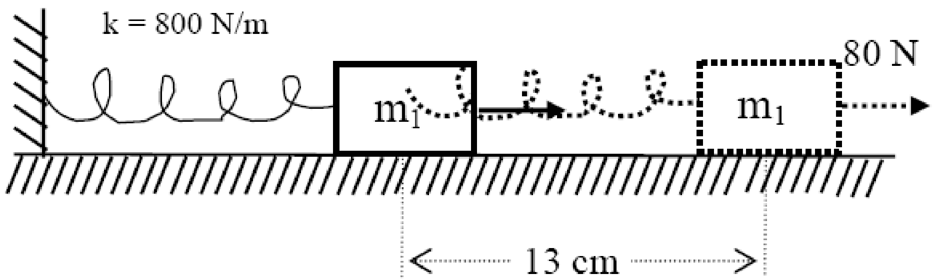
Selected Problems from Chapter 8

1) A 12-kg block is resting on a horizontal frictionless surface. The block is attached to an unstretched spring (k= 800 N/m) (see Fig.). A force F = 80 N parallel to the surface is applied to the block. What is the speed of the block when it is displaced by 13 cm from its initial position?

A1 0.78 m/s
A2 0.85 m/s
A3 1.1 m/s
A4 0.58 m/s
A5 0.64 m/s



$$W_{ext} = \Delta E \quad (no\ friction)$$

$$Fx = \Delta K + \Delta U$$

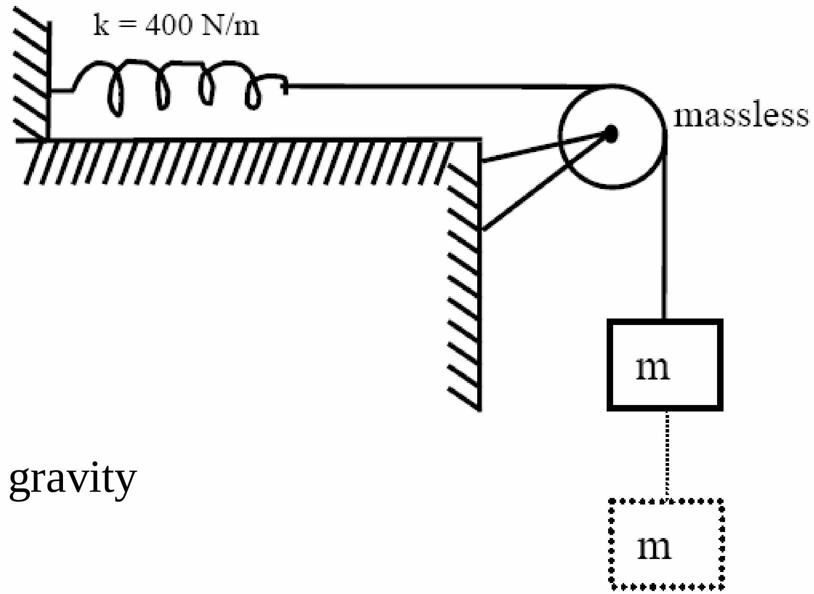
$$Fx = (K_f - K_i) + (U_f - U_i)$$

$$Fx = \frac{1}{2}mv^2 - 0 + \frac{1}{2}kx^2 - 0$$

$$\frac{1}{2}mv^2 = Fx - \frac{1}{2}kx^2$$

$$v = \sqrt{\frac{2Fx - kx^2}{m}} = \sqrt{\frac{2\times 80\times 0.13 - 800\times 0.13^2}{12}} = 0.78\ m/s$$

2) A block of mass m = 10 kg is connected to unstretched spring (k=400 N/m) (see Fig.). The block is released from rest. If the pulley is massless and frictionless, what is the maximum extension of the spring?



