Selected Problems from Chapter 10

1) The angular speed in rad/s of the minute hand of a watch is:
(Note that PI = 3.14159..)

A1 PI/1800
A2 PI/60
A3 PI/3600
A4 2*PI
A5 60

$$\omega = \frac{\theta}{t} = \frac{2\pi}{60\times 60} = \frac{\pi}{1800} \quad rad/s$$

2) A wheel of radius 0.10 m has a 2.5 m cord wrapped around its outside edge. Starting from rest, the wheel is given a constant angular acceleration of 2.0 rad/s**2. The cord will unwind in:

A1 5.0 s
A2 2.0 s
A3 8.0 s
A4 0.82 s
A5 130 s

$$\Delta\theta = \frac{s}{R} = \frac{2.5}{0.10} = 25\,rad$$

$$\omega_0 = 0, \alpha = 2.0\,rad/s^2, \quad t = ?$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow 25 = 0 + \frac{1}{2} \times 2.0 \times t^2$$

$$t = 5.0\,s$$

3) A rotating wheel has an initial angular velocity Wo. After 3.00 s its angular velocity is 98 rad/s. If it completes 37 revolutions during this 3.00 s interval, find Wo (assume constant angular acceleration).

A1 57.0 rad/s
A2 88.0 rad/s
A3 108 rad/s
A4 41.0 rad/s
A5 32.0 rad/s

$$t = 3.00s, \omega = 98\, rad/s,$$

$$\Delta\theta = 37\, rev \times \frac{2\pi\, rad}{1\, rev} = 232.5\, rad$$

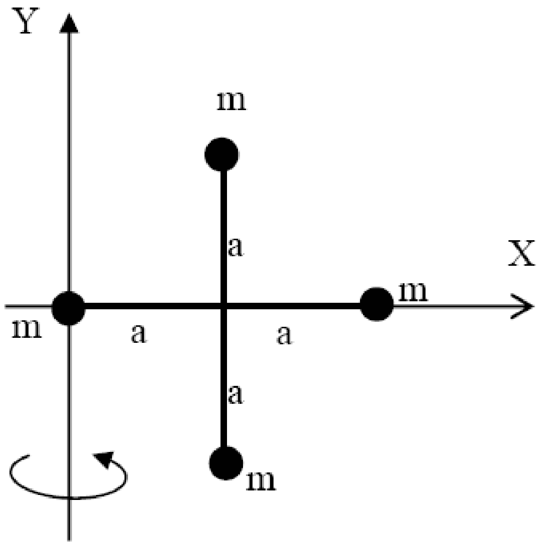
$$\Delta\theta = \frac{\omega + \omega_0}{2} t$$

$$232.5 = \frac{98 + \omega_0}{2} \times 3.00$$

$$\omega_0 = \frac{2 \times 232.5}{3.00} - 98 = 57.0\, rad/s$$

4) Four identical particles, each with mass m, are arranged in the x, y plane as shown in Fig. . They are connected by massless rods to form a rigid body. If m =2.0 kg and a =1.0 m, the rotational inertia of this array about the y-axis is:

A1 12 kg.m**2
A2 4.0 kg.m**2
A3 9.6 kg.m**2
A4 4.8 kg.m**2
A5 16 kg.m**2

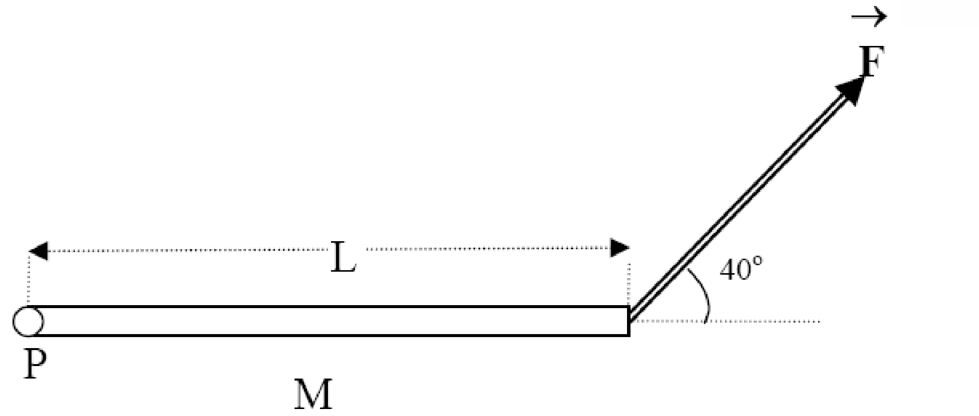


$$I = \sum m_i r^2_i = m \times 0 + 2m \times a^2 + m \times (2a)^2 = 6ma^2$$

$$= 6 \times 2.0 \times 1.0^2 = 12\ kg.m^2$$

5) A uniform rod of mass M = 1.2 kg and length L = 0.80 m is pivoted at point P and rests on a horizontal smooth surface (Fig.). If a force (F =5.0 N, theta = 40 degrees) is applied as shown, find its angular a

