Selected Problems from Chapter 3

1) The angle between the two vectors A = 2 i + 4 j and B = 4 i - 2 j is:

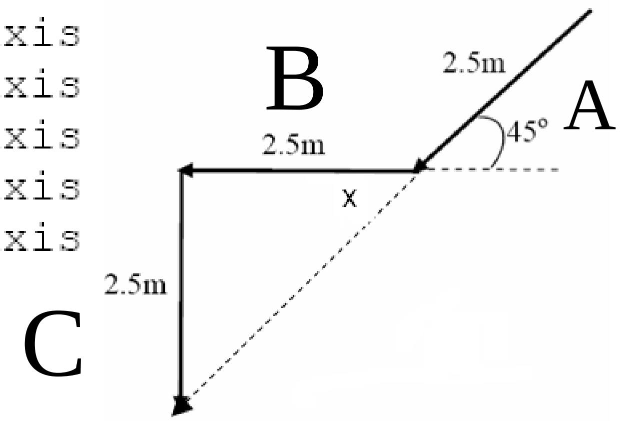
A1 90 degrees
A2 27 degrees
A3 39 degrees
A4 180 degrees
A5 0 degrees

$$A.B = (2i + 4j).(4i - 2j) = 8 - 8 = 0$$

$$\cos\Phi = \frac{A.B}{AB} = 0 \Rightarrow \Phi = 90^o$$

2) As shown in Fig. , a block moves down on a 45-degree inclined plane of 2.5 m length, then horizontally for another 2.5 m, and then falls down vertically a height of 2.5 m. Find the magnitude and direction of the resultant displacement vector of the block.

A1 6.0 m and 45 degrees below horizontal axis
A2 3.5 m and 30 degrees below horizontal axis
A3 6.0 m and 30 degrees below horizontal axis
A4 3.5 m and 45 degrees below horizontal axis
A5 5.5 m and 60 degrees below horizontal axis



Let's call the dissplacements A, B and C respectively.

The resultant $R_{B,C} = B + C$

where $|R_{B,C}| = |B + C| = \sqrt{2.5^2 + 2.5^2} = \sqrt{2 \times 2.5^2} = 2.5\sqrt{2} \approx 3.5m$

$R_{B,C}$ makes an angle x =45 with B so it is along A

the resultant $R = A + R_{B,C} = A + B + C$

$R = 2.5 + 3.5 = 6.0\ m$ in the direction of A

3) Given the vectors A = 3 j + 6 k , B = 15 i + 21 k. Find the magnitude of vector C that satisfies equation 2A + 3C - B = 0.

A1 6.16
A2 5.48
A3 18.5
A4 6.71
A5 8.60

rearange to get

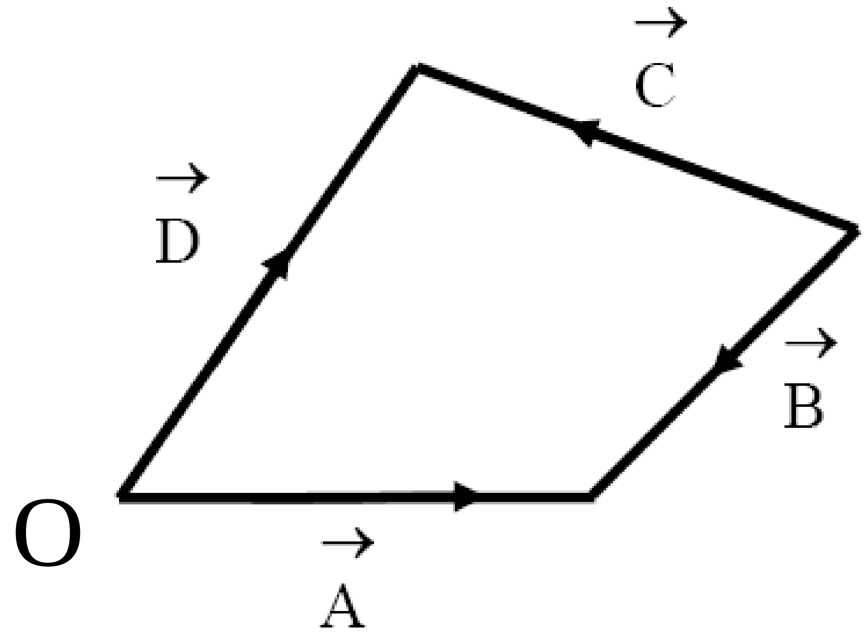
$$\vec{C} = \tfrac{1}{3}(\vec{B} - 2\vec{A}) = \tfrac{1}{3}\left[(15i + 21k) - 2(3j + 6k)\right]$$

