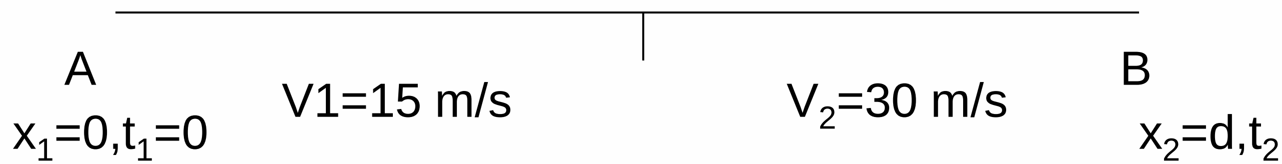
Selected Problems
from
Chapter 2

1) A car travels in a straight road with a velocity of v1=15 m/s for half the distance between two cities and with a velocity v2=30 m/s for the other half. What is the average velocity of the car for the entire trip?

A1	20.0 m/s
A2	22.5 m/s
A3	25.0 m/s
A4	18.5 m/s
A5	24.0 m/s



$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \quad (1)$$

$$x_1 = 0, t_1 = 0$$

$$let \;\; x_2 = d$$

$$t_2 = \frac{d/2}{15} + \frac{d/2}{30} = \frac{3d}{60}$$

$$Subsitute \text{ in } (1) v_{avg} = \frac{d-0}{3d/60} = \frac{60}{3} = 20 \; m/s$$

2) An object moving along the x axis has a position given by x = (3 t - t**3) m, where t is measured in s. What is the acceleration of the object when its velocity is zero?

A1 -6.0 m/s**2
A2 Zero
A3 4.0 m/s**2
A4 -3.5 m/s**2
A5 3.5 m/s**2

$$v = \frac{dx}{dt} = 3 - 3t^2 \Rightarrow$$

$$v = 0 = 3 - 3t^2 \quad \Rightarrow t^2 = 1$$

$$\Rightarrow t = \pm 1\, s$$

$$a = \frac{dv}{dt} = 0 - 6t = -6t$$

$$a(+1) = -6(1) = -6\ m/s^2$$

3) A particle moving with a constant acceleration has a velocity of 10 cm/s when its position is x0 =10 cm. Its position 4.0 s later is x= -14 cm. What is the acceleration of the particle?

A1 -8.0 cm/s**2
A2 -5.5 cm/s**2
A3 5.5 cm/s**2
A4 8.4 cm/s**2
A5 -2.0 cm/s**2

$$v_0 = 10\ \text{cm/s};\ \text{x}_0 = 10\,cm;\ t = 4.0s\ ,x = -14\,cm$$

$$a = ?$$

$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2$$

$$-14 - 10 = 10(4.0) - \tfrac{1}{2} a (4.0)^2 \Rightarrow a = \frac{2}{16} \times (-64)$$

$$a = -8.0\,cm/s^2$$

4) A stone is thrown vertically upward such that it has a speed of 9.0 m/s when it reaches one half of its maximum height above the launch point. Determine the maximum height.

