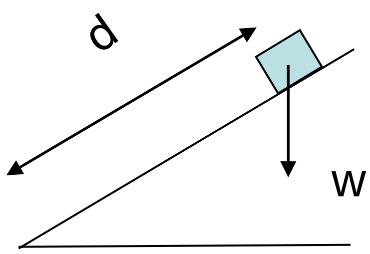
2nd major 062

Q1.

A 10.0 *kg* box slides with a constant speed a distance of 5.00 *m* downward along a rough slope that makes an angle of 30.0° with the horizontal. The work done by the force of gravity is:

A) 245 *J*
B) -490 *J*
C) -960 *J*
D) 424 *J*
E) 400 *J*



$$W_g = mgd\cos(60) = mgd\sin(30) = 10.0 \times 9.80 \times 5.00 \times \tfrac{1}{2} = 245J$$

Q2.

A block is attached to the end of an ideal spring and moved from coordinate x_i to coordinate x_f . The relaxed position is at $x = 0$. For which values of x_i and x_f that are given below, the work done by spring is positive?

A) x_i = -4 *cm* and x_f = -2 *cm*
B) x_i =-2 *cm* and x_f = 4 *cm*
C) x_i = -2 *cm* and x_f = -4 *cm*
D) x_i = 2 *cm* and x_f = -4 *cm*
E) x_i = 2 *cm* and x_f = 4 *cm*

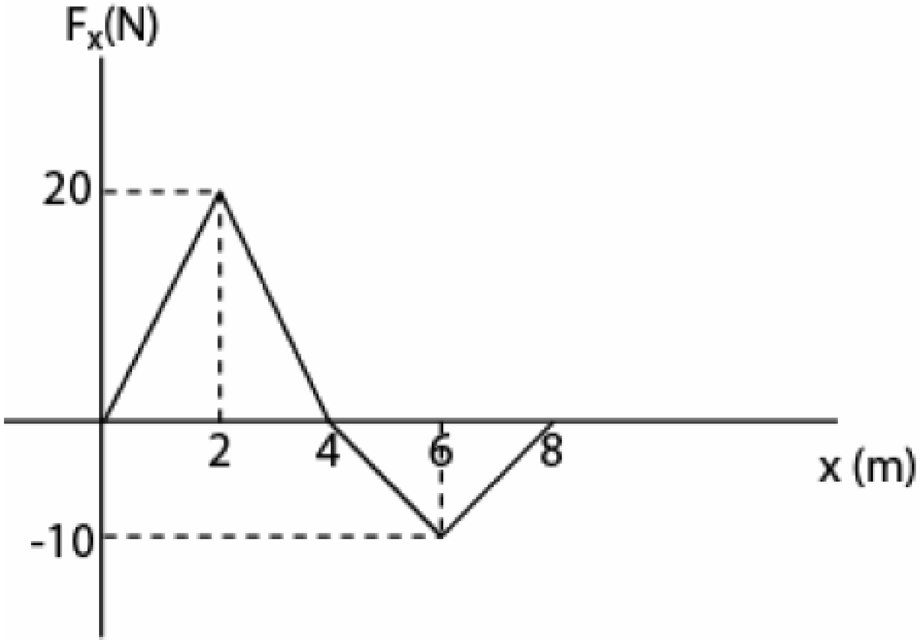
$$W_s = -\Delta U = U_i - U_f = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 = \tfrac{1}{2}k(x_i^2 - x_f^2)$$

$$W_s > 0 \; if \;\; x_i^2 > x_f^2$$

Q3.

Fig. 1 gives the only force F_x that can act on a particle. If the particle has a kinetic energy of 10 J at $x = 0$, find the kinetic energy of the particle when it is at $x = 8.0$ m.