$$= \tfrac{1}{3}\left[15i + 21k - 6j - 12k\right] = \tfrac{1}{3}(15i - 6j + 9k)$$

$$C = \tfrac{1}{3}\sqrt{15^2 + (-6)^2 + 9^2} = \tfrac{1}{3}\sqrt{225 + 36 + 81} = 6.16$$

4) Fig· shows four vectors A, B, C, D. Which of the following statements is correct:

A1 C = D + B - A
A2 C = A + B + D
A3 C =-D - B + A
A4 C = A - B + D
A5 C =-A - B - D



you could start from point O and return to it counterclockwise:

$$\vec{R} = \vec{A} + (-\vec{B}) + \vec{C} + (-\vec{D}) = 0$$

put $\vec{C}$ on the left-hand side and rest on the right-hand side:

you get $\vec{C} = \vec{D} + \vec{B} - \vec{A}$

5) Unit vectors i,j,k have magnitudes of unity and are directed in the positive directions of the x,y,z axes.
The value of k.(k x i) is:

A1 0
A2 -1
A3 +1
A4 i
A5 j

$$k.(k \times i) = 0$$

because :

$$k \times i = j$$

$$k.j = 0$$

6) If we have two vectors A = (a i - 2 j) and B = (2 i + 3 j) such that A.B = 4, find the value of a.

A1	5
A2	4
A3	0
A4	-5
A5	-4

$$\overrightarrow{A}.\overrightarrow{B} = (ai - 2j).(2i + 3j) = 2a - 6$$

$$\because \quad \overrightarrow{A}.\overrightarrow{B} = 4$$

$$\Rightarrow 2a - 6 = 4 \Rightarrow a = 10/2 = 5$$

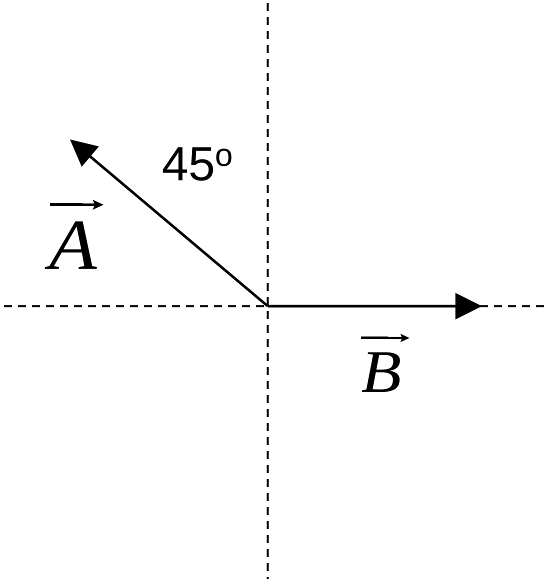
7) Three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are such that: $\vec{C} = \vec{A} + \vec{B}$, $\vec{B} = 5\hat{i}$ and $\vec{C} = 5\hat{j}$ Find the angle between $\vec{A}$ and $\vec{B}$.

A) 135°
B) 120°
C) 270°
D) 150°
E) 45°

$$\vec{C} = \vec{A} + \vec{B} \Rightarrow \vec{A} = \vec{C} - \vec{B} = 5j - 5i = -5i + 5j$$