A1 8.3 m
A2 2.8 m
A3 5.3 m
A4 6.5 m
A5 17 m

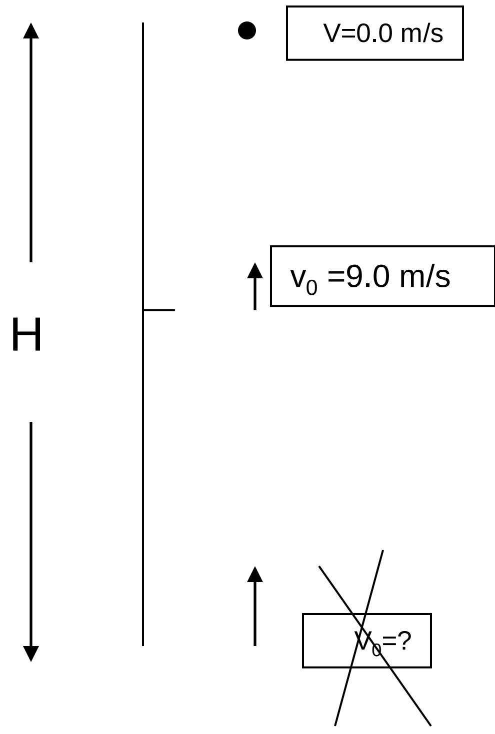
$$v_0 = 9.0\, m/s\ ; v = 0.0\, m/s$$

$$y - y_0 = \tfrac{1}{2} H\ ; a = -g = -9.80\, m/s^2$$

$$v^2 = v_0^2 + 2a(y - y_0)$$

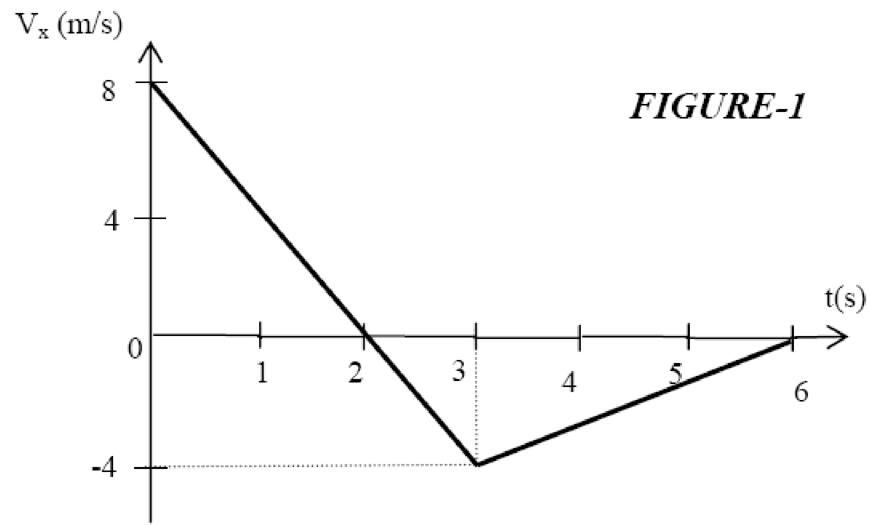
$$0 = v_0^2 - 2g(\tfrac{1}{2} H)$$

$$H = \frac{v_0^2}{g} = \frac{9.0^2}{9.8} = 8.2653.... \approx 8.3\, m$$



5) Fig (1) shows the velocity (Vx) of a particle moving along x axis as a function of time (t). What is the acceleration of the particle at t= 2.0 s?

A1 -4 m/s**2
A2 +4 m/s**2
A3 -1 m/s**2
A4 +1 m/s**2
A5 0 m/s**2



$$a = slope\,(2.0s) = -\frac{8}{2} = -4\ m/s^2$$

6) A particle moving along the x axis has a position given by

x =(24 t - 2 t**3) meters,

where t is measured in seconds. How far is the particle from the origin (x=0) when the particle stops momentarily?

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A1 32 m

A2 23 m

A3 40 m

A4 17 m

A5 98 m

$$find\ x\ when\ v = 0$$

$$v = \frac{dx}{dt} = 24 - 6t^2 = 0$$

$$t^2 = 4 \Rightarrow t = 2.0\ s$$

$$x(2s) = (24 \times 2 - 2(2.0)^3) = 48 - 16 = 32\ m$$

7) In 2.0 seconds, a particle moving with constant acceleration along the x axis goes from x=10 m to x=50 m. The velocity at the end of this time interval is 10 m/s. What is the acceleration of the particle?

