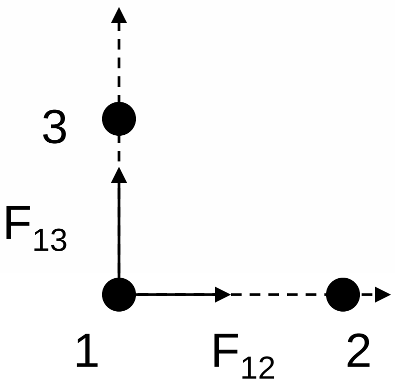
Selected Problems from Chapter 13

1) Three particles with equal mass M = 2.0 kg are located at (0,0), (4,0) and (0,3) where the x and y coordinates are in meters. Find the magnitude of the gravitational FORCE exerted on the particle located at the origin by the other two particles.

A1 3.4* 10**(-11) N
A2 4.6* 10**(-11) N
A3 5.2* 10**(-12) N
A4 1.7* 10**(-10) N
A5 2.6* 10**(-11) N



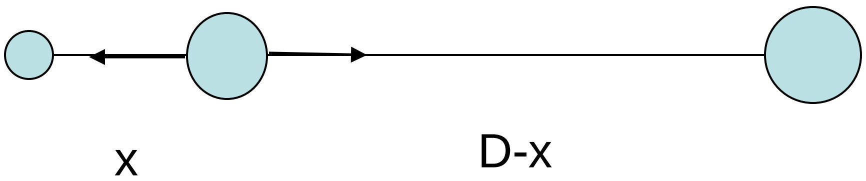
$$\vec{F}_{12} = G\frac{m^2}{r^2_{12}} = 6.67\times10^{-11}\times\frac{4}{16} = 1.67\times10^{-11} i\ N$$

$$\vec{F}_{13} = G\frac{m^2}{r^2_{13}} = 6.67\times10^{-11}\times\frac{4}{9} = 2.96\times10^{-11} j\ N$$

$$F = \sqrt{1.67^2 + 2.96^2}\times10^{-11} = 3.4\times10^{-11}\ N$$

2) Two stars of masses M and 6M are separated by a distance D. Calculate the distance (measured from M) to a point at which the net gravitational force on a third mass would be zero.

A1 0.29 D
A2 0.41 D
A3 0.33 D
A4 0.37 D
A5 0.14 D



$$F_{21} = F_{23}$$

$$G\frac{mM}{x^2} = G\frac{m(6M)}{(D-x)^2}$$

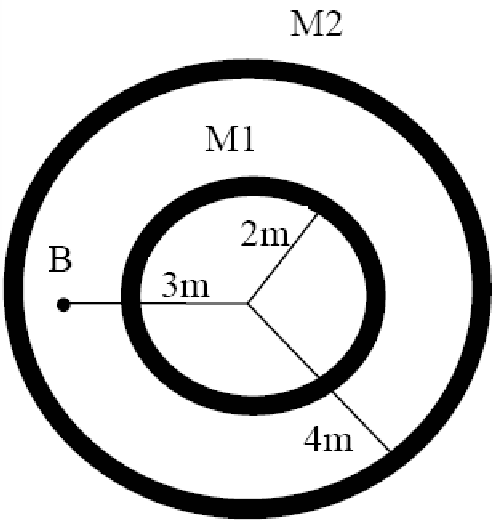
$$6x^2 = (D-x)^2$$

$$2.45x = D - x$$

$$x = \frac{D}{3.45} = 0.29D$$

3) Two concentric shells of uniform density having masses M1 and M2 and Radii R1 =2.0 m, R2 = 4.0 m are situated as shown in FIGURE 4. Find the gravitational FORCE on a particle of mass m placed at point B at a distance of 3.0 m from the center :