A) 30 J
B) 20 J
C) 0 J
D) 60 J
E) 10 J



$$W = \textit{area under the curve}$$

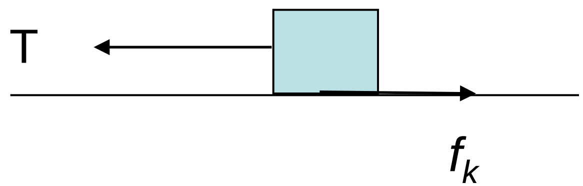
$$= \tfrac{1}{2}(4\times 20) + \tfrac{1}{2}(4\times -10) = \tfrac{1}{2}\times 40 = 20J$$

$$W = \Delta K = K_f - K_i = K_f = 20J$$

Q4.

A 200 *kg* box is pulled along a horizontal surface by an engine. The coefficient of friction between the box and the surface is 0.400. The power the engine delivers to move the box at constant speed of 5.00 *m/s* is:

A) 3920 *W*
B) 1960 *W*
C) 980 *W*
D) 490 *W*
E) 0 *W*



$$T = f_k \ (constant\ speed)$$

$$f_k = \mu_k N = \mu_k mg$$

$$Power = Tv = \mu_k mgv = 0.400 \times 200 \times 9.80 \times 5.00 = 3920\, W$$

Q5.

A 2.0 *kg* object is connected to one end of an unstretched spring which is attached to the ceiling by the other end and then the object is allowed to drop. The spring constant of the spring is 196 *N/m*. How far does it drop before coming to rest momentarily?

A) 0.20 *m*
B) 0.10 *m*
C) 0.40 *m*
D) 0.80 *m*
E) 0.50 *m*

$$E_i = E_f$$

$$K_i + U_{si} + U_{gi} = K_f + U_{sf} + U_{gf}$$

$$K_i = 0, K_f = 0,$$

$$U_{si} = 0, take\, U_{gi} = 0$$

$$\rightarrow U_{sf} + U_{gf} = 0$$

$$\tfrac{1}{2}kx^2 - mgx = 0$$

$$x = \frac{2mg}{k} = \frac{2 \times 2.0 \times 9.80}{196} = 0.20\ m$$

Q6.
A 2.0 *kg* block is thrown upward from the ground. At what height above the ground will the gravitational potential energy of the Earth-block system have increased by 490 *J*?

A) 25 *m*
B) 50 *m*
C) 12 *m*
D) 8.0 *m*
E) 18 *m*

$$mgh = 490$$

$$h = \frac{490}{2.0 \times 9.80} = 25\,m$$

Q7.

An ideal spring (compressed by 7.00 *cm* and initially at rest,) fires a 15.0 *g* block horizontally across a frictionless table top. The spring has a spring constant of 20.0 *N/m*. The speed of the block as it leaves the spring is:

A) 2.56 *m/s*
B) 1.90 *m/s*
C) 3.64 *m/s*
D) 8.12 *m/s*
E) 5.25 *m/s*

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$K_i = 0,\ K_f = \tfrac{1}{2}mv^2,$$

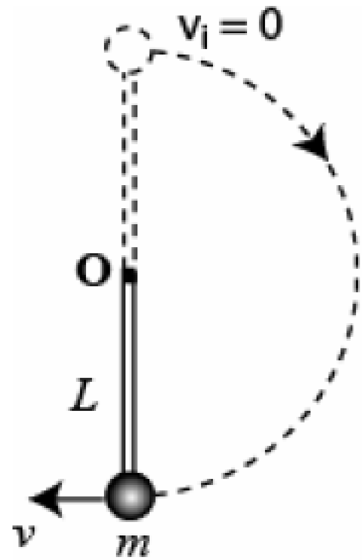
$$U_i = \tfrac{1}{2}kx^2,\ take\ U_f = 0$$

$$\rightarrow \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2$$

$$v = \sqrt{\frac{k}{m}}\,x = \sqrt{\frac{20.0}{0.015}} \times 0.0700 = 2.56\ m/s$$

Q8.

