KFUPM
Department of Physics
Phys530

Homework # 1

Due on Saturday, September 30, 2006

Problem 1

Stirling's formula is widely used in statistical mechanics. According to this formula for n >>1

$$\ln n! \approx n\ln - n.$$

Check the accuracy of this formula by comparing the two sides of the formula for n = 10, 10^2, 10^3, 10^4, 10^5, and 10^6.

Problem 2

Consider a system of a single particle of mass m confined to a cubical box of volume V.

(a) Write a computer code to find for integer values of ε* between 200 and 300 the number of microstates Ω(ε*) consistent with macrostates of the system having well defined energy ε. ε* is defined as $\varepsilon^* \equiv \frac{8mV^{2/3}\varepsilon}{h^2}$, where h is the Planck's constant.

(b) Plot Ω(ε*) versus ε* for the values you find in (a). Comment on the behavior of Ω(ε*).

(c) Modify your code to find the number of microstates Σ(ε*) consistent with all macrostates of the system with energies less than ε. Evaluate Σ(ε*) for values of ε* between 200 and 300 twice; in one time, include lattice points with coordinates equal to zero, and, in the second time, do not include them.

(d) Plot Σ(ε*) versus ε* for the values you find in (c) and eqs. 14, 15, 16 on page 18 of your textbook. Comment on the behavior of Σ(ε*).and compare your result with Figure 1.2 of your textbook.

Problem 3

Problem 1.9 from your textbook (Pathria).

Problem 4

Consider a system of N non-interacting particles. The energy of any particle can assume one of the two fixed energy values + ε or – ε.

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(a) What is the number of microstates of the system expressed as a function of N and the total energy of the system E?

(b) For the case N>>1 and $E/\varepsilon \ll N$, show that the distribution of the microstates is a Gaussian function of E. What is the width of this distribution?

(c) Suppose you have two isolated systems of these particles in equilibrium, one with $N_1=10$ and $E_1 = 6\ \varepsilon$ and the other with $N_2 = 20$ and $E_2 = -2\ \varepsilon$. Suppose you allow these systems to exchange energy with each other. Based on your physical intuition, guess the new equilibrium value of E_1. Give a simple argument to support your expectation.

(d) Plot the number of microstates of the composite system as a function of E_1 and verify that it peaks at the position you guessed in (d)

(e) With the help of the formula you obtained in (b), what is the width of this distribution of the microstates of the composite system as a function E_1? Comment on the width as you increase N_2