$$\vec{A} = -5i + 5j \quad \Rightarrow \quad A = \sqrt{5^2 + 5^2} = 5\sqrt{2}$$

$$\vec{B} = 5i \quad \Rightarrow \quad B = 5$$



you can say from drawing $\vec{A}$ and $\vec{B}$

$\Phi = 90° + 45° = 135°$

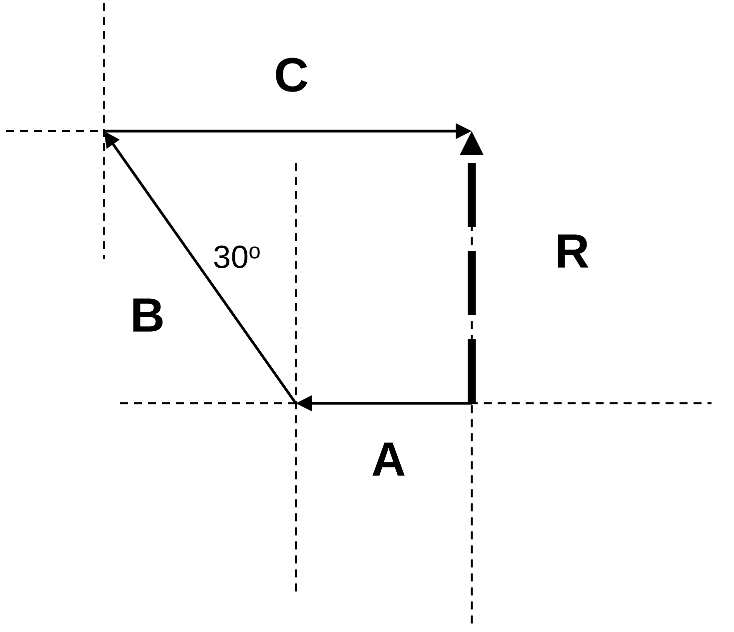
or continue:

$$\cos\Phi = \frac{\vec{A}.\vec{B}}{AB} \qquad \vec{A}.\vec{B} = (-5i + 5j).5i = -25$$

$$\cos\Phi = \frac{-25}{5\sqrt{2}\times 5} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \Rightarrow \Phi = 135°$$

8) A man walks 4.65 km West, then 12.7 km in the direction 30° West of North and finally 11.0 km due East. The man is now at

A) 11.0 km due North
B) 12.7 km due West
C) 4.65 km due South
D) 15.6 km in the direction 45° West of North
E) back to where he started



take i (east) j(north) -i (west) -j (south)

A = -4.65 i

B = -(12.7 sin 30º)i + (12.7 cos 30º)j = -6.35 i + 11.0 j

C = 11.0 i

R = A+B+C= -4.65 i + (- 6.35 i + 11.0 j) + 11.0 i

= -11.0 i +11.0i + 11.0j = 11.0 j (11.0 km due north)

9) If vector $\vec{A}$ has the magnitude of 3.0 m and makes an angle 30° with the +x-axis, then the vector $\vec{B} = -2\vec{A}$ is:

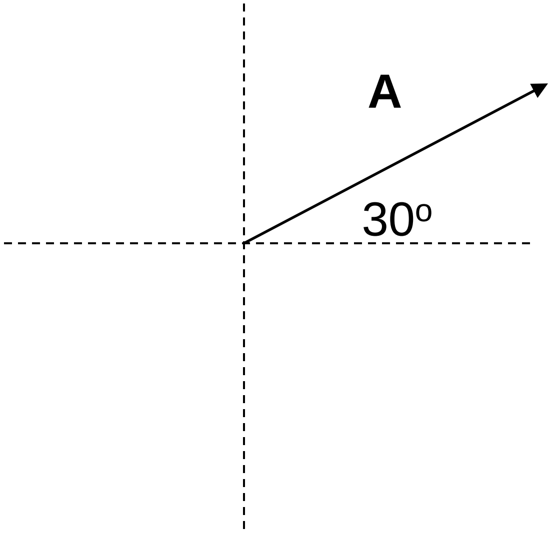
A) $\vec{B} = -5.2\hat{i} - 3.0\hat{j}$ (m)

B) $\vec{B} = 5.2\hat{i} + 3.0\hat{j}$ (m)

C) $\vec{B} = -5.2\hat{i} + 3.0\hat{j}$ (m)

D) $\vec{B} = 5.2\hat{i} - 3.0\hat{j}$ (m)

E) $\vec{B} = -3.0\hat{i} - 5.2\hat{j}$ (m)