A small object of mass *m* on the end of a massless rod of length *L* is held vertically, initially. The rod is pivoted at the other end **O**. The object is then released from rest and allowed to swing down in a circular path as shown in Fig. 2. What is the speed (v) of the object at the lowest point of its swing? (Assume no friction at the pivot)



A) $\sqrt{4gL}$

B) $\sqrt{2gL}$

C) $\sqrt{gL}$

D) $\sqrt{gL/2}$

E) $\sqrt{gL/4}$

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$K_i = 0,\ K_f = \tfrac{1}{2} I\omega^2,$$

$$U_i = 0,\ take\ U_f = -mg(2L)$$

$$\rightarrow 0 + 0 = \tfrac{1}{2} I\omega^2 - 2mgL$$

$$\omega = \sqrt{\frac{4mgL}{I}}$$

$$I = mL^2 \rightarrow \omega = \sqrt{\frac{4mgL}{mL^2}} = \sqrt{\frac{4g}{L}},$$

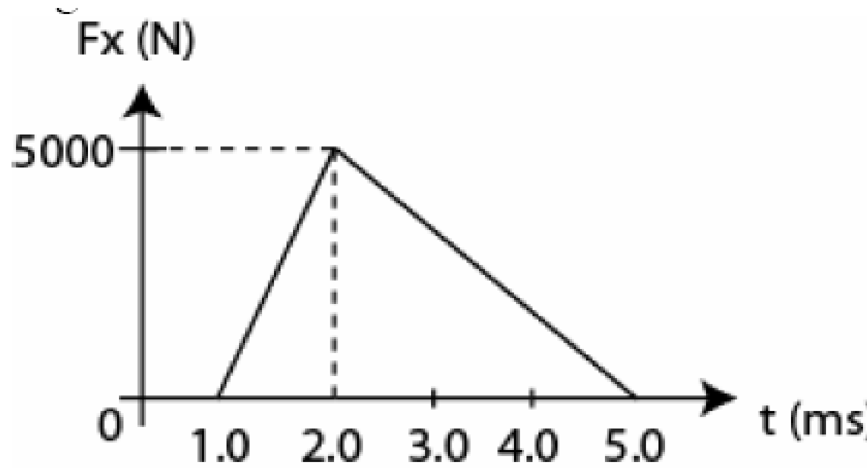
$$but\ \omega = \frac{v}{L}$$

$$\therefore v = \sqrt{4gL}$$

Q9.

An impulsive force F_x as a function of time (in ms) is shown in the Fig. 3 as applied to an object ($m = 5.0\ kg$) at rest. What will be its final speed?

A) 2.0 m/s.
B) - 3.2 m/s.
C) 8.0 m/s.
D) 16 m/s.
E) 4.2 m/s.



$$the\ \text{Im}\ pulse\ J = area\ under\ the\ curve = \tfrac{1}{2} \times 4.0 \times 10^{-3} \times 5000 = 10\ N.s$$

$$but\ J = \Delta p = mv_f - mv_i = m(v_f - v_i) = mv_f$$

$$\therefore 10 = 5.0 v_f$$

$$v_f = 2.0\ m/s$$

Q10.

Each object in Fig. 4 has a mass of 2.0 kg. The mass m_1 is at rest, m_2 has a speed of 3.0 m/s in the direction of +ve x-axis and m_3 has a speed of 6.0 m/s in the direction of +ve y-axis. The momentum of the center of mass of the system is:

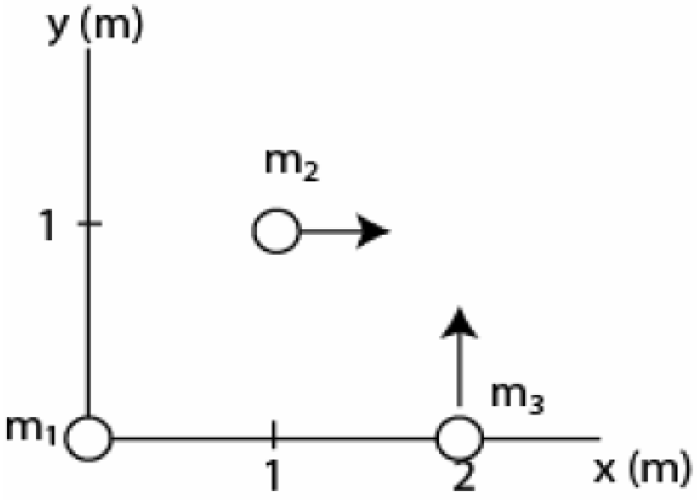
A) $\left(6.0\hat{i}+12\hat{j}\right)kg\cdot m/s$