A1 49 cm
A2 25 cm
A3 33 cm
A4 55 cm
A5 11 cm

We have only conservative force viz. spring & gravity

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

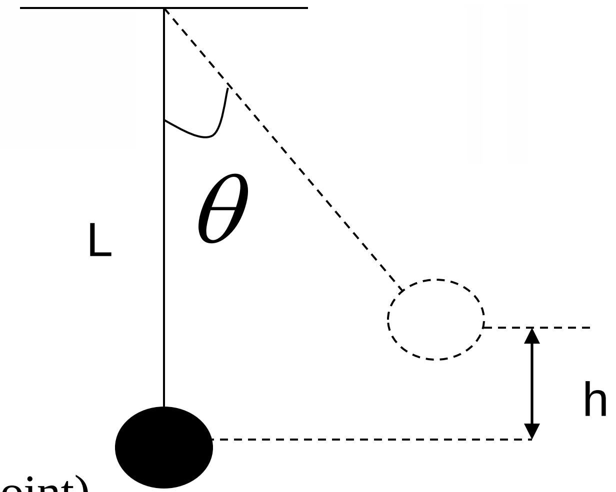
$$0+0+0 = 0 + \frac{1}{2}kx^2 - mgx \quad \text{(I took the zero at the original position of the block)}$$

$$\frac{1}{2}kx^2 = mgx$$

$$x = \frac{2mg}{k} = \frac{2\times 10\times 9.8}{400} = 0.49\ m$$

3) A 0.6-kg ball is suspended from the ceiling at the end of a 2.0-m string. As this ball swings, it has a speed of 4.0 m/s at the lowest point of its path. What maximum angle does the string make with the vertical as the ball swings?

A1 54 degrees
A2 61 degrees
A3 69 degrees
A4 77 degrees
A5 47 degrees



only gravity is doing work, the tension is not. Hence,

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = 0 + mgh$$ (I took the zero level ath the lowest point)

$$\text{h} = \frac{v^2}{2g} = \frac{4^2}{2 \times 9.8} = 0.816\ m$$

$$\cos\theta = \frac{L-h}{L} = \frac{2-0.816}{2} = 0.592$$

$$\theta = \cos^{-1} 0.592 = 54$$

4) When applied to a single object, a force is conservative if:

A1 its work done for motion in closed paths is equal to zero.
A2 its work done for motion in closed paths is greater than zero.
A3 it is parallel to the displacement always.
A4 it does equal work in equal displacement.
A5 its work done for motion in closed paths is less than zero.

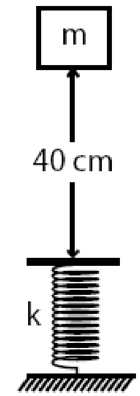
5) A 3.00 kg block is dropped from a height of 40 cm onto a spring of spring constant k (see Fig). If the maximum distance the spring is compressed = 0.130 m, find k.

A1	1840 N/m
A2	980 N/m
A3	490 N/m
A4	1250 N/m
A5	2800 N/m

$$initialy: K = 0, U_s = 0, U_g = 0$$

$$finaly: K = 0, U_s = \frac{1}{2}kx^2, U_g = -mg(h+x)$$

(I took the zero level at the intial position of the block, h =0.40 m, x=0.130 is the compression in the spring)



only conseravtive forces viz. gravity and spring.

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f,$$

$$0 + 0 + 0 = 0 - mg(h+x) + \frac{1}{2}kx^2$$

$$\frac{1}{2}kx^2 = mg(h+x)$$

$$k = \frac{2mg(h+x)}{x^2} = \frac{2 \times 3.00 \times 9.80 \times 0.530}{0.130^2} = 1840 \text{ N/m}$$

6) As a particle moves from point A to point B only two forces act on it: one force is non-conservative and does work = -30 J, the other force is conservative and does +50 J work. The change of the kinetic energy of the particle is:

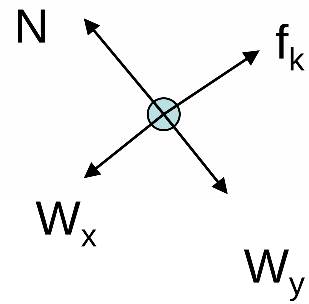
A1 20 J
A2 0 J
A3 30 J
A4 50 J
A5 80 J

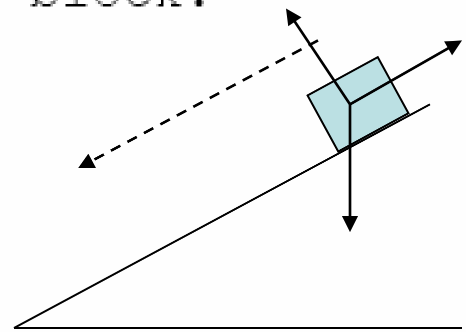
$$\Delta K = W_{net} = W_{con} + W_{noncon}$$

$$= 50 - 30 = 20\ J$$

7) A 2.2-kg block starts from rest on a rough inclined plane that makes an angle of 25 degrees with the horizontal. The coefficient of kinetic friction is 0.25. As the block goes 2.0 m down the plane, find the change in the mechanical energy of the block.

A1 −9.8 J
A2 9.8 J
A3 19.6 J
A4 −19.6 J
A5 0.0 J





$$W_y = mg\cos 25 = N \, ,$$

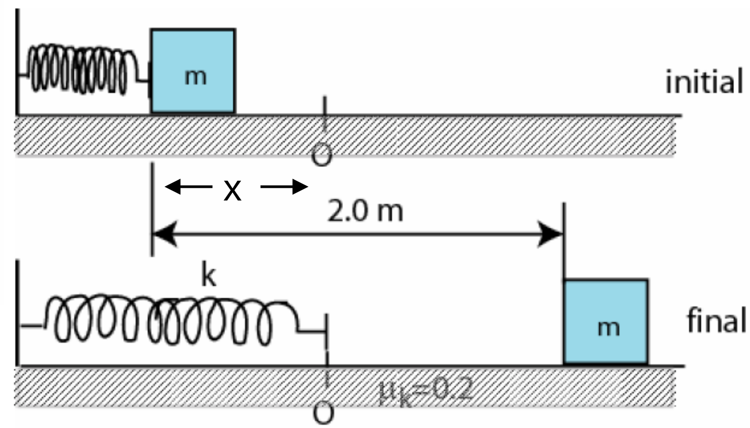
$$f_k = \mu_k N$$

$$\Delta E = -E_{th} = -f_k d$$

$$= -\mu_k mg\cos 25 d = -0.25 \times 2.2 \times 9.8 \times \cos 25 \times 2.0 = -9.8\ J$$

8) A 3.0 kg block is released from a compressed spring (k=120 N/m). It travels over a horizontal surface (mu =0.20) for a distance of 2.0 m before coming to rest, Fig . How far was the spring compressed before being released ?

A1 0.44 m
A2 0.39 m
A3 0.23 m
A4 0.13 m
A5 0.56 m



$$\Delta E = -E_{th} = -f_k d = -\mu_k N d = -\mu_k mgd \quad (d = 2.0\ m)$$

$$E_i = K_i + U_i = 0 + \frac{1}{2}kx^2$$

$$E_f = K_f + U_f = 0 + 0$$

$$\Delta E = E_f - E_i = -\mu_k mgd$$

$$-\frac{1}{2}kx^2 = -\mu_k mgd$$

$$x = \sqrt{\frac{2\mu_k mgd}{k}} = \sqrt{\frac{2 \times 0.20 \times 3.0 \times 9.80 \times 2.0}{120}} = 0.44\ m$$

••7 In Fig. 8-33, a small block of mass $m = 0.032$ kg can slide along the frictionless loop-the-loop, with loop radius $R = 12$ cm. The block is released from rest at point P, at height $h = 5.0R$ above the bottom of the loop. How much work does the gravitational force do on the block as the block travels from point P to (a) point Q and (b) the top of the loop? If the gravitational potential energy of the block–Earth system is taken to be zero at the bottom of the loop, what is that potential energy when the block is (c) at point P, (d) at point Q, and (e) at the top of the loop? (f) If, instead of merely being released, the block is given some initial speed downward along the track, do the answers to (a) through (e) increase, decrease, or remain the same?