A1	(G*M1*m)/9
A2	G*(M1+M2)*m/9
A3	G*(M1+M2)*m/3
A4	(G*M2)*m/16
A5	G*(M1+M2)*m/4



as B inside the big shell

the shell is not exerting any force on m :

only the inner shell is exerting :

$$F_B = G\frac{M_1 m}{3.0^2} = \frac{GM_1 m}{9}$$

5) A 1000-kg rocket is fired vertically from Earth's surface with zero total mechanical energy. With what KINETIC energy was it fired?
(Mass of Earth = 6.0* 10**24 kg, Re = 6.4* 10**6 m)

A1 6.3* 10**10 J
A2 3.1* 10**10 J
A3 5.2* 10**6 J
A4 1.0* 10**9 J
A5 9.8* 10**7 J

$$K + U = 0$$

$$K = -U = G\frac{Mm}{R} = 6.67\times10^{-11}\frac{6.0\times10^{24}\times1000}{6.4\times10^{6}} = 6.3\times10^{10}J$$

6) A planet has a mass of 5.0 x 10**23 kg and radius of 2.0 x 10**6 m. A rocket is fired vertically from the surface of the planet with an initial speed of 4.0 km/s. What is the speed of the rocket when it is 1.0 x 10**6 m from the surface of the planet?

A1 2.2 km/s
A2 3.0 km/s
A3 1.6 km/s
A4 5.9 km/s
A5 3.7 km/s

$$r_i = R = 2.0\times10^6 m, \quad r_f = 3.0\times10^6 m, \quad M = 5.0\times10^{23} kg$$

$$v_i = 4000\ m/s$$

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2_i - G\frac{Mm}{r_i} = \frac{1}{2}mv^2_f - G\frac{Mm}{r_f}$$

$$solve\ for\ v_f$$

7) A satellite circles a planet every 2.8 h in an orbit of radius 1.2 x 10**7 m. If the radius of the planet is 5.0 x 10**6 m, what is the mass of the planet?

A1 1.0 x 10**25 kg
A2 3.1 x 10**26 kg
A3 3.4 x 10**24 kg
A4 4.0 x 10**27 kg
A5 1.9 x 10**23 kg

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

4) A spaceship of mass m circles a planet (mass = M) in an orbit of radius R. How much energy is required to transfer the spaceship to a circular orbit of radius 3R?

A1 GmM/(3R)
A2 GmM/(6R)
A3 GmM/(2R)
A4 GmM/(4R)
A5 3GmM/(4R)

$$E = -\frac{1}{2}U = -G\frac{Mm}{2R}$$

$$E_i = -G\frac{Mm}{2R}, \quad E_f = -G\frac{Mm}{6R}$$

$$\Delta E = E_f - E_i = -G\frac{Mm}{6R} + G\frac{Mm}{2R} = G\frac{Mm}{2R}\left(1 - \frac{1}{3}\right) = G\frac{Mm}{3R}$$

••3 A mass M is split into two parts, m and $M - m$, which are then separated by a certain distance. What ratio m/M maximizes the magnitude of the gravitational force between the parts? ILW

3. The gravitational force between the two parts is

$$F = \frac{Gm\left(M - m\right)}{r^2} = \frac{G}{r^2}\left(mM - m^2\right)$$

which we differentiate with respect to m and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}\left(M - 2m\right) \Rightarrow M = 2m$$

which leads to the result $m/M = 1/2$.

•5 How far from Earth must a space probe be along a line toward the Sun so that the Sun's gravitational pull on the probe balances Earth's pull? SSM WWW

5. At the point where the forces balance $GM_em/r_1^2 = GM_sm/r_2^2$, where M_e is the mass of Earth, M_s is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_e}{r_1^2} = \frac{M_s}{(d-r_1)^2}.$$

Taking the positive square root of both sides, we solve for r_1. A little algebra yields

$$r_1 = \frac{d\sqrt{M_e}}{\sqrt{M_s}+\sqrt{M_e}} = \frac{(150\times10^9\ \text{m})\sqrt{5.98\times10^{24}\ \text{kg}}}{\sqrt{1.99\times10^{30}\ \text{kg}}+\sqrt{5.98\times10^{24}\ \text{kg}}} = 2.60\times10^8\ \text{m}.$$

Values for M_e, M_s, and d can be found in Appendix C.

