Selected Problems from Chapter 11

1) A disk has a mass of 32 kg and a radius of 25 cm. It rolls without slipping along a level ground at 5.0 m/s. Find the total kinetic energy of the disk.

A1 600 J
A2 400 J
A3 800 J
A4 200 J
A5 100 J

$$I = \tfrac{1}{2}MR^2, \quad v_{com} = 5.0\, m/s$$

$$K = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}Mv^2_{com}$$

$$\omega = \frac{v_{com}}{R} \quad (rolling)$$

$$K = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_{com}}{R}\right)^2 + \tfrac{1}{2}Mv^2_{com}$$

$$= \tfrac{1}{4}Mv_{com}^2 + \tfrac{1}{2}Mv^2_{com} = \tfrac{3}{4}Mv^2_{com} = \tfrac{3}{4}\times 32\times(5.0)^2 = 600\, J$$

2) A solid cylinder of mass M and radius R starts from rest and rolls down an incline plane making an angle of 30 degrees with the horizontal. The linear speed of its center, after it has travelled 5 m down the incline, is:
(Icm = 1/2* M* R**2)

A1 5.7 m/s
A2 3.8 m/s
A3 2.5 m/s
A4 4.9 m/s
A5 1.3 m/s

$$a = \frac{g \sin\theta}{1 + I_{com}/MR^2},$$

$$I_{com} = \frac{1}{2}MR^2, \sin 30 = \frac{1}{2}$$

$$a = \frac{\frac{1}{2}g}{1+\frac{1}{2}} = \frac{1}{3}g$$

$$v^2 = v^2 + 2ad = 2ad = 2 \times \frac{1}{3}g \times (5.0)$$

$$v = 5.7 m/s$$

$$h = 5.0 \sin 30 = 2.5\ m$$

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$0 + Mgh = \left(\frac{1}{2}I_{com}\omega^2 + \frac{1}{2}Mv^2_{com}\right) + 0 \quad (1)$$

$$\frac{1}{2}I_{com}\omega^2 = \frac{1}{2} \times \left(\frac{1}{2}MR^2\right) \times \left(\frac{v_{com}}{R}\right)^2 = \frac{1}{4}Mv^2_{com}$$

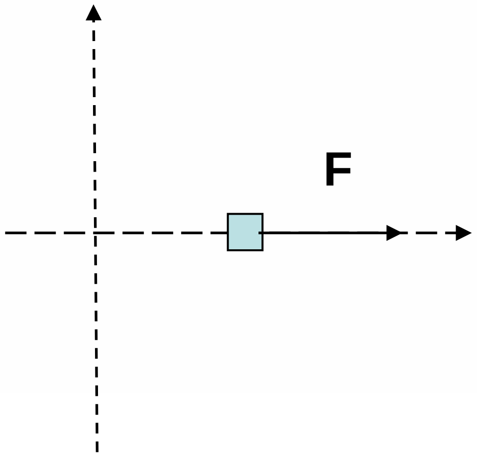
$Eq.1\ becomes:$

$$mgh = \left(\frac{1}{4}Mv^2_{com} + \frac{1}{2}Mv^2_{com}\right) = \frac{3}{4}Mv^2_{com}$$

$$v_{com} = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4 \times 9.8 \times 2.5}{3}} = 5.7\ m/s$$

3) A 2.0-kg block is located on the x-axis 3.0 m from the origin and is acted upon by a force F = 8.0i N. Find the net torque acting on the block relative to the origin.

A1	0.0	N.m
A2	-12 k	N.m
A3	-24 k	N.m
A4	18 k	N.m
A5	24 k	N.m



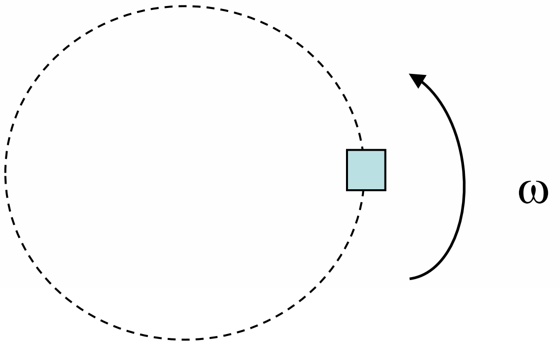
$$\vec{r} = 3.0i \ m$$

$$\vec{F} = 8.0i \ N$$

$$\vec{\tau} = \vec{r} \times \vec{F} = 3.0i \times 8.0i = 0 \ N.m$$

4) A 2.5 kg block travels around a 0.50 m radius circle with an angular velocity of 12 rad/s. Find the magnitude of the angular momentum of the block about the center of the circle.