A1	10	rad/s**2
A2	16	rad/s**2
A3	12	rad/s**2
A4	8.0	rad/s**2
A5	33	rad/s**2



$$\tau = I\alpha$$

$$\tau = FL\sin 40 = 5.0\times 0.80\times \sin 4 = 2.57\,N.m$$

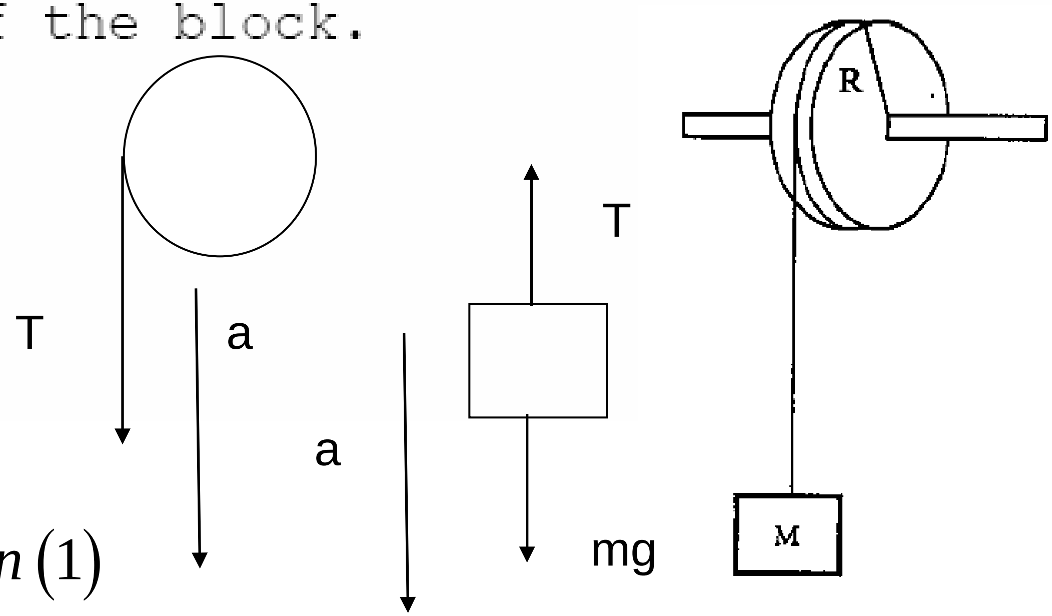
$$I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2$$

$$= \frac{1}{3}\times 1.2\times 0.80^2 = 0.256\;kg.m^2$$

$$\alpha = \frac{\tau}{I} = \frac{2.57}{0.256} = 10\;rad/s^2$$

6) A 10-kg block is attached to a cord that is wrapped around the rim of a flywheel of radius 0.5 m and hangs vertically (see Fig.). If the moment of inertia of the flywheel is 2.0 kg.m**2, find the magnitude of the linear acceleration of the block.

A1 5.4 m/s**2
A2 9.8 m/s**2
A3 0.0 m/s**2
A4 2.0 m/s**2
A5 3.5 m/s**2



$$mg - T = ma \quad (1)$$

$$\tau = I\alpha$$

$$TR = I \times \left(\frac{a}{R}\right)$$

$$T = \frac{I}{R^2} a \quad (2)$$

put (2) in (1)

$$mg - \frac{I}{R^2} a = ma$$

$$a = \frac{m}{m + \frac{I}{R^2}} g = \frac{10}{10 + \frac{2.0}{0.5^2}} 9.80 = 5.4\, m/s^2$$

7) A disk has a rotational inertia of 6.0 kg.m**2 and a constant angular acceleration of 2.0 rad/s**2. If it starts from rest, the work done by the net torque on it during the first 5.0 seconds is:

A1 300 J
A2 0 J
A3 60 J
A4 600 J
A5 30 J

$$W = \Delta K = \frac{1}{2} I\omega^2 - \frac{1}{2} I\omega_0{}^2 = \frac{1}{2} I\omega^2$$

$$\omega = \omega_0 + \alpha t = \alpha t = 2.0 \times 5.0 = 10\ rad/s$$

$$W = \frac{1}{2} \times 6.0 \times 10^2 = 300\ J$$

8) The rigid body shown in Fig. is rotated about an axis perpendicular to the paper and passing through point P. If M = 0.40 kg, a = 30 cm, b = 50 cm, find the work required to increase the angular velocity of the body from rest to 5.0 rad/s. (Neglect the force of friction, mass of the connecting rods and treat the particles as point masses).