••17 One model for a certain planet has a core of radius R and mass M surrounded by an outer shell of inner radius R, outer radius $2R$, and mass $4M$. If $M = 4.1 \times 10^{24}$ kg and $R = 6.0 \times 10^{6}$ m, what is the gravitational acceleration of a particle at points (a) R and (b) $3R$ from the center of the planet?

17. (a) The gravitational acceleration is

$$a_g = \frac{GM}{R^2} = 7.6 \text{ m/s}^2.$$

(b) Note that the total mass is 5*M*. Thus,

$$a_g = \frac{G(5M)}{(3R)^2} = 4.2 \text{ m/s}^2.$$

••32 Zero, a hypothetical planet, has a mass of 5.0×10^{23} kg, a radius of 3.0×10^6 m, and no atmosphere. A 10 kg space probe is to be launched vertically from its surface. (a) If the probe is launched with an initial energy of 5.0×10^7 J, what will be its kinetic energy when it is 4.0×10^6 m from the center of Zero? (b) If the probe is to achieve a maximum distance of 8.0×10^6 m from the center of Zero, with what initial kinetic energy must it be launched from the surface of Zero?

32. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2$$

$$K_1 - \frac{GmM}{r_1} = K_2 - \frac{GmM}{r_2}$$

where $M = 5.0 \times 10^{23}$ kg, $r_1 = R = 3.0 \times 10^6$ m and $m = 10$ kg.

(a) If $K_1 = 5.0 \times 10^7$ J and $r_2 = 4.0 \times 10^6$ m, then the above equation leads to

$$K_2 = K_1 + GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 2.2 \times 10^7 \text{ J}.$$

(b) In this case, we require $K_2 = 0$ and $r_2 = 8.0 \times 10^6$ m, and solve for K_1:

$$K_1 = K_2 + GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 6.9 \times 10^7 \text{ J}.$$

•39 The Martian satellite Phobos travels in an approximately circular orbit of radius 9.4×10^6 m with a period of 7 h 39 min. Calculate the mass of Mars from this information.

39. The period T and orbit radius r are related by the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of Mars. The period is 7 h 39 min, which is 2.754×10^4 s. We solve for M:

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (9.4 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})\left(2.754 \times 10^4 \text{ s}\right)^2} = 6.5 \times 10^{23} \text{ kg}.$$

•56 Two Earth satellites, A and B, each of mass m, are to be launched into circular orbits about Earth's center. Satellite A is to orbit at an altitude of 6370 km. Satellite B is to orbit at an altitude of 19 110 km. The radius of Earth R_E is 6370 km. (a) What is the ratio of the potential energy of satellite B to that of satellite A, in orbit? (b) What is the ratio of the kinetic energy of satellite B to that of satellite A, in orbit? (c) Which satellite has the greater total energy if each has a mass of 14.6 kg? (d) By how much?

56. Although altitudes are given, it is the orbital radii which enter the equations. Thus, r_A = (6370 + 6370) km = 12740 km, and r_B = (19110 + 6370) km = 25480 km

(a) The ratio of potential energies is

$$\frac{U_B}{U_A} = \frac{-\frac{GmM}{r_B}}{-\frac{GmM}{r_A}} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(b) Using Eq. 13-38, the ratio of kinetic energies is

$$\frac{K_B}{K_A} = \frac{\frac{GmM}{2r_B}}{\frac{GmM}{2r_A}} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(c) From Eq. 13-40, it is clear that the satellite with the largest value of r has the smallest value of $|E|$ (since r is in the denominator). And since the values of E are negative, then the smallest value of $|E|$ corresponds to the largest energy E. Thus, satellite B has the largest energy.

(d) The difference is

$$\Delta E = E_B - E_A = -\frac{GmM}{2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

Being careful to convert the r values to meters, we obtain $\Delta E = 1.1 \times 10^8$ J. The mass M of Earth is found in Appendix C.