A1 -10 m/s**2
A2 +15 m/s**2
A3 -15 m/s**2
A4 +20 m/s**2
A5 -20 m/s**2

$$t = 2.0\,s;\, x_0 = 10\,m;\, x = 50\,m;\, v = 10\,m/s$$

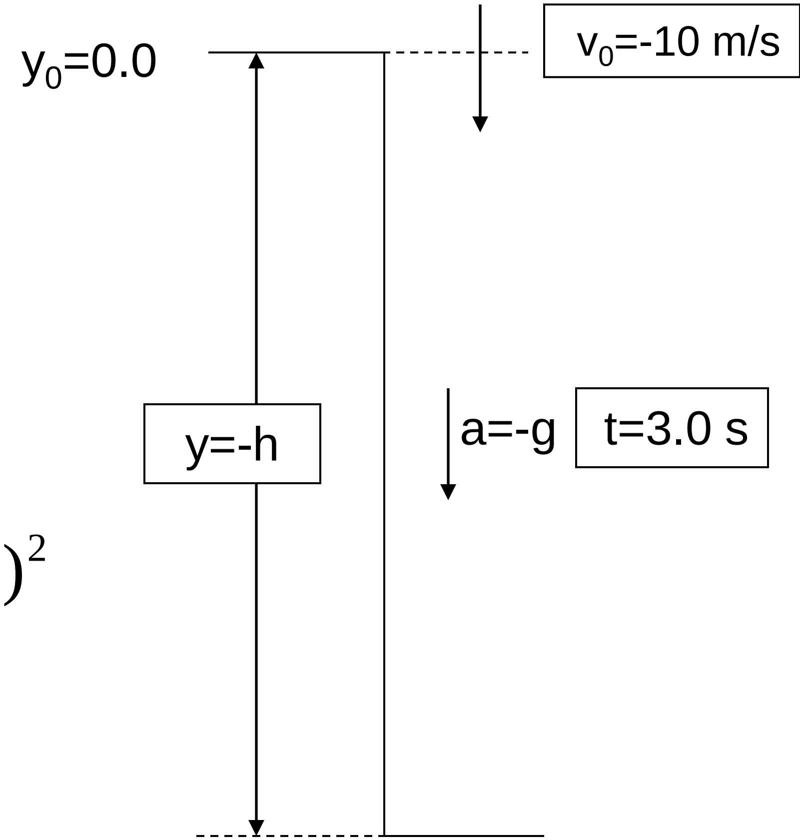
$$a = ?$$

$$x - x_0 = vt - \tfrac{1}{2}at^2$$

$$50 - 10 = 10(2) - \tfrac{1}{2}a(2.0)^2 \Rightarrow a = -\frac{40-20}{2} = -10\,m/s^2$$

8) A stone is thrown downward from height (h) above the ground with an initial speed of 10 m/s. It strikes the ground 3.0 seconds later. Determine h.

A1 74 m
A2 44 m
A3 14 m
A4 90 m
A5 60 m



$$y - y_0 = v_0 t + \tfrac{1}{2} a t^2$$

$$-h = (-10)(3.0) - \tfrac{1}{2}(9.80)(3.0)^2$$

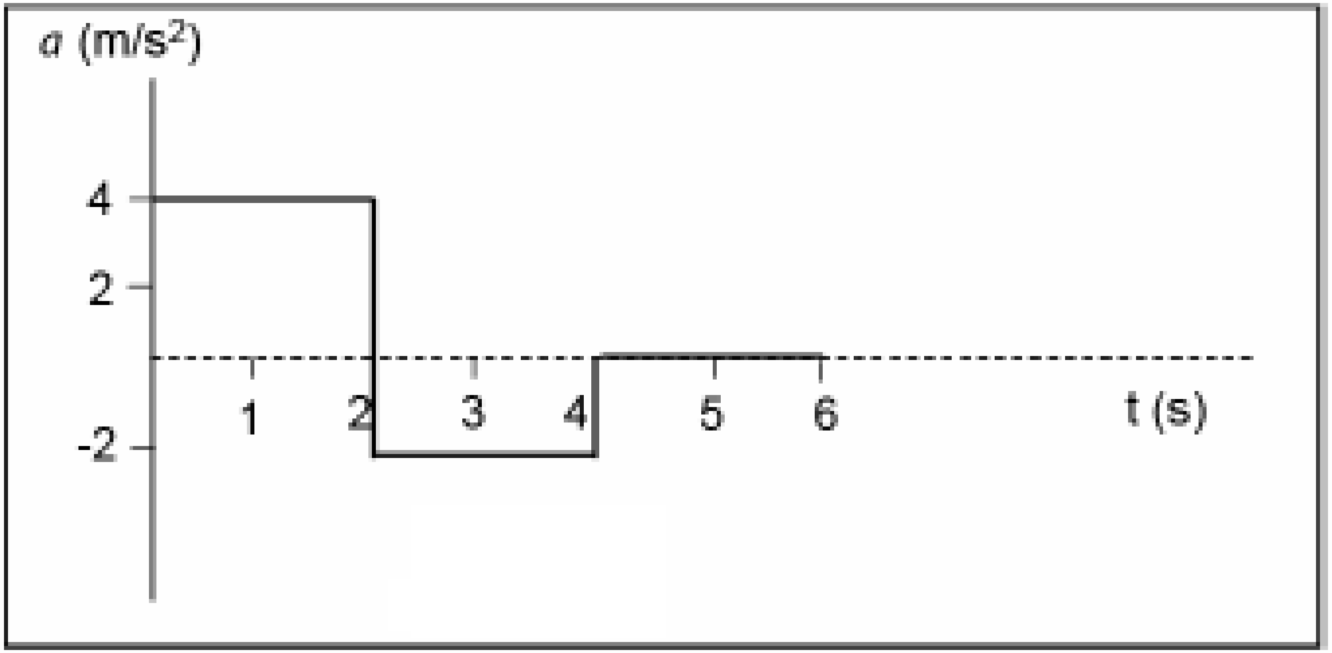
$$-h = -30 - 44.1 = -74$$

$$h = 74\ m$$

9) A helicopter at height h (m) from the surface of the sea is descending at a CONSTANT SPEED v (m/s). The time it takes to reach the surface of the sea can be found from:

A) $-h = -v\,t$
B) $h = \frac{1}{2}\,g\,t^2$
C) $-h = \frac{1}{2}\,g\,t^2$
D) $h = v\,t - \frac{1}{2}\,g\,t^2$
E) $-h = -v\,t - \frac{1}{2}\,g\,t^2$

10) A particle starts from rest at t = 0 s. Its acceleration as a function of time is shown in Fig. What is its speed at the end of the 6.0 s?



A) 4.0 m/s
B) 0 m/s
C) 12 m/s
D) 2.0 m/s
E) -12 m/s

$$v = \int a\,dx = area\ under\ the\ curve$$

$$= 4.0 \times 2.0 - 2.0 \times 2.0 + 0 = 8.0 - 4.0 = 4.0\ m/s$$

11) The position of a particle x(t) as a function of time (t) is described by the equation: $x(t) = 2.0 + 3.0\,t - t^3$, where x is in m and t is in s. What is the maximum positive position of the particle on the x axis?

A) 4.0 m
B) 2.0 m
C) 3.0 m
D) 1.0 m
E) 5.0 m

$$\frac{dx}{dt} = 0 = 3.0 - 3t^2 \Rightarrow t^2 = 1.0 \Rightarrow t = 1.0\,s$$

$$x(1) = 2.0 + 3.0(1.0) - (1.0)^2 = +4.0\ m$$

•3 An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during this 80 km trip? (Assume that it moves in the positive x direction.) (b) What is the average speed? (c) Graph x versus t and indicate how the average velocity is found on the graph. SSM WWW

3. We use Eq. 2-2 and Eq. 2-3. During a time t_c when the velocity remains a positive constant, speed is equivalent to velocity, and distance is equivalent to displacement, with $\Delta x = v\, t_c$.

(a) During the first part of the motion, the displacement is Δx_1 = 40 km and the time interval is

$$t_1 = \frac{(40\text{ km})}{(30\text{ km/h})} = 1.33\text{ h}.$$

During the second part the displacement is Δx_2 = 40 km and the time interval is

$$t_2 = \frac{(40\text{ km})}{(60\text{ km/h})} = 0.67\text{ h}.$$

Both displacements are in the same direction, so the total displacement is

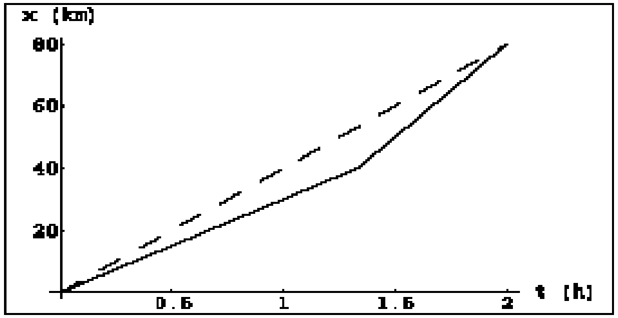
$$\Delta x = \Delta x_1 + \Delta x_2 = 40\text{ km} + 40\text{ km} = 80\text{ km}.$$

The total time for the trip is $t = t_1 + t_2$ = 2.00 h. Consequently, the average velocity is

$$v_{avg} = \frac{(80\text{ km})}{(2.0\text{ h})} = 40\text{ km/h}.$$

(b) In this example, the numerical result for the average speed is the same as the average velocity 40 km/h.

(c) As shown below, the graph consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to (t_1, x_1) = (1.33 h, 40 km) and the second having a slope of 60 km/h and connecting (t_1, x_1) to (t, x) = (2.00 h, 80 km). From the graphical point of view , the slope of the dashed line drawn from the origin to (t, x) represents the average velocity.



•4 A car travels up a hill at a constant speed of 40 km/h and returns down the hill at a constant speed of 60 km/h. Calculate the average speed for the round trip.

4. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion) we have speed = D/t. Thus, the average speed is

$$\frac{D_{up} + D_{down}}{t_{up} + t_{down}} = \frac{2D}{\frac{D}{v_{up}} + \frac{D}{v_{down}}}$$

which, after canceling D and plugging in $v_{up} = 40$ km/h and $v_{down} = 60$ km/h, yields 48 km/h for the average speed.

