

2nd major 061

Q1.

A 16 kg crate falls from rest from a height of 1.0 m onto a spring scale with a spring constant of $2.74 \times 10^3 \text{ N/m}$. Find the maximum distance the spring is compressed.

- A) 40 cm
- B) 2.0 cm
- C) 60 cm
- D) 7.0 cm
- E) 5.0 cm

$$E_i = E_f$$

$$K_i + U_{si} + U_{gi} = K_f + U_{sf} + U_{gf} \quad (s : \text{spring}, g : \text{gravity}) \quad (1)$$

$$K_i = 0, K_f = 0$$

$$U_{si} = 0, \text{ take } U_{gi} = 0$$

$$U_{sf} = \frac{1}{2}kx^2, U_{gf} = -mg(h + x) \quad (h = 1.0\text{m}, x : \text{max. distance the spring is compressed})$$

$$\text{Eq. } \rightarrow 0 + 0 + 0 = 0 + \frac{1}{2}kx^2 - mg(h + x)$$

$$\frac{1}{2}kx^2 = mg(h + x) \rightarrow \frac{1}{2} \times 2.74 \times 10^3 \times x^2 = 16 \times 9.80 \times (h + x)$$

$$8.74x^2 - x - 1 = 0 \quad x = 0.40 \text{ m}$$

Q2.

A 4.0 kg cart starts up an incline with a speed of 3.0 m/s and comes to rest 2.0 m up the incline. The net work done on the cart is:

A) -18 J

B) 8.0 J

C) -12 J

D) 6.0 J

E) -20 J

$$W_{net} = \Delta K = K_f - K_i = -K_i \quad (\text{as } K_f = 0)$$

$$= -\frac{1}{2}mv_i^2 = -\frac{1}{2} \times 4.0 \times 3.0^2 = -18 \text{ J}$$

Q3.

A net horizontal force of 50 N is acting on a 2.0 kg crate that starts from rest on a horizontal frictionless surface. At the instant the object has traveled 2.0 m , the rate at which this net force doing work is:

- A) 500 W
- B) 25 W
- C) 75 W
- D) 100 W
- E) 2.5 W

$$a = \frac{F}{m} = \frac{50}{2.0} = 25\text{ m/s}^2$$

$$v_f^2 = v_i^2 + 2ad = 0 + 2 \times 25 \times 2.0 = 100\text{ m}^2/\text{s}^2$$

$$v_f = 10\text{ m/s}$$

$$P = Fv = 50 \times 10 = 500\text{ W}$$

Q4.

At time $t = 0$ a single force (\vec{F}) acts on a 2.0 kg particle and changes its velocity from $\vec{v}_i = (4.0\hat{i} - 3.0\hat{j}) \text{ m/s}$ at $t = 0$ to $\vec{v}_f = (4.0\hat{i} + 3.0\hat{j}) \text{ m/s}$ at $t = 3.0 \text{ s}$. During this time the work done by \vec{F} on the particle is:

- A) 0
- B) 6.0 J
- C) 2.0 J
- D) 50 J
- E) 10 J

$$W = \Delta K = K_f - K_i$$

$$v_i = \sqrt{4.0^2 + (-3.0)^2} = 5.0 \text{ m/s}$$

$$v_f = \sqrt{4.0^2 + (3.0)^2} = 5.0 \text{ m/s}$$

$$K_i = K_f$$

$$\therefore W = 0$$

Q5.

A 2.2 kg block starts from rest on a rough inclined plane that makes an angle of 25° with the horizontal. The coefficient of kinetic friction is 0.25. As the block slides 2.0 m down the plane, the mechanical energy of the Earth-block system changes by:

A) $-9.8 J$

B) $0 J$

C) $9.8 J$

D) $-18 J$

E) $-11 J$

No external forces :

$$\begin{aligned}\Delta E &= -\Delta E_{th} = -f_k d = -\mu_k N d = -\mu_k (mg \cos(25)) d \\ &= -0.25 \times 2.2 \times 9.80 \times \cos(25) \times 2.0 = -9.8 J\end{aligned}$$

Q6.