A1 7.5 kg.m**2/s
A2 1.5 kg.m**2/s
A3 6.0 kg.m**2/s
A4 9.0 kg.m**2/s
A5 12 kg.m**2/s



$$\ell = I\,\omega$$

$$I = mr^2 = 2.5 \times (0.50)^2 = 0.625\,kg.m^2$$

$$\ell = 0.625 \times 12 = 7.5\,kg.m^2/s$$

5) A student in a class demonstration is sitting on a frictionless rotating chair with his arms by the side of his body. The chair-student system is rotating with an angular speed w. The student suddenly extends his arms horizontally. The angular velocity of the system:

A1 decreases
A2 increases
A3 remains the same
A4 may increase or decrease depending on the mass of the student
A5 may increase or decrease depending on the mass of the chair

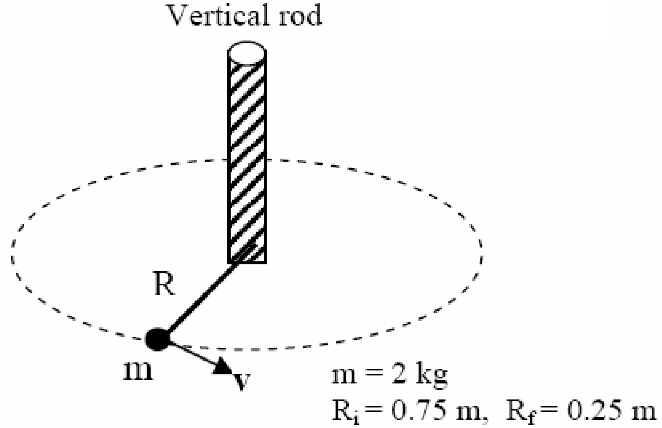
$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0 \quad \Rightarrow \vec{L} = \text{constant},$$

$$(L_i = L_f\,, \quad I_i\,\omega_i = I_f\,\omega_f\,)$$

$$I\,\omega = \text{constant}$$

$$I\uparrow \Rightarrow \omega\downarrow$$

6) A 2.0 kg mass is attached to a string and fixed to a vertical rod Fig . The mass is initially orbiting with a speed of 5.0 m/s in a circle of radius 0.75 m. The string is then slowly winding around the vertical rod. What is the speed of the mass at the moment the string reaches a length of 0.25 m?



A1 15 m/s
A2 3.9 m/s
A3 45 m/s
A4 75 m/s
A5 12 m/s

$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0 \quad \Rightarrow \vec{L} = \text{constant},$$

$$L_i = L_f \,,$$

$$I_i \omega_i = I_f \omega_f \qquad (1)$$

$$I_i = m r_i^2 , \quad \omega_i = \frac{v_i}{r_i}$$

$$I_f = m r_f^2 , \quad \omega_f = \frac{v_f}{r_f}$$

$$Eq.1 becomes:$$

$$\left(m r_i^2\right) \times \left(\frac{v_i}{r_i}\right) = \left(m r_f^2\right) \times \left(\frac{v_f}{r_f}\right)$$