B) $\left(1.0\hat{i}+2.0\hat{j}\right)kg\cdot m/s$

C) $\left(3.0\hat{i}+6.0\hat{j}\right)kg\cdot m/s$

D) $3.0\,kg\cdot m/s$

E) $\left(-3.0\hat{i}+6.0\hat{j}\right)kg\cdot m/s$



$$\vec{P}_{com}=\vec{P}=\vec{p}_1+\vec{p}_2+\vec{p}_3$$

$$=m_1\vec{v}_1+m_2\vec{v}_2+m_3\vec{v}_3$$

$$=2.0\times0+2.0\times3.0i+2.0\times6.0j$$

$$=(6.0i+12j)kg.m/s$$

Q11.
A 0.20 *kg* steel ball, travels along the *x*-axis at 10 *m/s*, undergoes an elastic collision with a 0.50 *kg* steel ball traveling along the *y*-axis at 4.0 *m/s*. The total kinetic energy of the two balls after collision is:

A) 14 *J*.
B) 18 *J*.
C) 4.0 *J*.
D) 10 *J*.
E) $\left(10\hat{i} + 4.0\hat{j}\right) J$

$$K_i = K_f \quad \left(elastic\ collision\right)$$

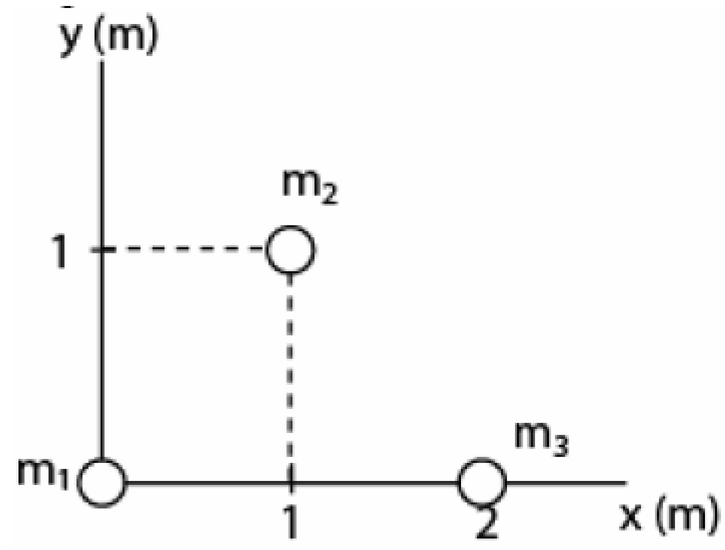
$$K_i = \tfrac{1}{2} m_1 v_{1i}^{\ 2} + \tfrac{1}{2} m_2 v_{2i}^{\ 2} = \tfrac{1}{2} \times 0.20 \times 10^2 + \tfrac{1}{2} \times 0.50 \times 4.0^2 = 14J$$

$$\therefore K_f = 14J$$

Q12.

If the masses of m_1 and m_3 in Fig. 5 are 1.0 *kg* each and m_2 is 2.0 *kg*, what are the coordinates of the center of mass?

A) (1.00, 0.50) *m*
B) (0.50, 1.00) *m*
C) (1.25, 0.50) *m*
D) (0.75, 1.00) *m*
E) (0.50, 0.75) *m*



$$x_{com} = \frac{\sum m_i x_i}{\sum m_i} = \frac{1.0\times0+2.0\times1+1.0\times2}{1.0+2.0+1.0} = \frac{4.0}{4.0} = 1.0\ m$$

$$y_{com} = \frac{\sum m_i y_i}{\sum m_i} = \frac{1.0\times0+2.0\times1+1.0\times0}{1.0+2.0+1.0} = \frac{2.0}{4.0} = 0.5\ m$$

Q13.