An ideal spring with a 20 N/m spring constant is compressed by a 10 N force. The potential energy stored in the spring is:

- A) 2.5 J
- B) 0.50 J
- C) 5.0 J
- D) 10 J
- E) 200 J

$$x = \frac{10}{20} = 0.50 \text{ m}$$

$$U = \frac{1}{2} kx^2 = \frac{1}{2} \times 20 \times 0.50^2 = 2.5 \text{ J}$$

Q7.

An object of mass m , attached to a light cord of length L , is held horizontally from a fixed support as shown in Fig 1. The object is then released from rest. What is the tension force in the cord when the object is at the lowest point of its swing?

- A) $3 mg$
- B) mg
- C) $2 mg$
- D) $mg / 2$
- E) mgL

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f \quad (1)$$

$$K_i = 0, \text{ take } U_i = 0$$

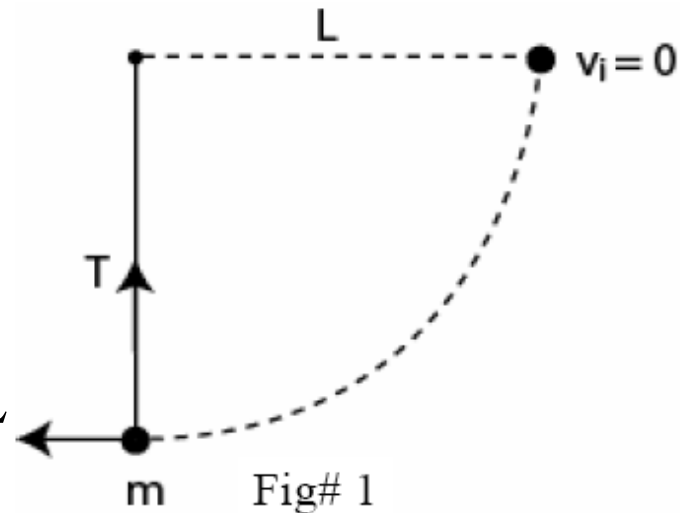
$$K_f = \frac{1}{2}mv^2, U_f = -mgL$$

$$\text{Eq.1} \rightarrow 0 + 0 = \frac{1}{2}mv^2 - mgL$$

$$\frac{1}{2}mv^2 = mgL \rightarrow v = \sqrt{2gL}$$

$$T - mg = \frac{mv^2}{L} \quad (\text{centripital acceleration})$$

$$\rightarrow T = mg + \frac{mv^2}{L} = m\left(g + \frac{v^2}{L}\right) = m\left(g + \frac{2gL}{L}\right) = 3mg$$



Q8.

A block of mass 2.0 kg is initially moving to the right on a horizontal frictionless surface at a speed 5.0 m/s . It then compresses a spring of spring constant 100 N/m . At the instant when the kinetic energy of the block is equal to the potential energy of the spring, the spring is compressed a distance of:

A) 0.50 m

B) 0.25 m

C) 1.0 m

D) 0.75 m

E) 0.10 m

$$E_i = K_i = \frac{1}{2}mv^2 = \frac{1}{2} \times 2.0 \times 5.0^2 = 25 \text{ J} = E_f$$

because in this case : $K_f = U_f$

$$E_f = 2U_f$$

$$25 = 2\left(\frac{1}{2}kx^2\right) = 100 \times x^2$$

$$x = 0.50 \text{ m}$$

Q9.

Two identical 1500 kg cars are moving perpendicular to each other. One moves with a speed of 25 m/s due north and the other moves at 15 m/s due east. What is the total momentum of the system?