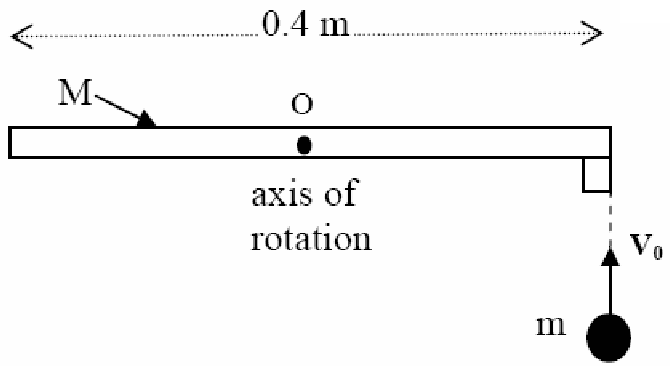
$$r_i \times v_i = r_f \times v_f$$

$$0.75 \times 5.0 = 0.25 \times v_f$$

$$v_f = 15 \quad m/s$$

7) Fig shows an object of mass m=100 g and velocity =Vo is fired onto one end of a uniform thin rod (L=0.4 m, M = 1.0 kg) initially at rest. The rod can rotate freely about an axis through its center (O). The object sticks to the rod after collision. The angular velocity of the system (rod + object) is 10 rad/s immediately after the collision. Calculate Vo.

A1 8.7 m/s
A2 4.0 m/s
A3 1.8 m/s
A4 2.2 m/s
A5 9.5 m/s



$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0 \quad \Rightarrow \vec{L} = \text{constant},$$

$$L_i = L_f \,,$$

$$L_f = I_f\,\omega_f = \left(\frac{1}{12}ML^2 + m\left(\frac{1}{2}L\right)^2\right)\omega_f = \left(\frac{M+3m}{12}\right)\omega_f\,L^2$$

$$L_i = mv_0\left(\frac{L}{2}\right)$$

$$mv_0\left(\frac{L}{2}\right) = \left(\frac{M+3m}{12}\right)\omega_f\,L^2$$

$$v_0 = \left(\frac{M+3m}{6m}\right)\omega_f\,L$$

•3 A 140 kg hoop rolls along a horizontal floor so that the hoop's center of mass has a speed of 0.150 m/s. How much work must be done on the hoop to stop it? SSM

3. By Eq. 10-52, the work required to stop the hoop is the negative of the initial kinetic energy of the hoop. The initial kinetic energy is $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$ (Eq. 11-5), where $I = mR^2$ is its rotational inertia about the center of mass, m = 140 kg, and v = 0.150 m/s is the speed of its center of mass. Eq. 11-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K = \frac{1}{2}mR^2\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2 = mv^2 = (140)(0.150)^2$$

which implies that the work required is – 3.15 J.

••10 Figure 11-34 gives the speed v versus time t for a 0.500 kg object of radius 6.00 cm that rolls smoothly down a 30° ramp. What is the rotational inertia of the object?

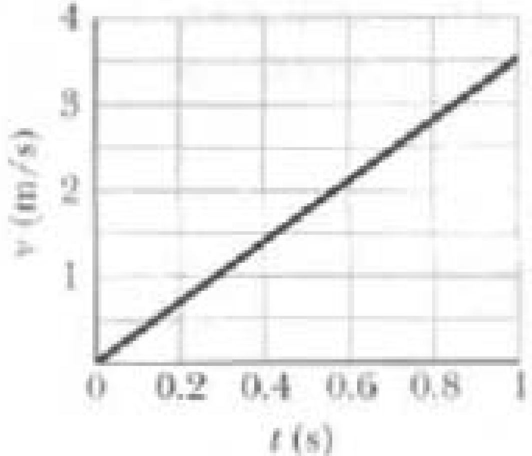


Fig. 11-34 Problem 10.

10. We plug $a = -3.5$ m/s^2 (where the magnitude of this number was estimated from the "rise over run" in the graph), $\theta = 30^\circ$, $M = 0.50$ kg and $R = 0.060$ m into Eq. 11-10 and solve for the rotational inertia. We find $I = 7.2 \times 10^{-4}$ kg·m^2.

•6 In Fig. 11-30, a constant horizontal force $\vec{F}_{app}$ of magnitude 10 N is applied to a wheel of mass 10 kg and radius 0.30 m. The wheel rolls smoothly on the horizontal surface, and the acceleration of its center of mass has magnitude 0.60 m/s^2. (a) In unit-vector notation, what is the frictional force on the wheel? (b) What is the rotational inertia of the wheel about the rotation axis through its center of mass?