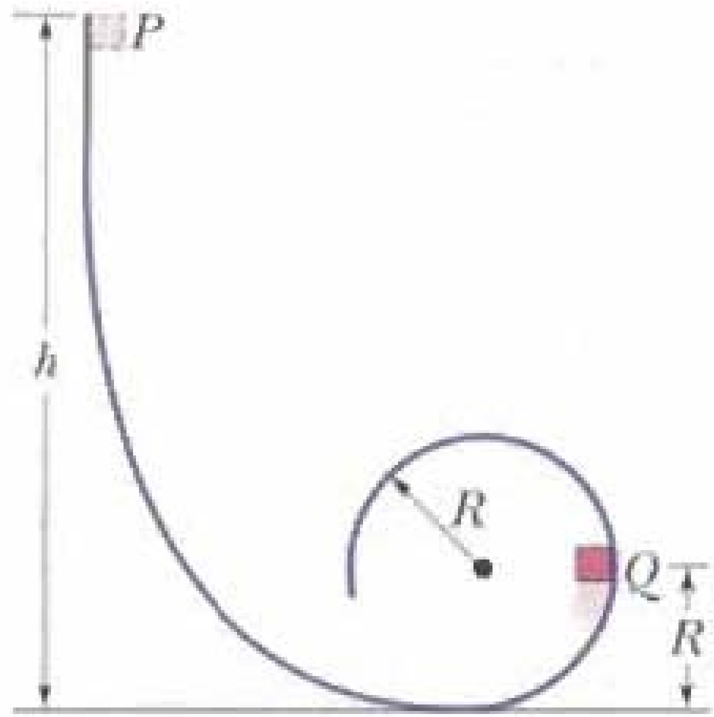


Fig. 8-33 Problems 7 and 21.

7. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and Q has a vertical component of $h - R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 4mgR = 4(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.15\ \text{J}.$$

(b) The displacement between the initial point and the top of the loop has a vertical component of $h - 2R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 3mgR = 3(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.11\ \text{J}.$$

(c) With $y = h = 5R$, at P we find

$$U = 5mgR = 5(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.19\ \text{J}.$$

(d) With $y = R$, at Q we have

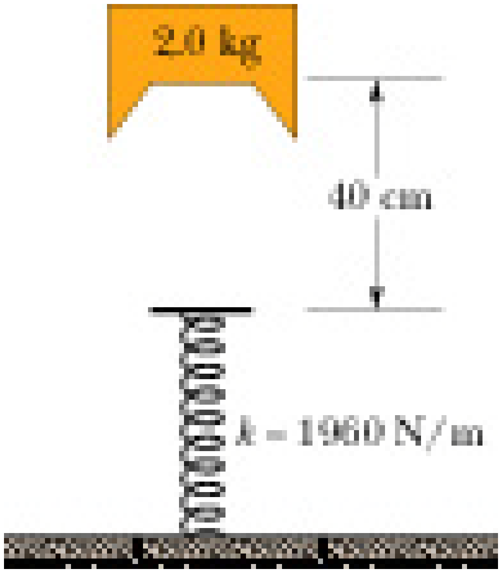
$$U = mgR = (3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.038\ \text{J}$$

(e) With $y = 2R$, at the top of the loop, we find

$$U = 2mgR = 2(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.075\ \text{J}$$

(f) The new information $(v_i \neq 0)$ is not involved in any of the preceding computations; the above results are unchanged.

•18 A block of mass $m = 2.0$ kg is dropped from height $h =$ 40 cm onto a spring of spring constant $k = 1960$ N/m (Fig. 8-36). Find the maximum distance the spring is compressed.