- A) $4.4 \times 10^4 \text{ kg}\cdot\text{m/s}$ at 59° North of East
- B) $4.4 \times 10^4 \text{ kg}\cdot\text{m/s}$ at 31° North of East
- C) $6.0 \times 10^4 \text{ kg}\cdot\text{m/s}$ at 31° North of East
- D) $6.0 \times 10^4 \text{ kg}\cdot\text{m/s}$ at 59° North of East
- E) $4.0 \times 10^4 \text{ kg}\cdot\text{m/s}$ at 59° North of East

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2 = 1500(25j + 15i) \\ &= 7500(3i + 5j)\end{aligned}$$

$$P = 7500\sqrt{3^2 + 5^2} = 4.4 \times 10^4$$

$$\text{arc tan}(\theta) = \tan^{-1}\left(\frac{5}{3}\right) = 59^\circ$$

Q10.

A small object with linear momentum $5.0 \text{ kg}\cdot\text{m/s}$ makes a head-on collision with a large object at rest. The small object bounces straight back with a momentum of magnitude $4.0 \text{ kg}\cdot\text{m/s}$. What is the magnitude of the change in momentum of the large object?

A) $9.0 \text{ kg}\cdot\text{m/s}$

B) $5.0 \text{ kg}\cdot\text{m/s}$

C) $4.0 \text{ kg}\cdot\text{m/s}$

D) $1.0 \text{ kg}\cdot\text{m/s}$

E) $3.0 \text{ kg}\cdot\text{m/s}$

$$\vec{J} = \vec{\Delta p}$$

$$\vec{J}_1 = -\vec{J}_2$$

$$\vec{\Delta p}_1 = -\vec{\Delta p}_2$$

$$\vec{\Delta p}_2 = -\vec{\Delta p}_1 = p_{1f} - p_{1i} = -4.0 - 5.0 = -9.0 \text{ kg}\cdot\text{m/s}$$

$$\Delta p_2 = 9.0 \text{ kg}\cdot\text{m/s}$$

Q11.

A 1500 kg car traveling at 90.0 km/h east collides with a 3000 kg car traveling at 60.0 km/h south. The two cars stick together after the collision (see Fig 2). What is the speed of the cars after collision?

- A) 13.9 m/s
- B) 8.33 m/s
- C) 17.4 m/s
- D) 21.7 m/s
- E) 50.0 m/s

$$\vec{v}_1 = 90.0 \times \frac{1000}{60 \times 60} = 25.0 \text{ m/s}$$

$$\vec{v}_2 = 60.0 \times \frac{1000}{60 \times 60} = 16.7 \text{ m/s}$$

$$\vec{P}_i = \vec{P}_f$$

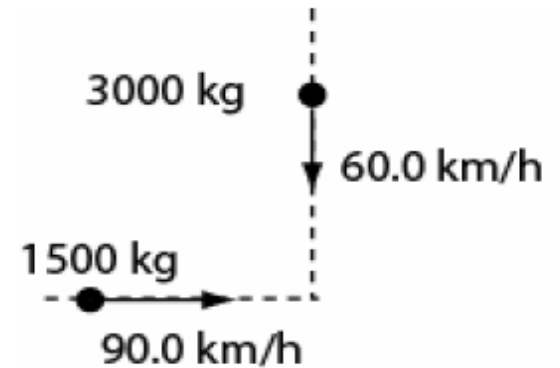
$$\vec{p}_1 + \vec{p}_2 = \vec{P}_f$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{V}$$

$$1500 \times 25.0i + 3000 \times 16.7j = 4500\vec{V}$$

$$\vec{V} = 0.333 \times (25.0i + 33.4j) = 8.33i + 11.1j$$

$$V = \sqrt{8.33^2 + 11.1^2} = 13.9 \text{ m/s}$$



Fig# 2 Before Collision

Q12.

