Selected Problems from Chapter 15

1) What happens to the FREQUENCY if the length of a simple pendulum is INCREASED by a factor of FOUR?

A1 it decreases by a factor of TWO.
A2 it increases by a factor of TWO.
A3 it remains constant(i.e. does not change).
A4 it increases by a factor of FOUR.
A5 it decreases by a factor of FOUR.

$$T' = 2\pi\sqrt{\frac{L'}{g}}$$

$$f' = \frac{1}{T'} = \frac{1}{2\pi}\sqrt{\frac{g}{L'}} = \frac{1}{2\pi}\sqrt{\frac{g}{4L}} = \frac{1}{2}\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right) = \frac{1}{2}f$$

2) A 0.65 kg block is fastened to a spring whose spring constant is 65 N/m. The block is pulled a distance 20 cm from its equilibrium position on a frictionless horizontal surface and released from rest. What is the angular frequency of the resulting motion?

A1 10 rad/s
A2 20 rad/s
A3 65 rad/s
A4 0.65 rad/s
A5 2.0 rad/s

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65}{0.65}} = \sqrt{100} = 10\ rad\ /\ s$$

3) A 0.25-kg block oscillates on the end of the spring with a spring constant of 200 N/m. When t=0, the position and velocity of the block are x=0.15 m and v=3.0 m/s. What is the maximum speed of the block?

A1 5.2 m/s
A2 0.18 m/s
A3 3.7 m/s
A4 0.13 m/s
A5 13 m/s

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0.25}} = \sqrt{800} = 20\sqrt{2}\ rad/s$$

$$x = x_m \cos(\omega t + \phi)$$

$$x(0) = x_m \cos(\phi) = 0.15\ m \quad (1)$$

$$v = -v_m \sin(\omega t + \phi) \qquad (v_m = x_m \omega)$$

$$v(0) = -x_m \omega \sin(\phi) = 3.0\ m/s \quad (2)$$

divide (2) by (1)

$$\frac{v(0)}{x(0)} = \frac{3.0}{0.15} = 20 = -\omega \tan(\phi)$$

$$\tan(\phi) = -\frac{20}{28.28} = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \phi = -35^\circ \ or\ 180 - 35 = 145\ (rejected\ gives\ x_m\ (-ve))$$

$$from\ (1) \quad x_m = \frac{x(0)}{\cos(\phi)} = \frac{0.15}{\cos(-35)} = 0.18\ m$$

$$v_m = x_m \omega = 0.18 \times 20\sqrt{2} = 5.2\ m/s$$

4) A particle of mass 0.10 kg is vibrating with simple harmonic motion with a period of 0.20 s and a maximum speed of 10 m/s. Find the maximum DISPLACEMENT of the particle.

A1 0.32 m
A2 0.12 m
A3 o.53 m
A4 0.98 m
A5 0.00 m

$$T = 0.20\, s \Rightarrow \omega = \frac{2\pi}{T} = 31.4\, rad/s$$

$$v_m = x_m \omega$$

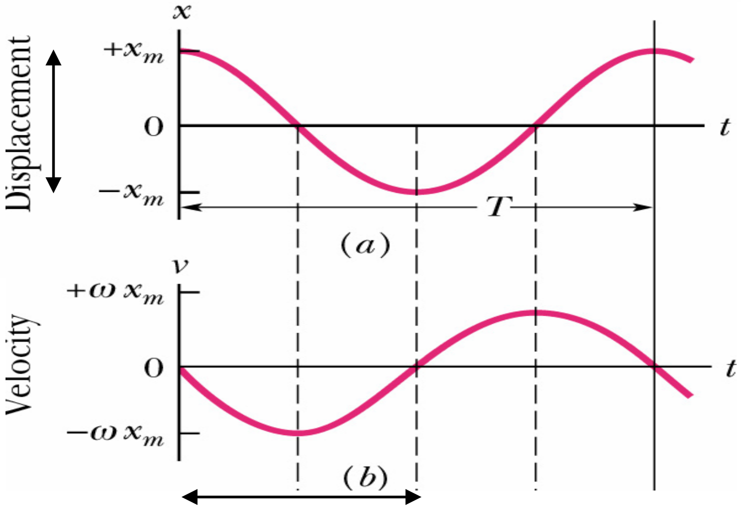
$$10 = 31.4 \times x_m$$

$$x_m = 0.32\, m$$

5) An object undergoing simple harmonic motion takes 0.25 s to travel from one point of zero velocity to the next such point. The distance between those points is 40 cm. The amplitude and frequency of the motion are:

A1 20 cm, 2 Hz
A2 40 cm, 2 Hz
A3 30 cm, 2 Hz
A4 30 cm, 4 Hz
A5 20 cm, 4 Hz

$2x_m$=40 cm



T/2=0.25 s

$$\frac{1}{2}T = 0.25 \Rightarrow T = 0.5\,s \Rightarrow f = \frac{1}{T} = 2\,Hz$$

$$2x_m = 40 \Rightarrow x_m = 20\,cm$$

6) The displacement of a particle moving with simple harmonic motion is given by: x = 0.02 cos (300 t - pi/3) , where x is in m and t is in sec. What is the maximum speed of the particle?

A1 6 m/s
A2 3 m/s
A3 300 m/s
A4 0.02 m/s
A5 pi/3 m/s

$$x = x_m \cos(\omega t + \phi)$$

$$x = 0.02 \cos(300t - \frac{\pi}{3})$$

$$x_m = 0.02\, m,\ \omega = 300\ rad/s,\ \phi = -\frac{\pi}{3}\ rad$$

$$v_m = x_m \omega = 0.02 \times 300 = 6\ m/s$$

7) A 3-kg block, attached to a spring, executes simple harmonic motion on a horizontal frictionless surface according to x = 2 cos(10 t + 3.14) where x is in meters and t is in seconds. Find the magnitude of the maximum ACCELERATION.

A1 200 m/s**2
A2 400 m/s**2
A3 20 m/s**2
A4 500 m/s**2
A5 00 m/s**2

$$x = x_m \cos(\omega t + \phi)$$

$$x = 2\cos(10t + 3.14)$$

$$x_m = 2\,m,\ \omega = 10\ rad/s,\ \phi = 3.14\ rad$$

$$a_m = x_m \omega^2 = 2 \times 10^2 = 200\ m/s^2$$

8) A block-spring system is set in a simple harmonic motion. The block has a kinetic energy of 6 J and an elastic potential energy of 2 J when the displacement of the block is 2.0 cm from the equilibrium point. What is the amplitude of the simple harmonic motion?

A1 4 cm
A2 2 cm
A3 8 cm
A4 10 cm
A5 6 cm

$$U = \frac{1}{2}kx^2 \Rightarrow 2 = \frac{1}{2}k(0.02)^2 \Rightarrow k = 10000\ N/m$$

$$E = K + U = 6 + 2 = 8\ J$$

$$E = \frac{1}{2}kx_m^2 \Rightarrow 8 = \frac{1}{2} \times 10000 x_m^2 \Rightarrow x_m = 0.04\ m = 4\ cm$$

9) Consider a simple harmonic motion, say as described by a mass-spring system. The ACCELERATION of the mass will be maximum when the

A1 displacement of the mass is maximum
A2 velocity of the mass is maximum
A3 displacement of the mass is minimum
A4 potential energy is minimum
A5 kinetic energy is maximum

$$a = -\omega^2 x\,, \quad a_m = \omega^2 x_m$$

$$when\ |x| = x_m \Rightarrow |a| = a_m$$

A simple harmonic oscillator is oscillating with an amplitude A. For what value of the DISPLACEMENT does the kinetic energy equal the potential energy?

A1 0.707 * A
A2 0.500 * A
A3 1.414 * A
A4 0.816 * A
A5 1.633 * A

$$K = U$$

$$\frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}kx^2$$

$$\frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}kx^2$$

$$\frac{1}{2}kA^2 = kx^2$$

$$A^2 = 2x^2$$

$$x = \sqrt{\frac{A^2}{2}} = \frac{1}{\sqrt{2}}A = 0.707A$$