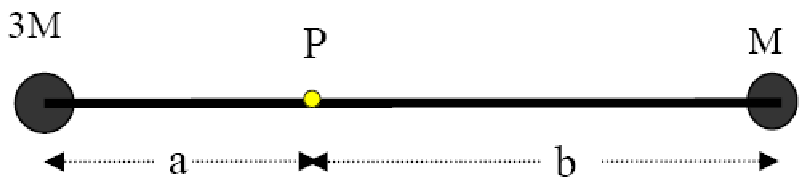
A1 2.6 J

A2 2.9 J

A3 3.4 J

A4 1.2 J

A5 4.3 J



$$I = \sum m_i r_i = 3M \times (0.3)^2 + M \times \left(0.5\right)^2$$

$$= 0.52M = 0.52 \times 0.40 = 0.208\, kg.m^2$$

$$W = \Delta K = \frac{1}{2} I\omega^2 + \frac{1}{2} I\omega_0{}^2 = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.208 \times 5.0^2 = 2.6\ J$$

9) A disk starts from rest and rotates around a fixed axis, subject to a constant net torque. The work done by the torque from t=0 to t=3.0 s is W1 and the work done from t=0 s to t=6 s is W2. The value of W1/W2 is:

A1 1/4
A2 2
A3 1/2
A4 1
A5 4

$$W_1 = \tau\,\theta_1$$

$$W_2 = \tau\,\theta_2$$

$$\frac{W_1}{W_2} = \frac{\tau\,\theta_1}{\tau\,\theta_2}$$

$$= \frac{\theta_1}{\theta_2} = \frac{\frac{1}{2}\alpha t^2{}_1}{\frac{1}{2}\alpha t^2{}_2}$$

$$= (\frac{t_1}{t_2})^2 = (\frac{3}{6})^2 = \frac{1}{4}$$

••4 The angular position of a point on the rim of a rotating wheel is given by $\theta = 4.0t - 3.0t^2 + t^3$, where θ is in radians and t is in seconds. What are the angular velocities at (a) $t =$ 2.0 s and (b) $t = 4.0$ s? (c) What is the average angular acceleration for the time interval that begins at $t = 2.0$ s and ends at $t = 4.0$ s? What are the instantaneous angular accelerations at (d) the beginning and (e) the end of this time interval?

4. If we make the units explicit, the function is

$$\theta = \left(4.0 \text{ rad/s}\right)t - \left(3.0 \text{ rad/s}^2\right)t^2 + \left(1.0 \text{ rad/s}^3\right)t^3$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Eq. 10-6 leads to

$$\omega = \frac{d}{dt}\left(4t - 3t^2 + t^3\right) = 4 - 6t + 3t^2.$$

Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

(b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

(c) Consequently, Eq. 10-7 gives

$$\alpha_{avg} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad/s}^2.$$

(d) And Eq. 10-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(4 - 6t + 3t^2\right) = -6 + 6t.$$

Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s^2.

(e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

•11 Starting from rest, a disk rotates about its central axis with constant angular acceleration. In 5.0 s, it rotates 25 rad. During that time, what are the magnitudes of (a) the angular acceleration and (b) the average angular velocity? (c) What is the instantaneous angular velocity of the disk at the end of the 5.0 s? (d) With the angular acceleration unchanged, through what additional angle will the disk turn during the next 5.0 s?

11. We assume the sense of rotation is positive, which (since it starts from rest) means all quantities (angular displacements, accelerations, etc.) are positive-valued.

