 g from the string over the pulley. Record the total hanging mass, including the mass hanger.
2. Adjust the frequency of the Sine Wave Generator so that the string vibrates in *four* segments. As before, adjust the driving amplitude and frequency to obtain a large-amplitude wave, *and* clean nodes, including the node at the end of the blade. Record the frequency.

Note: For this part of the experiment, you will always adjust the frequency so that the wave vibrates in *four* segments.

3. Add 50 g to the hanging mass and repeat steps 1 and 2.
4. Repeat at intervals of 50 g up to at least 250 g. Record your data in a table.

ANALYSIS

1. For this part of the experiment, you always adjusted the frequency so that the wave vibrated in *four* segments, and thus the length of the string was always equal to two wavelengths ($L = 2\lambda$).



Use this information to show that equations (1) and (2) can be combined to yield

$$f^2 = \frac{4g}{\mu L^2} m \qquad (3)$$

where: f = driving frequency of the Sine Wave Generator
g = acceleration due to gravity

m = total hanging mass
L = length of string (vibrating part only)
μ = linear density of the string (mass/length)

2. Make a graph of the square of the frequency (f^2) versus hanging mass, *m*. (The units will be easier to work with later if you graph the mass in kilograms.) Is the graph linear?
3. Find the slope of the best-fit line through this data.
4. As you can determine from Equation *3*, the slope of the f^2 vs. *m* graph is:

$$\text{slope} = \frac{4g}{\mu L^2}$$

From the slope of your graph, calculate the density (μ) of the string.

5. Compare the density that you measured from the graph to the *actual* (direct measurement) density that you determined when you weighed the string. Calculate

$$\%\ \text{Deviation} = \frac{\text{Measured} - \text{Actual}}{\text{Actual}} \times 100\%$$

the percent deviation.

FURTHER INVESTIGATIONS

1. Repeat the procedure (for Part II only) using the yellow cord. Make sure you use the same length L as for the braided string. Put the data from the string and cord on the same graph to show the difference in their densities.

CONCLUSION

Summarize the quantities that affect the speed of a wave in a string: Consider the number of segments, the frequency, the tension in the string, and the length and density of the string.