•12 (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where t is in seconds and x is in meters), what is its velocity at $t = 1$ s? (b) Is it moving in the positive or negative direction of x just then? (c) What is its speed just then? (d) Is the speed increasing or decreasing just then? (Try answering the next two questions without further calculation.) (e) Is there ever an instant when the velocity is zero? If so, give the time t; if not, answer no. (f) Is there a time after $t = 3$ s when the particle is moving in the negative direction of x? If so, give the time t; if not, answer no.

12. We use Eq. 2-4. to solve the problem.

(a) The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(4 - 12t + 3t^2\right) = -12 + 6t.$$

Thus, at $t = 1$ s, the velocity is $v = (-12 + (6)(1)) = -6$ m/s.

(b) Since $v < 0$, it is moving in the negative x direction at $t = 1$ s.

(c) At $t = 1$ s, the *speed* is $|v| = 6$ m/s.

(d) For $0 < t < 2$ s, $|v|$ decreases until it vanishes. For $2 < t < 3$ s, $|v|$ increases from zero to the value it had in part (c). Then, $|v|$ is larger than that value for $t > 3$ s.

(e) Yes, since v smoothly changes from negative values (consider the $t = 1$ result) to positive (note that as $t \to +\infty$, we have $v \to +\infty$). One can check that $v = 0$ when $t = 2$ s.

(f) No. In fact, from $v = -12 + 6t$, we know that $v > 0$ for $t > 2$ s.

•14 (a) If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds, when, if ever, is the particle's velocity zero? (b) When is its acceleration a zero? (c) For what time range (positive or negative) is a negative? (d) Positive? (e) Graph $x(t)$, $v(t)$, and $a(t)$.

14. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

with SI units understood. These expressions are used in the parts that follow.

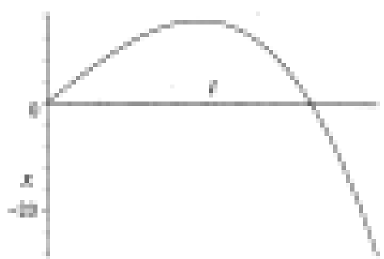
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

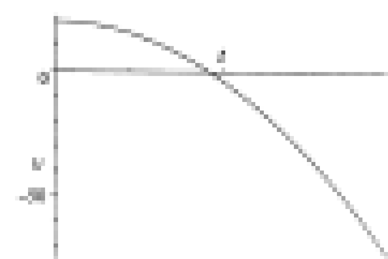
(b) From $0 = -30t$, we find $a(0) = 0$ (that is, it vanishes at $t = 0$).

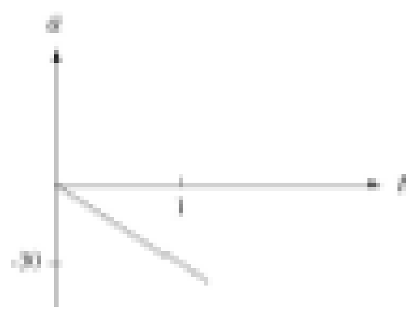
(c) It is clear that $a(t) = -30t$ is negative for $t > 0$

(d) The acceleration $a(t) = -30t$ is positive for $t < 0$.

(e) The graphs are shown below. SI units are understood.







•24 An electric vehicle starts from rest and accelerates at a rate of 2.0 m/s^2 in a straight line until it reaches a speed of 20 m/s. The vehicle then slows at a constant rate of 1.0 m/s^2 until it stops. (a) How much time elapses from start to stop? (b) How far does the vehicle travel from start to stop?

24. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given $v_0 = 0$; $v = 20$ m/s and $a = 2.0$ m/s^2. In part 2, the vehicle decelerates from its highest speed to a halt; we are given $v_0 = 20$ m/s; $v = 0$ and $a = -1.0$ m/s^2 (negative because the acceleration vector points opposite to the direction of motion).

(a) From Table 2-1, we find t_1 (the duration of part 1) from $v = v_0 + at$. In this way, $20 = 0 + 2.0t_1$ yields $t_1 = 10$ s. We obtain the duration t_2 of part 2 from the same equation. Thus, $0 = 20 + (-1.0)t_2$ leads to $t_2 = 20$ s, and the total is $t = t_1 + t_2 = 30$ s.