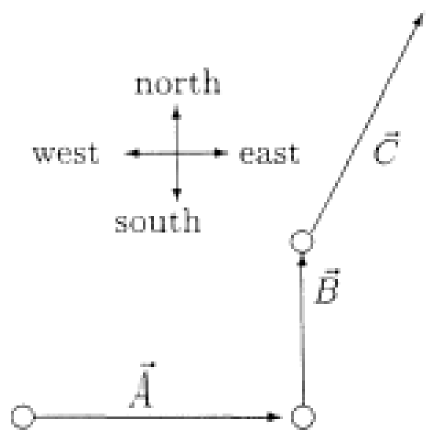


A = (3.0 cos 30^o) i + (3.0 sin 30^o) j = 2.6 i + 1.5 j

B = - 2 A = -2 (2.6 i + 1.5 j) = - 5.2 i - 3.0 j

•10 A car is driven east for a distance of 50 km, then north for 30 km, and then in a direction 30° east of north for 25 km. Sketch the vector diagram and determine (a) the magnitude and (b) the angle of the car's total displacement from its starting point.

10. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction (+x direction) and *north* as the $\hat{\mathrm{j}}$ direction (+y direction). All distances are understood to be in kilometers. We note that the angle between $\vec{C}$ and the x axis is 60°. Thus,



$$\vec{A} = 50\,\hat{\mathrm{i}}$$
$$\vec{B} = 30\,\hat{\mathrm{j}}$$
$$\vec{C} = 25\cos(60°)\,\hat{\mathrm{i}} + 25\sin(60°)\,\hat{\mathrm{j}}$$

(a) The total displacement of the car from its initial position is represented by

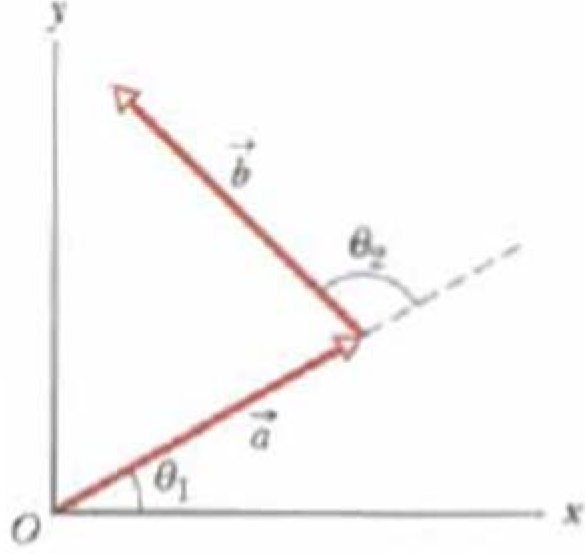
$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = 62.5\,\hat{\mathrm{i}} + 51.7\,\hat{\mathrm{j}}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{(62.5)^2 + (51.7)^2} = 81 \text{ km.}$$

(b) The angle (counterclockwise from +x axis) is $\tan^{-1}(51.7/62.5) = 40°$, which is to say that it points 40° *north of east*. Although the resultant $\vec{r}$ is shown in our sketch, it would be a direct line from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

•19 The two vectors $\vec{a}$ and $\vec{b}$ in Fig. 3-29 have equal magnitudes of 10.0 m and the angles are $\theta_1 = 30°$ and $\theta_2 = 105°$. Find the (a) x and (b) y components of their vector sum $\vec{r}$, (c) the magnitude of $\vec{r}$, and (d) the angle $\vec{r}$ makes with the positive direction of the x axis.



19. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}, \vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the +x axis is 30° +105° = 135° and apply Eq. 3-5 and Eq. 3-6 where appropriate.

(a) The x component of $\vec{r}$ is $r_x = 10\cos 30° + 10\cos 135° = 1.59$ m.

(b) The y component of $\vec{r}$ is $r_y = 10\sin 30° + 10\sin 135° = 12.1$ m.

(c) The magnitude of $\vec{r}$ is $\sqrt{(1.59)^2 + (12.1)^2} = 12.2$ m.

(d) The angle between $\vec{r}$ and the +x direction is $\tan^{-1}(12.1/1.59) = 82.5°$.

•18 In the sum $\vec{A} + \vec{B} = \vec{C}$, vector $\vec{A}$ has a magnitude of 12.0 m and is angled 40.0° counterclockwise from the $+x$ direction, and vector $\vec{C}$ has a magnitude of 15.0 m and is angled 20.0° counterclockwise from the $-x$ direction. What are (a) the magnitude and (b) the angle (relative to $+x$) of $\vec{B}$?