A torque of 0.80 *N·m* applied to a pulley increases its angular speed from 45.0 *rev/min* to 180 *rev/min* in 3.00 *s*. Find the moment of inertia of the pulley.

A) 0.17 $kg \cdot m^2$
B) 0.21 $kg \cdot m^2$
C) 0.54 $kg \cdot m^2$
D) 0.42 $kg \cdot m^2$
E) 0.30 $kg \cdot m^2$

$$\tau = I\alpha$$

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$\omega_i = 45.0 rev/\min \times \frac{1\min}{60s} \times \frac{2\pi}{1rev} = 4.71\, rad/s$$

$$\omega_i = 180 rev/\min \times \frac{1\min}{60s} \times \frac{2\pi}{1rev} = 18.8\, rad/s$$

$$\alpha = \frac{18.8 - 4.71}{3.00} = 4.70\, rad/s^2$$

$$I = \frac{\tau}{\alpha} = \frac{0.80}{4.70} = 0.17\, kg.m^2$$

Q14.

A thin rod of mass 0.23 *kg* and length 1.00 *m* is rotated in a horizontal circle about a fixed axis passing through a point 20.0 *cm* from one of the edges of the rod. If it has a constant angular acceleration of 3.0 rad/s^2, find the net torque acting on the rod?

A) 0.12 $N\cdot m$
B) 0.085 $N\cdot m$
C) 0.028 $N\cdot m$
D) 0.15 $N\cdot m$
E) 0.077 $N\cdot m$

$$\tau_{net} = I\alpha$$

$$I = I_{com} + mh^2 \left(parallel\ axis\ theorem \right)$$

$$I_{com} = \tfrac{1}{12} ML^2, \quad h = 0.50m - 0.20m = 0.30m$$

$$I = \tfrac{1}{12} \times 0.23 \times 1.00^2 + 0.23 \times 0.30^2 = 0.040\ kg.m^2$$

$$\tau_{net} = 0.040 \times 3.0 = 0.12\ N.m$$

Q15.

A disk starts from rest at $t = 0$, and rotates about a fixed axis (moment of inertia = 0.030 $kg{\cdot}m^2$) with an angular acceleration of 7.5 rad/s^2. What is the rate at which work is being done on the disk when its angular velocity is 32 rad/s?

A) 7.2 W
B) 5.5 W
C) 3.1 W
D) 8.7 W
E) 2.2 W

$$P = \tau\omega = I\alpha\omega = 0.030 \times 7.5 \times 32 = 7.2\,W$$

Q16.
A disk has a rotational inertia of 4.0 $kg{\cdot}m^2$ and a constant angular acceleration of 2.0 rad/s^2. If it starts from rest the work done during the first 5.0 s by the net torque acting on it is:

A) 200 J
B) 100 J
C) 40 J
D) 0 J
E) 400 J

$$W = \Delta K = K_f - K_i = K_f = \frac{1}{2} I\omega^2$$

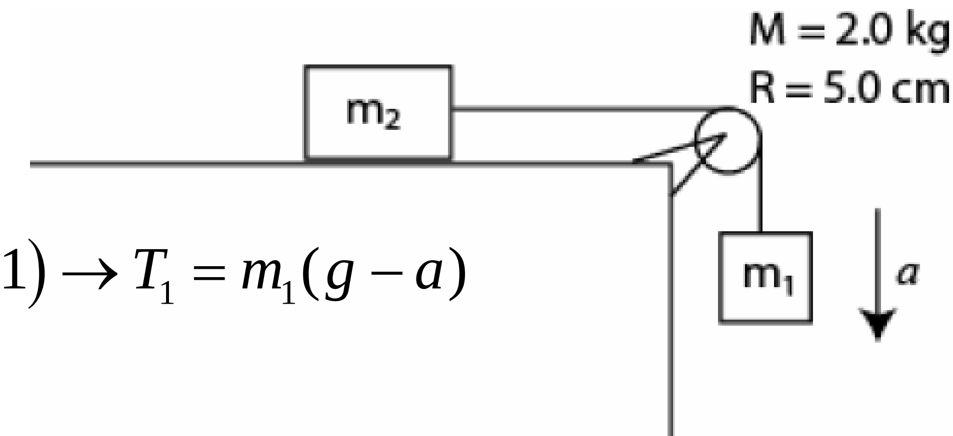
$$\omega = \alpha t = 2.0 \times 5.0 = 10\ rad/s$$

$$\therefore W = \frac{1}{2} \times 4.0 \times 10^2 = 200J$$

Q17.

A mass, $m_1 = 5.0\ kg$, hangs from a string and descends with an acceleration = a. The other end is attached to a mass $m_2 = 4.0\ kg$ which slides on a frictionless horizontal table. The string goes over a pulley (a uniform disk) of mass $M = 2.0\ kg$ and radius $R = 5.0\ cm$ (see Fig. 6). The value of a is:

A) $4.9\ m/s^2$
B) $5.4\ m/s^2$
C) $9.8\ m/s^2$
D) $2.0\ m/s^2$
E) $1.0\ m/s^2$



$$m_1 g - T_1 = m_1 a \quad (1) \rightarrow T_1 = m_1(g - a)$$

$$T_2 = m_2 a \quad (2)$$

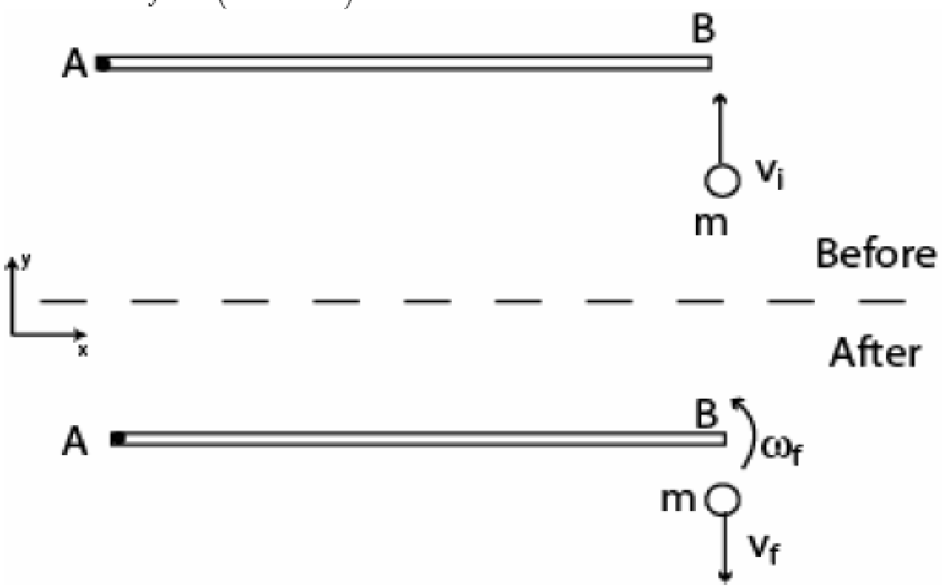
$$T_1 R - T_2 R = I\alpha = \tfrac{1}{2} M R^2 \left(\frac{a}{R}\right) \quad (3)$$

$$\rightarrow T_1 - T_2 = \tfrac{1}{2} M a$$

$$m_1(g - a) - m_2 a = \tfrac{1}{2} M a$$

$$\rightarrow a = \frac{m_1}{\tfrac{1}{2} M + m_1 + m_2} = 4.9\ m/s^2$$

Q18.
Fig. 7 shows an overhead view of a thin rod of mass M (=2.0 kg) and length L = 2.0 m which can rotate horizontally about a vertical axis through the end A. A particle of mass m = 2.0 kg traveling horizontally with a velocity $\vec{v}_i = \left(10\,\hat{j}\right) m/s$ strikes the rod (which was initially at rest) at point B. The particle rebounds with a velocity $\vec{v}_f = \left(-6.0\,\hat{j}\right) m/s$. Find the angular speed (ω_f) of the rod just after collision.