(a) The angular acceleration satisfies Eq. 10-13:

$$25\ \text{rad} = \frac{1}{2}\alpha(5.0\ \text{s})^2 \;\Rightarrow\; \alpha = 2.0\ \text{rad/s}^2.$$

(b) The average angular velocity is given by Eq. 10-5:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{25\ \text{rad}}{5.0\ \text{s}} = 5.0\ \text{rad/s}.$$

(c) Using Eq. 10-12, the instantaneous angular velocity at $t = 5.0$ s is

$$\omega = \left(2.0\ \text{rad/s}^2\right)(5.0\ \text{s}) = 10\ \text{rad/s}.$$

(d) According to Eq. 10-13, the angular displacement at $t = 10$ s is

$$\theta = \omega_0 + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(2.0)(10)^2 = 100\ \text{rad}.$$

Thus, the displacement between $t = 5$ s and $t = 10$ s is $\Delta\theta = 100 - 25 = 75$ rad.

•22 An astronaut is being tested in a centrifuge. The centrifuge has a radius of 10 m and, in starting, rotates according to $\theta = 0.30t^2$, where t is in seconds and θ is in radians. When t = 5.0 s, what are the magnitudes of the astronaut's (a) angular velocity, (b) linear velocity, (c) tangential acceleration, and (d) radial acceleration?

22. (a) Using Eq. 10-6, the angular velocity at $t = 5.0$s is

$$\omega = \left.\frac{d\theta}{dt}\right|_{t=5.0} = \frac{d}{dt}\left(0.30t^2\right)\Big|_{t=5.0} = 2(0.30)(5.0) = 3.0 \text{ rad / s}.$$

(b) Eq. 10-18 gives the linear speed at $t = 5.0$s: $v = \omega r = (3.0 \text{ rad/s})(10 \text{ m}) = 30 \text{ m/s}$.

(c) The angular acceleration is, from Eq. 10-8,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(0.60t) = 0.60 \text{ rad / s}^2.$$

Then, the tangential acceleration at $t = 5.0$s is, using Eq. 10-22,

$$a_t = r\alpha = (10 \text{ m})\left(0.60 \text{ rad / s}^2\right) = 6.0 \text{ m/ s}^2.$$

(d) The radial (centripetal) acceleration is given by Eq. 10-23:

$$a_r = \omega^2 r = \left(3.0 \text{ rad / s}\right)^2 \left(10 \text{ m}\right) = 90 \text{ m / s}^2.$$

*34 Figure 10-32 gives angular speed versus time for a thin rod that rotates around one end. (a) What is the magnitude of the rod's angular acceleration? (b) At t = 4.0 s, the rod has a rotational kinetic energy of 1.60 J. What is its kinetic energy at t = 0?

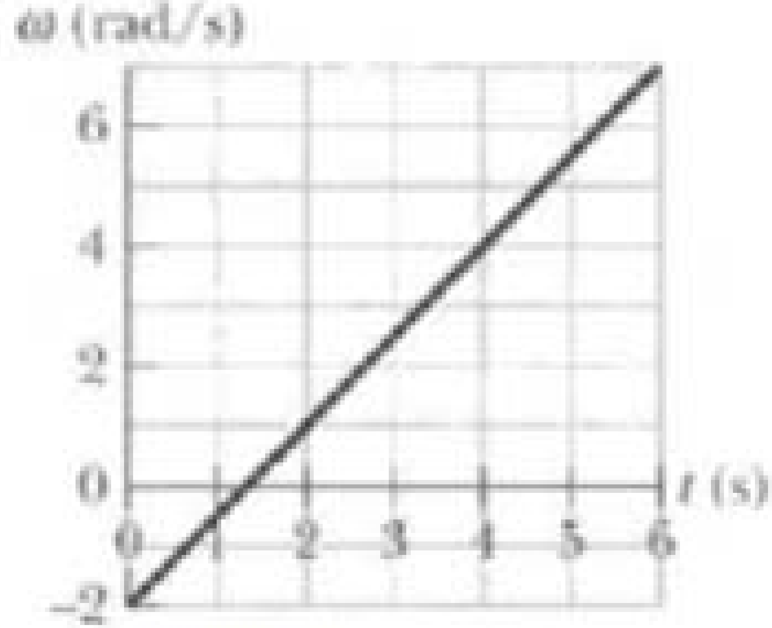


Fig. 10-32 Problem 34.

34. (a) Eq. 10-12 implies that the angular acceleration α should be the slope of the ω vs t graph. Thus, $\alpha = 9/6 = 1.5$ rad/s^2.