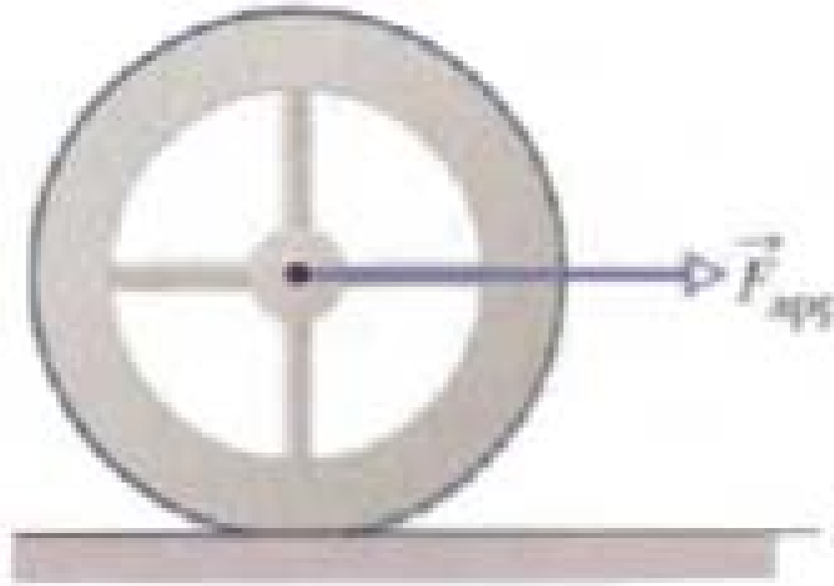


Fig. 11-30 Problem 6.

6. With $\vec{F}_{app} = (10\ \text{N})\hat{\text{i}}$, we solve the problem by applying Eq. 9-14 and Eq. 11-37.

(a) Newton's second law in the x direction leads to

$$F_{app} - f_s = ma \quad \Rightarrow f_s = 10\text{N} - (10\text{kg})\left(0.60\ \text{m/s}^2\right) = 4.0\ \text{N}.$$

In unit vector notation, we have $\vec{f}_s = (-4.0\ \text{N})\hat{\text{i}}$ which points leftward.

(b) With $R = 0.30$ m, we find the magnitude of the angular acceleration to be

$$|\alpha| = |a_{com}| / R = 2.0\ \text{rad/s}^2,$$

from Eq. 11-6. The only force not directed towards (or away from) the center of mass is $\vec{f}_s$, and the torque it produces is clockwise:

$$|\tau| = I|\alpha| \quad \Rightarrow \quad (0.30\,\text{m})(4.0\,\text{N}) = I\left(2.0\,\text{rad/s}^2\right)$$

which yields the wheel's rotational inertia about its center of mass: $I = 0.60\,\text{kg}\cdot\text{m}^2$.

•19 In unit-vector notation, what is the net torque about the origin on a flea located at coordinates (0, −4.0 m, 5.0 m) when forces $\vec{F}_1 = (3.0\text{ N})\hat{k}$ and $\vec{F}_2 = (-2.0\text{ N})\hat{j}$ act on the flea?

19. If we write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\mathrm{i}} + \left(zF_x - xF_z\right)\hat{\mathrm{j}} + \left(xF_y - yF_x\right)\hat{\mathrm{k}}.$$

With (using SI units) $x = 0$, $y = -4.0$, $z = 5.0$, $F_x = 0$, $F_y = -2.0$ and $F_z = 3.0$ (these latter terms being the individual forces that contribute to the net force), the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = (-2.0\,\mathrm{N \cdot m})\hat{\mathrm{i}}.$$

•26 A 2.0 kg particle-like object moves in a plane with velocity components $v_x = 30$ m/s and $v_y = 60$ m/s as it passes through the point with (x, y) coordinates of $(3.0, -4.0)$ m. Just then, in unit-vector notation, what is its angular momentum relative to (a) the origin and (b) the point $(-2.0, -2.0)$ m?