18. We denote m as the mass of the block, $h = 0.40$ m as the height from which it dropped (measured from the relaxed position of the spring), and x the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block. The block drops a total distance $h + x$, and the final gravitational potential energy is $-mg(h + x)$. The spring potential energy is $\frac{1}{2}kx^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

$$K_i + U_i = K_f + U_f$$

$$0 = -mg(h + x) + \frac{1}{2}kx^2$$

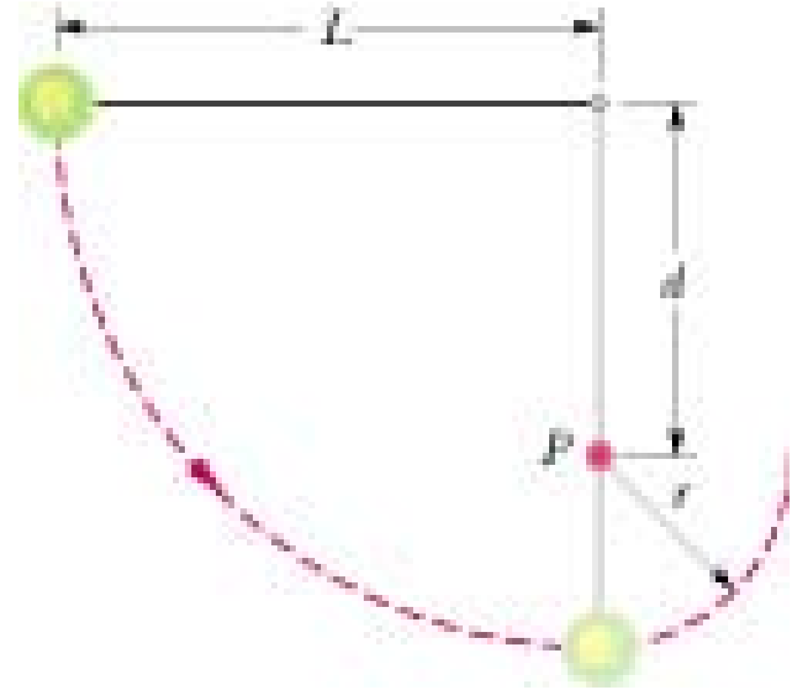
which is a second degree equation in x. Using the quadratic formula, its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k} .$$

Now $mg = 19.6$ N, $h = 0.40$ m, and $k = 1960$ N/m, and we choose the positive root so that $x > 0$.

$$x = \frac{19.6 + \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10 \text{ m} .$$

••23 The string in Fig. 8-38 is $L = 120$ cm long, has a ball attached to one end, and is fixed at its other end. The distance d from the fixed end to a fixed peg at point P is 75.0 cm. When the initially stationary ball is released with the string horizontal as shown, it will swing along the dashed arc. What is its speed when it reaches (a) its lowest point and (b) its highest point after the string catches on the peg? ILW



23. (a) As the string reaches its lowest point, its original potential energy $U = mgL$ (measured relative to the lowest point) is converted into kinetic energy. Thus,

$$mgL = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gL} \ .$$

With $L = 1.20$ m we obtain $v = 4.85 \text{ m/s}$.

(b) In this case, the total mechanical energy is shared between kinetic $\frac{1}{2}mv_b^2$ and potential mgy_b. We note that $y_b = 2r$ where $r = L - d = 0.450$ m. Energy conservation leads to

$$mgL = \frac{1}{2}mv_b^2 + mgy_b$$

which yields $v_b = \sqrt{2gL - 2g(2r)} = 2.42 \text{ m/s}$.