A 3.0 kg mass is positioned at (0, 8.0) m, and a 1.0 kg mass is positioned at (12, 0) m. What are the coordinates of a 4.0 kg mass which will result in the center of mass of the system of three masses being located at the origin (0, 0)?

- A) (-3.0, -6.0) m
- B) (-12, -8.0) m
- C) (3.0, 6.0) m
- D) (-6.0, -3.0) m
- E) (-12, 0) m

$$x_{com} = \frac{\sum m_i x_i}{\sum m_i} = \frac{3.0 \times 0 + 1.0 \times 12 + 4.0x}{3.0 + 1.0 + 4.0} = \frac{12 + 4.0x}{8.0} = 0$$

$$\rightarrow 12 + 4.0x = 0 \rightarrow x = -3.0 \text{ m}$$

$$y_{com} = \frac{\sum m_i y_i}{\sum m_i} = \frac{3.0 \times 8.0 + 1.0 \times 0 + 4.0y}{3.0 + 1.0 + 4.0} = \frac{24 + 4.0y}{8.0} = 0$$

$$\rightarrow 24 + 4.0y = 0 \rightarrow y = -6.0 \text{ m}$$

Q13.

A string (one end attached to the ceiling) is wound around a uniform solid cylinder of mass $M = 2.0 \text{ kg}$ and radius $R = 10 \text{ cm}$ (see Fig 3). The cylinder starts falling from rest as the string unwinds. The linear acceleration of the cylinder is:

- A) 6.5 m/s^2
- B) 9.8 m/s^2
- C) 3.2 m/s^2
- D) 4.9 m/s^2
- E) 2.1 m/s^2

$$mg - T = ma \quad (1)$$

$$TR = I\alpha = \left(\frac{1}{2}mR^2\right) \times \left(\frac{a}{R}\right)$$

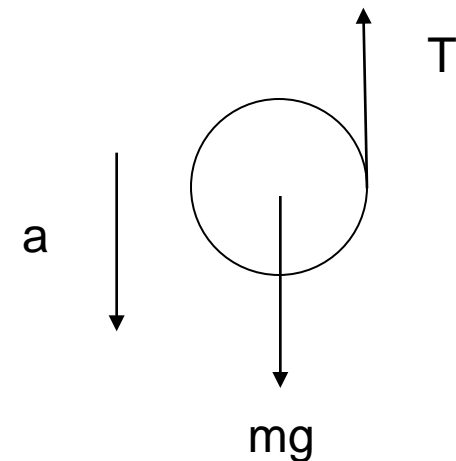
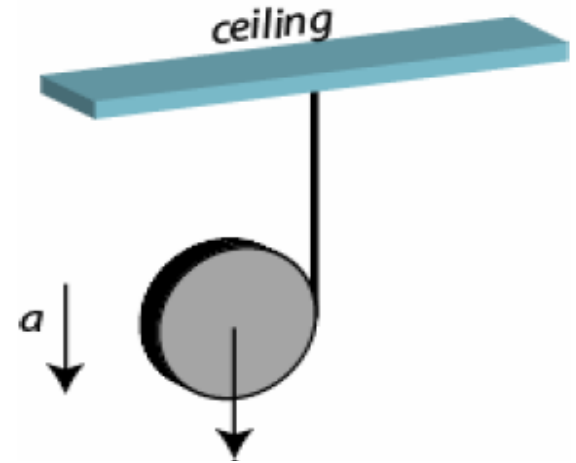
$$\rightarrow T = \frac{1}{2}ma \quad (2)$$

from Eq.1 & 2:

$$mg - \frac{1}{2}ma = ma$$

$$a = \frac{2}{3}g = \frac{2}{3} \times 9.8 = 6.5 \text{ m/s}^2$$

Fig# 3



Q14.