18. If we wish to use Eq. 3-5 in an unmodified fashion, we should note that the angle between $\vec{C}$ and the $+x$ axis is 180° + 20.0° = 200°.

(a) The x component of $\vec{B}$ is given by $C_x - A_x = 15.0 \cos 200° - 12.0 \cos 40° = -23.3$ m, and the y component of $\vec{B}$ is given by $C_y - A_y = 15.0 \sin 200° - 12.0 \sin 40° = -12.8$ m. Consequently, its magnitude is $\sqrt{(-23.3)^2 + (-12.8)^2} = 26.6 \text{ m}$.

(b) The two possibilities presented by a simple calculation for the angle between $\vec{B}$ and the $+x$ axis are $\tan^{-1}[(-12.8)/(-23.3)] = 28.9°$, and 180° + 28.9° = 209°. We choose the latter possibility as the correct one since it indicates that $\vec{B}$ is in the third quadrant (indicated by the signs of its components). We note, too, that the answer can be equivalently stated as $-151°$.

••24 An explorer is caught in a whiteout (in which the snowfall is so thick that the ground cannot be distinguished from the sky) while returning to base camp. He was supposed to travel due north for 5.6 km, but when the snow clears, he discovers that he actually traveled 7.8 km at 50° north of due east. (a) How far and (b) in what direction must he now travel to reach base camp?

24. The desired result is the displacement vector, in units of km, $\vec{A} = 5.6, 90^{\circ}$ (measured counterclockwise from the +x axis), or $\vec{A} = 5.6\,\hat{\mathrm{j}}$, where $\hat{\mathrm{j}}$ is the unit vector along the positive y axis (north). This consists of the sum of two displacements: during the whiteout, $\vec{B} = 7.8,\ 50^{\circ}$, or $\vec{B} = 7.8(\cos 50^{\circ}\,\hat{\mathrm{i}} + \sin 50^{\circ}\,\hat{\mathrm{j}}) = 5.01\,\hat{\mathrm{i}} + 5.98\,\hat{\mathrm{j}}$ and the unknown $\vec{C}$. Thus, $\vec{A} = \vec{B} + \vec{C}$.

(a) The desired displacement is given by $\vec{C} = \vec{A} - \vec{B} = -5.01\,\hat{\mathrm{i}} - 0.38\,\hat{\mathrm{j}}$. The magnitude is $\sqrt{(-5.01)^2 + (-0.38)^2} = 5.0$ km.

(b) The angle is $\tan^{-1}(-0.38/-5.01) = 4.3^{\circ}$, south of due west.

41 If $\vec{B}$ is added to $\vec{A}$, the result is $6.0\hat{i} + 1.0\hat{j}$. If $\vec{B}$ is subtracted from $\vec{A}$, the result is $-4.0\hat{i} + 7.0\hat{j}$. What is the magnitude of $\vec{A}$?

41. Given: $\vec{A} + \vec{B} = 6.0\,\hat{i} + 1.0\,\hat{j}$ and $\vec{A} - \vec{B} = -4.0\,\hat{i} + 7.0\,\hat{j}$. Solving these simultaneously leads to $\vec{A} = 1.0\,\hat{i} + 4.0\,\hat{j}$. The Pythagorean theorem then leads to $A = \sqrt{(1.0)^2 + (4.0)^2} = 4.1$.

** What angle does **a** = -5**i** + 3 **j** – 2 **k** make with the z- axis

$$k.\vec{a} = \left|\hat{k}\right|\left|\vec{a}\right| \cos\phi,$$

$$\hat{k}\,.\,\vec{a} = (\hat{k}).\left(-5.00\,\hat{i} + 3.00\,\hat{j} - 2.00\,\hat{k} \right) = -2.00$$

$$\left|\hat{k}\right| = 1,$$

$$\left|\vec{a}\right| = \sqrt{25+9+4} = 6.16$$

$$\cos\phi = -\frac{2.00}{6.16} = -0.324,$$

$$\phi = \cos(-0.324) = 109^{\circ}$$