••43 A rope is used to pull a 3.57 kg block at constant speed 4.06 m along a horizontal floor. The force on the block from the rope is 7.68 N and directed 15.0° above the horizontal. What are (a) the work done by the rope's force, (b) the increase in thermal energy of the block–floor system, and (c) the coefficient of kinetic friction between the block and floor? SSM

43. (a) The work done on the block by the force in the rope is, using Eq. 7-7,

$$W = Fd\cos\theta = (7.68\ \text{N})(4.06\ \text{m})\cos 15.0° = 30.1\ \text{J}.$$

(b) Using f for the magnitude of the kinetic friction force, Eq. 8-29 reveals that the increase in thermal energy is

$$\Delta E_{th} = fd = (7.42\ \text{N})(4.06\ \text{m}) = 30.1\ \text{J}.$$

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/F_N$ to obtain the coefficient of friction. Place the x axis along the path of the block and the y axis normal to the floor. The x and the y component of Newton's second law are

$$\begin{aligned} x&: \quad F\cos\theta - f = 0 \\ y&: \quad F_N + F\sin\theta - mg = 0, \end{aligned}$$

where m is the mass of the block, F is the force exerted by the rope, and θ is the angle between that force and the horizontal. The first equation gives

$$f = F\cos\theta = (7.68)\cos 15.0° = 7.42\ \text{N}$$

and the second gives

$$F_N = mg - F\sin\theta = (3.57)(9.8) - (7.68)\sin 15.0° = 33.0\ \text{N}.$$

Thus

$$\mu_k = \frac{f}{F_N} = \frac{7.42\ \text{N}}{33.0\ \text{N}} = 0.225.$$

••54 A 4.0 kg bundle starts up a 30° incline with 128 J of kinetic energy. How far will it slide up the incline if the coefficient of kinetic friction between bundle and incline is 0.30?

54. We look for the distance along the incline d which is related to the height ascended by $\Delta h = d \sin\theta$. By a force analysis of the style done in Ch. 6, we find the normal force has magnitude $F_N = mg\cos\theta$ which means $f_k = \mu_k mg\cos\theta$. Thus, Eq. 8-33 (with $W = 0$) leads to

$$\begin{aligned} 0 &= K_f - K_i + \Delta U + \Delta E_{th} \\ &= 0 - K_i + mgd\sin\theta + \mu_k mgd\cos\theta \end{aligned}$$

which leads to

$$d = \frac{K_i}{mg(\sin\theta + \mu_k\cos\theta)} = \frac{128}{(4.0)(9.8)(\sin 30° + 0.30\cos 30°)} = 4.3\ \text{m}.$$

••31 A block with mass $m = 2.00$ kg is placed against a spring on a frictionless incline with angle $\theta = 30.0°$ (Fig. 8-43). (The block is not attached to the spring.) The spring, with spring constant $k = 19.6$ N/cm, is compressed 20.0 cm and then released. (a) What is the elastic potential energy of the compressed spring? (b) What is the change in the gravitational potential energy of the block–Earth system as the block moves from the release point to its highest point on the incline? (c) How far along the incline is the highest point from the release point? ILW

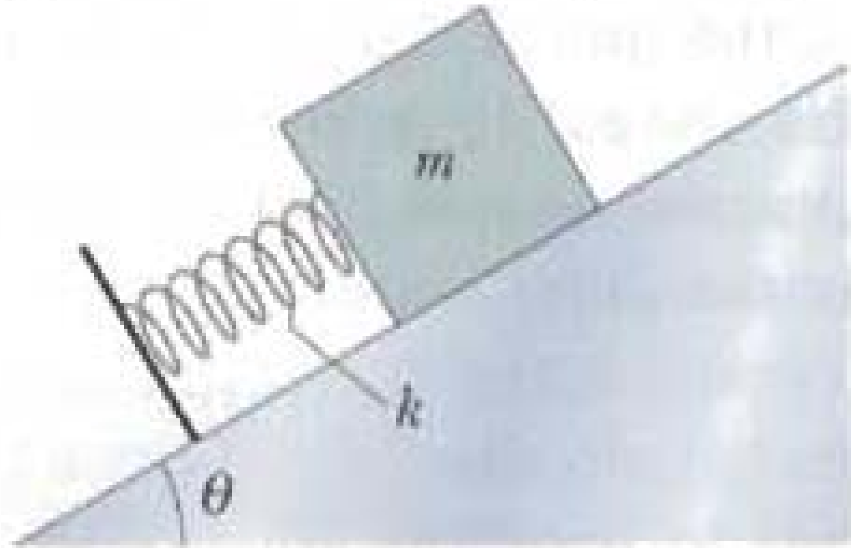


Fig. 8-43 Problem 31.