(b) For part 1, taking $x_0 = 0$, we use the equation $v^2 = v_0^2 + 2a(x - x_0)$ from Table 2-1 and find

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(20)^2 - (0)^2}{2(2.0)} = 100 \text{ m}.$$

This position is then the *initial* position for part 2, so that when the same equation is used in part 2 we obtain

$$x - 100 = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (20)^2}{2(-1.0)}.$$

Thus, the final position is $x = 300$ m. That this is also the total distance traveled should be evident (the vehicle did not "backtrack" or reverse its direction of motion).

•43 A hot-air balloon is ascending at the rate of 12 m/s and is 80 m above the ground when a package is dropped over the side. (a) How long does the package take to reach the ground? (b) With what speed does it hit the ground? SSM

43. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. We are placing the coordinate origin on the ground. We note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12$ m/s and that its initial coordinate is $y_0 = +80$ m.

(a) We solve $y = y_0 + v_0 t - \frac{1}{2} g t^2$ for time, with $y = 0$, using the quadratic formula (choosing the positive root to yield a positive value for t).

$$t = \frac{v_0 + \sqrt{v_0^2 + 2 g y_0}}{g} = \frac{12 + \sqrt{12^2 + 2(9.8)(80)}}{9.8} = 5.4 \text{ s}$$

(b) If we wish to avoid using the result from part (a), we could use Eq. 2-16, but if that is not a concern, then a variety of formulas from Table 2-1 can be used. For instance, Eq. 2-11 leads to

$$v = v_0 - g t = 12 - (9.8)(5.4) = -41 \text{ m/s}.$$

Its final *speed* is 41 m/s.

**61 How far does the runner whose velocity–time graph is shown in Fig. 2-30 travel in 16 s? ILW

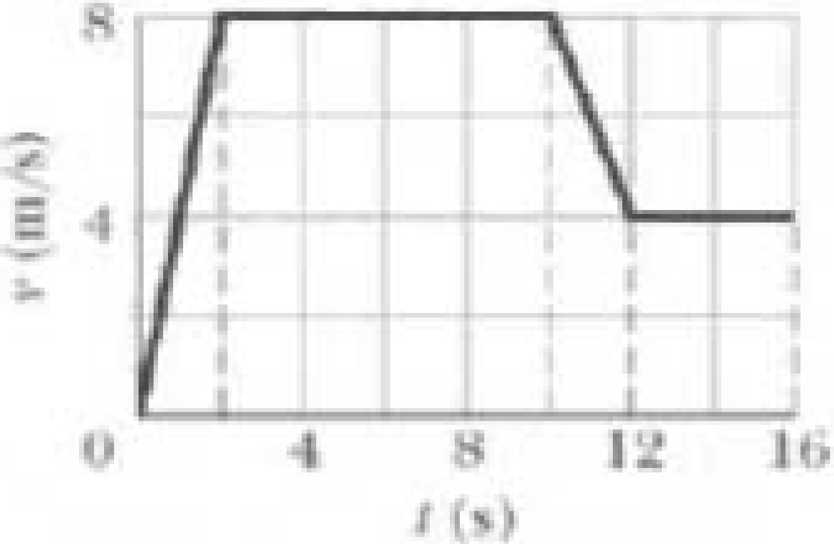


Fig. 2-30 Problem 61.

61. Since $v = \frac{dx}{dt}$ (Eq. 2-4), then $\Delta x = \int v\,dt$, which corresponds to the area under the v vs t graph. Dividing the total area A into rectangular (base × height) and triangular $\left(\frac{1}{2}\text{base} \times \text{height}\right)$ areas, we have

$$\begin{aligned} A &= A_{0<t<2} + A_{2<t<10} + A_{10<t<12} + A_{12<t<16} \\ &= \frac{1}{2}(2)(8) + (8)(8) + \left((2)(4) + \frac{1}{2}(2)(4)\right) + (4)(4) \end{aligned}$$

with SI units understood. In this way, we obtain $\Delta x = 100$ m.

75 The acceleration of a particle along an x axis is $a = 5.0t$, with t in seconds and a in meters per second squared. At $t = 2.0$ s, its velocity is $+17$ m/s. What is its velocity at $t = 4.0$ s?

75. Integrating (from $t = 2$ s to variable $t = 4$ s) the acceleration to get the velocity (and using the velocity datum mentioned in the problem, leads to

$$v = 17 + \frac{1}{2}(5)(4^2 - 2^2) = 47 \text{ m/s.}$$