A 16 kg block is attached to a cord that is wound around the rim of a flywheel of radius 0.20 m and hangs vertically, as shown in Fig 4. The rotational inertia of the flywheel is 0.50 kg·m². When the block is released and the cord unwinds, the acceleration of the block is:

- A) 5.5 m/s²
- B) 1.5 m/s²
- C) 8.2 m/s²
- D) 9.8 m/s²
- E) 13 m/s²

$$mg - T = ma \quad (1)$$

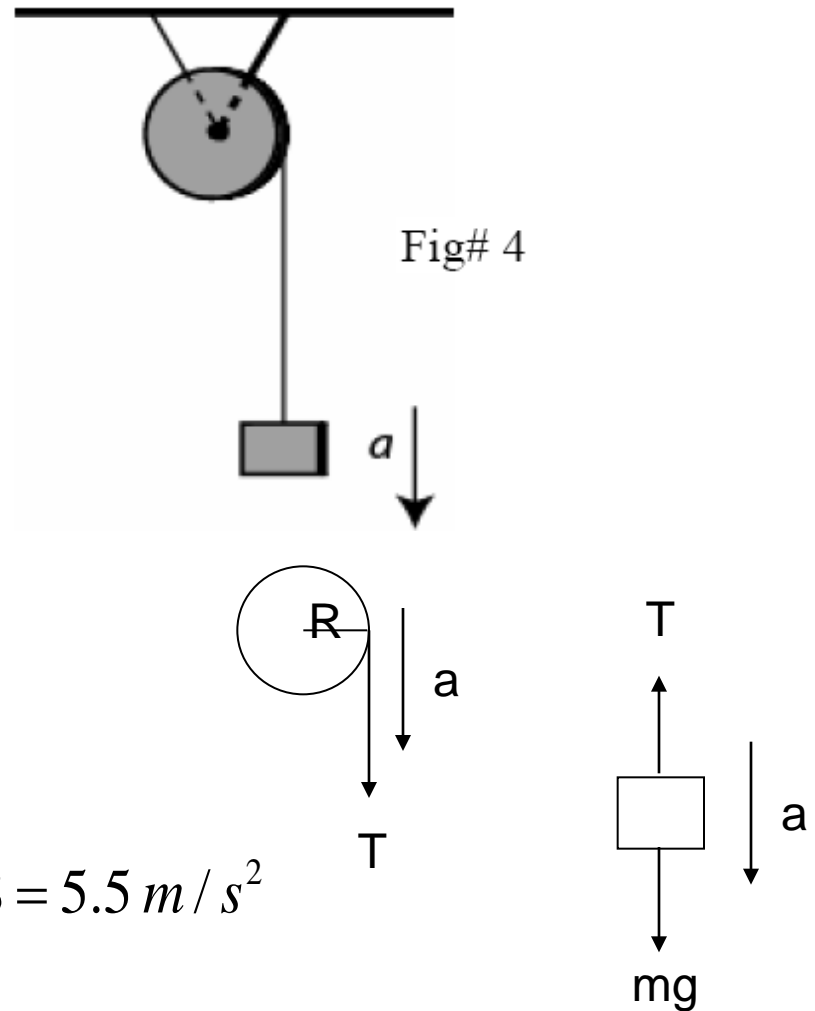
$$TR = I\alpha = I \times \left(\frac{a}{R}\right)$$

$$\rightarrow T = \frac{I}{R^2} a \quad (2)$$

from Eq.1 & 2:

$$mg - \frac{I}{R^2} a = ma$$

$$a = \frac{m}{m + \frac{I}{R^2}} g = \frac{16}{16 + \frac{0.50}{0.20^2}} \times 9.8 = 5.5 \text{ m/s}^2$$