A) 24 rad/s
B) 2.0 rad/s
C) 10 rad/s
D) 50 rad/s
E) 30 rad/s

no external τ :

$$\vec{L}_i = \vec{L}_f$$

$$\vec{l}_{ri} + \vec{l}_{pi} = \vec{l}_{rf} + \vec{l}_{pf} \quad (rod(r), particle(p)) \quad (1)$$

$$\vec{l}_{ri} = 0, l_{pi} = mv_i L = 2.0 \times 10 \times 2.0 = 40\ kg.m^2/s$$

$$l_{pf} = mv_i L = 2.0 \times (-6.0) \times 2.0 = -24\,j\ kg.m^2/s$$

$$l_{rf} = I\omega_f = (I_{com} + M\left(\frac{L}{2}\right)^2)\omega_f = (\frac{1}{12}ML^2 + \frac{1}{4}ML^2)\omega_f = \frac{1}{3}ML^2\omega_f = \frac{1}{3} \times 2.0 \times 2.0^2\omega_f = \frac{8.0}{3}\omega_f$$

$$Eq.1 \rightarrow 0 + 40 = -24 + \frac{8.0}{3}\omega_f \rightarrow \omega_f = 24\ rad/s$$

Q19.

A string is wrapped around a solid disk of mass m, radius R. The string is stretched in the vertical direction and the disk is released as shown in Fig. 8. Find the tension (T) in the string.

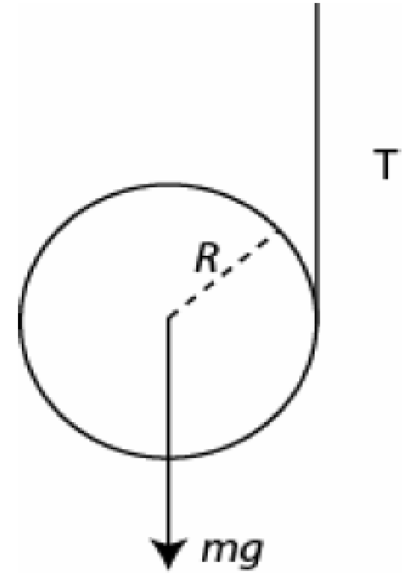
A) $\frac{1}{3}mg$

B) $\frac{3}{2}mg$

C) $\frac{2}{5}mg$

D) $\frac{2}{3}mg$

E) $\frac{3}{4}mg$



$$mg - T = ma \quad (1)$$

$$TR = I\alpha = \left(\tfrac{1}{2}mR^2\right)\left(\frac{a}{R}\right) \rightarrow T = \tfrac{1}{2}ma \quad (2)$$

$$Eq.\,1 \rightarrow mg - \tfrac{1}{2}ma = ma$$

$$a = \tfrac{2}{3}g$$

$$Eq.\,2 \rightarrow T = \tfrac{1}{2}m\left(\tfrac{2}{3}g\right) = \tfrac{1}{3}mg$$

Q20.
The engine delivers 1.20×10^5 W to a plane propeller at $\omega = 2400$ rev/min. How much work does the engine do in one revolution?

A) 3000 J
B) 4000 J
C) 5000 J
D) 2000 J
E) 1000 J

$$W = Pt$$

$$\omega = 2400\ rev/\text{min}$$

$$T = t(\ for\ one\ rev) = \frac{1}{2400}\text{min} = \frac{60}{2400} = \frac{1}{40}\text{sec}$$

$$W = 1.20 \times 10^5 \times \frac{1}{40} = 3000\ J$$