(b) By Eq. 10-34, K is proportional to ω^2. Since the angular velocity at $t = 0$ is -2 rad/s (and this value squared is 4) and the angular velocity at $t = 4$ s is 4 rad/s (and this value squared is 16), then the ratio of the corresponding kinetic energies must be

$$\frac{K_o}{K_4} = \frac{4}{16} \quad \Rightarrow \quad K_o = \tfrac{1}{4} K_4 = 0.40 \text{ J} .$$

**39 In Fig. 10-35, two particles, each with mass $m = 0.85$ kg, are fastened to each other, and to a rotation axis at O, by two thin rods, each with length $d = 5.6$ cm and mass $M = 1.2$ kg. The combination rotates around the rotation axis with angular speed $\omega = 0.30$ rad/s. Measured about O, what are the combination's (a) rotational inertia and (b) kinetic energy?

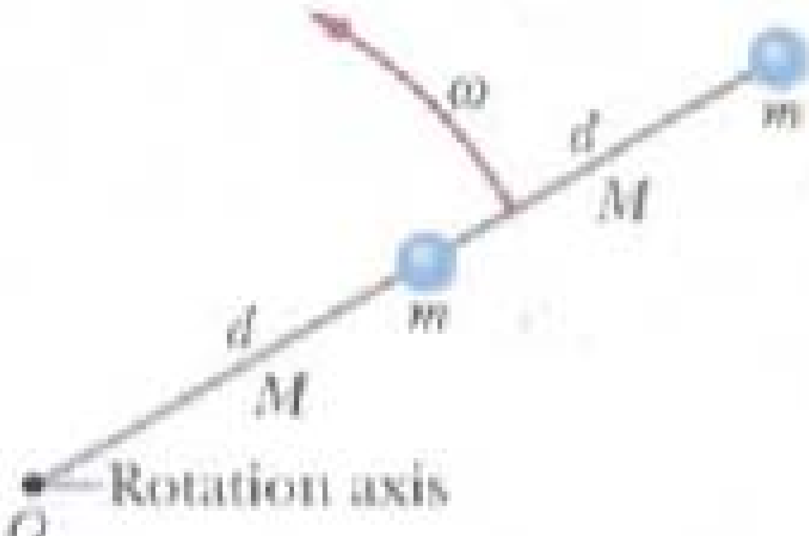


Fig. 10-35 Problem 39.

39. The particles are treated "point-like" in the sense that Eq. 10-33 yields their rotational inertia, and the rotational inertia for the rods is figured using Table 10-2(*e*) and the parallel-axis theorem (Eq. 10-36).

(a) With subscript 1 standing for the rod nearest the axis and 4 for the particle farthest from it, we have

$$I = I_1 + I_2 + I_3 + I_4 = \left(\frac{1}{12}Md^2 + M\left(\frac{1}{2}d\right)^2\right) + md^2 + \left(\frac{1}{12}Md^2 + M\left(\frac{3}{2}d\right)^2\right) + m(2d)^2$$
$$= \frac{8}{3}Md^2 + 5md^2 = \frac{8}{3}(1.2\ \text{kg})(0.056\ \text{m})^2 + 5(0.85\ \text{kg})(0.056\ \text{m})^2$$
$$= 0.023\ \text{kg}\cdot\text{m}^2.$$

(b) Using Eq. 10-34, we have

$$K = \frac{1}{2}I\omega^2 = \left(\frac{4}{3}M + \frac{5}{2}m\right)d^2\omega^2 = \left[\frac{4}{3}(1.2\ \text{kg}) + \frac{5}{2}(0.85\ \text{kg})\right](0.056\ \text{m})^2(0.30\ \text{rad/s})^2$$
$$= 1.1\times10^{-3}\ \text{J}.$$

••51 Figure 10-40 shows a uniform disk that can rotate around its center like a merry-go-round. The disk has a radius of 2.00 cm and a mass of 20.0 grams and is initially at rest. Starting at time $t = 0$, two forces are to be applied tangentially to the rim as indicated, so that at time $t = 1.25$ s the disk has an angular velocity of 250 rad/s counterclockwise. Force $\vec{F}_1$ has a magnitude of 0.100 N. What is magnitude F_2?

Fig. 10-40 Problem 51.

51. Combining Eq. 10-45 ($\tau_{net} = I\,\alpha$) with Eq. 10-38 gives $RF_2 - RF_1 = I\alpha$, where $\alpha = \omega / t$ by Eq. 10-12 (with $\omega_0 = 0$). Using item (c) in Table 10-2 and solving for F_2 we find

$$F_2 = \frac{MR\omega}{2t} + F_1 = \frac{(0.02)(0.02)(250)}{2(1.25)} + 0.1 = 0.140\,\text{N}.$$