26. If we write $\vec{r}\,' = x'\hat{\mathrm{i}} + y'\hat{\mathrm{j}} + z'\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' = \vec{v}$ is equal to

$$\left(y'v_z - z'v_y\right)\hat{\mathrm{i}} + \left(z'v_x - x'v_z\right)\hat{\mathrm{j}} + \left(x'v_y - y'v_x\right)\hat{\mathrm{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3.0\hat{\mathrm{i}} - 4.0\hat{\mathrm{j}}$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3.0, y = -4.0, z = 0, v_x = 30, v_y = 60$ and $v_z = 0$. Then (with $m = 2.0$ kg) we obtain

$$\vec{\ell} = m\left(\vec{r} \times \vec{v}\right) = (6.0\times10^2 \text{ kg}\cdot\text{m}^2/\text{s})\hat{\mathrm{k}}.$$

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = -2.0\hat{\mathrm{i}} - 2.0\hat{\mathrm{j}}$. Therefore, in the above expression, we set $x' = 5.0, y' = -2.0, z' = 0, v_x = 30, v_y = 60$ and $v_z = 0$. We get

$$\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = (7.2\times10^2 \text{ kg}\cdot\text{m}^2/\text{s})\hat{\mathrm{k}}.$$

•35 The angular momentum of a flywheel having a rotational inertia of 0.140 kg · m^2 about its central axis decreases from 3.00 to 0.800 kg · m^2/s in 1.50 s. (a) What is the magnitude of the average torque acting on the flywheel about its central axis during this period? (b) Assuming a constant angular acceleration, through what angle does the flywheel turn? (c) How much work is done on the wheel? (d) What is the average power of the flywheel? SSM

35. (a) Since $\tau = dL/dt$, the average torque acting during any interval Δt is given by $\tau_{avg} = \left(L_f - L_i\right)/\Delta t$, where L_i is the initial angular momentum and L_f is the final angular momentum. Thus

$$\tau_{avg} = \frac{0.800\ \text{kg}\cdot\text{m}^2/\text{s} - 3.00\ \text{kg}\cdot\text{m}^2/\text{s}}{1.50\text{s}} = -1.47\ \text{N}\cdot\text{m},$$

or $|\tau_{avg}| = 1.47\ \text{N}\cdot\text{m}$. In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) The angle turned is $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$. If the angular acceleration α is uniform, then so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_0 = L_i/I$, and we obtain

$$\theta = \frac{L_i t + \frac{1}{2}\tau t^2}{I} = \frac{\left(3.00\,\text{kg}\cdot\text{m}^2/\text{s}\right)\left(1.50\,\text{s}\right) + \frac{1}{2}\left(-1.467\,\text{N}\cdot\text{m}\right)\left(1.50\,\text{s}\right)^2}{0.140\,\text{kg}\cdot\text{m}^2} = 20.4\,\text{rad}.$$

(c) The work done on the wheel is

$$W = \tau\theta = \left(-1.47\ \text{N}\cdot\text{m}\right)\left(20.4\,\text{rad}\right) = -29.9\ \text{J}$$

where more precise values are used in the calculation than what is shown here. An equally good method for finding W is Eq. 10-52, which, if desired, can be rewritten as $W = \left(L_f^2 - L_i^2\right)/2I$.

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

$$P_{avg} = -\frac{W}{\Delta t} = -\frac{-29.8\ \text{J}}{1.50\text{s}} = 19.9\,\text{W}.$$

**53 Figure 11-51 is an overhead view of a thin uniform rod of length 0.800 m and mass M rotating horizontally at angular speed 20.0 rad/s about an axis through its center. A particle of mass $M/3.00$ initially attached to one end is ejected from the rod and travels along a path that is perpendicular to the rod at the instant of ejection. If the particle's speed v_p is 6.00 m/s greater than the speed of the rod end just after ejection, what is the value of v_p?

Fig. 11-51 Problem 53.

53. By angular momentum conservation (Eq. 11-33), the total angular momentum after the explosion must be equal to before the explosion:

$$L'_p + L'_r = L_p + L_r$$

$$\left(\frac{L}{2}\right) m v_p + \frac{1}{12} M L^2 \omega' = I_p \omega + \frac{1}{12} M L^2 \omega$$

where one must be careful to avoid confusing the length of the rod ($L = 0.800$ m) with the angular momentum symbol. Note that $I_p = m(L/2)^2$ by Eq.10-33, and

$$\omega' = v_{end}/r = (v_p - 6)/(L/2),$$

where the latter relation follows from the penultimate sentence in the problem (and "6" stands for "6.00 m/s" here). Since $M = 3m$ and $\omega = 20$ rad/s, we end up with enough information to solve for the particle speed: $v_p = 11.0$ m/s.