31. The reference point for the gravitational potential energy U_g (and height h) is at the block when the spring is maximally compressed. When the block is moving to its highest point, it is first accelerated by the spring; later, it separates from the spring and finally reaches a point where its speed v_f is (momentarily) zero. The x axis is along the incline, pointing uphill (so x_0 for the initial compression is negative-valued); its origin is at the relaxed position of the spring. We use SI units, so $k = 1960$ N/m and $x_0 = -0.200$ m.

(a) The elastic potential energy is $\frac{1}{2}kx_0^2 = 39.2\text{ J}$.

(b) Since initially $U_g = 0$, the change in U_g is the same as its final value mgh where m = 2.00 kg. That this must equal the result in part (a) is made clear in the steps shown in the next part. Thus, $\Delta U_g = U_g = 39.2$ J.

(c) The principle of mechanical energy conservation leads to

$$K_0 + U_0 = K_f + U_f$$
$$0 + \frac{1}{2}kx_0^2 = 0 + mgh$$

which yields $h = 2.00$ m. The problem asks for the distance *along the incline*, so we have $d = h/\sin 30° = 4.00$ m.

••55 A child whose weight is 267 N slides down a 6.1 m playground slide that makes an angle of 20° with the horizontal. The coefficient of kinetic friction between slide and child is 0.10. (a) How much energy is transferred to thermal energy? (b) If she starts at the top with a speed of 0.457 m/s, what is her speed at the bottom?

55. (a) Using the force analysis shown in Chapter 6, we find the normal force $F_N = mg\cos\theta$ (where mg = 267 N) which means $f_k = \mu_k F_N = \mu_k mg\cos\theta$. Thus, Eq. 8-31 yields

$$\Delta E_{th} = f_k d = \mu_k mgd\cos\theta = (0.10)(267)(6.1)\cos 20° = 1.5\times10^2\ \text{J}.$$

(a) The potential energy change is

$$\Delta U = mg(-d\sin\theta) = (267)(-6.1\sin 20°) = -5.6\times10^2\ \text{J}.$$

The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{267\ \text{N}}{9.8\ \text{m/s}^2}\right)(0.457\ \text{m/s}^2) = 2.8\ \text{J}.$$

Therefore, using Eq. 8-33 (with $W = 0$), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{th} = 2.8 - \left(-5.6\times10^2\right) - 1.5\times10^2 = 4.1\times10^2\ \text{J}.$$

Consequently, the final speed is $v_f = \sqrt{2K_f/m} = 5.5$ m/s.

68 A 30 g bullet moving a horizontal velocity of 500 m/s comes to a stop 12 cm within a solid wall. (a) What is the change in the bullet's mechanical energy? (b) What is the magnitude of the average force from the wall stopping it?

68. We use SI units so $m = 0.030$ kg and $d = 0.12$ m.

(a) Since there is no change in height (and we assume no changes in elastic potential energy), then $\Delta U = 0$ and we have

$$\Delta E_{\text{mech}} = \Delta K = -\frac{1}{2}mv_0^2 = -3.8\times10^3 \text{ J}.$$

where $v_0 = 500$ m/s and the final speed is zero.

(b) By Eq. 8-33 (with $W = 0$) we have $\Delta E_{th} = 3.8\times10^3$ J, which implies

$$f = \frac{\Delta E_{th}}{d} = 3.1\times10^4 \text{ N}$$

using Eq. 8-31 with f_k replaced by f (effectively generalizing that equation to include a greater variety of dissipative forces than just those obeying Eq. 6-2).