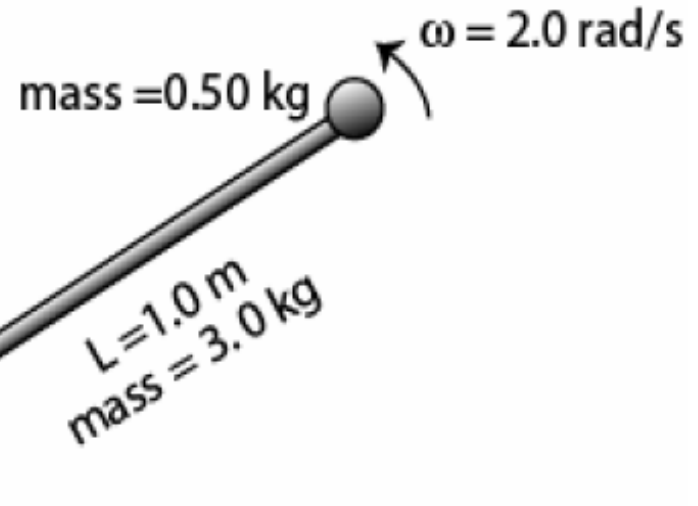


Q15.

A particle of mass 0.50 kg is attached to one end of a 1.0 m long rod of mass 3.0 kg (Fig 5). The rod and the particle are rotating around the other pivoted end of the rod with 2.0 rad/s . The kinetic energy of the system about the pivot is:

- A) 3.0 J
- B) 0.84 J
- C) 0.50 J
- D) 0.62 J
- E) 1.0 J

Fig# 5



$$K = \frac{1}{2} I \omega^2$$

$$I = I_r + I_p \quad (r: \text{rod}, p: \text{particle})$$

$$I_r = I_{com} + Mh^2 \quad (\text{par. axis theorem})$$

$$\rightarrow I_r = \left(\frac{1}{12} ML^2 \right) + M \left(\frac{L}{2} \right)^2 = \frac{1}{3} ML^2 = \frac{1}{3} \times 3.0 \times 1.0^2 = 1.0 \text{ kg.m}^2$$

$$I_p = mL^2 = 0.50 \times 1.0^2 = 0.50 \text{ kg.m}^2$$

$$I = 1.0 + 0.5 = 1.5 \text{ kg.m}^2$$

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2} \times 1.5 \times 2.0^2 = 3.0 \text{ J}$$

Q16.

A disk starts from rest and rotates around a fixed axis, subject to a constant net torque. The work done by the torque during the time interval from $t = 0$ to 2 s is W_1 and the work done during the time interval from $t = 0$ to 6 s is W_2 . The ratio $W_2/W_1 =$

A) 9

B) 3

C) 1 / 3

D) 1 / 9

E) 4

$$W_1 = \tau \theta_1$$

$$W_2 = \tau \theta_2$$

$$\frac{W_2}{W_1} = \frac{\tau \theta_2}{\tau \theta_1}$$

$$= \frac{\theta_2}{\theta_1} = \frac{\frac{1}{2} \alpha t_2^2}{\frac{1}{2} \alpha t_1^2}$$

$$= \left(\frac{t_2}{t_1}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$$

Q17.

A particle, held by a string whose other end is attached to a fixed point C , moves in a circle on a horizontal frictionless surface. If the string is cut, the angular momentum of the particle about the point C :

- A) does not change
- B) changes direction but not magnitude
- C) increases
- D) decreases
- E) becomes zero

no external torques :

angular momentum is conserved

Q18.

What is the net torque about the origin on an object located at $(0, -5.0, 5.0) \text{ m}$ when forces $\vec{F}_1 = (-3.0\hat{k})\text{N}$ and $\vec{F}_2 = (2.0\hat{j})\text{N}$ act on the object?

A) $(5.0\hat{i})\text{N} \cdot \text{m}$

B) $(10\hat{j})\text{N} \cdot \text{m}$

C) $(15\hat{i})\text{N} \cdot \text{m}$

D) $(-3.0\hat{k} + 2.0\hat{j})\text{N} \cdot \text{m}$

E) Zero

$$\vec{r} = -5.0\hat{j} + 5.0\hat{k}$$

$$\vec{\tau} = \vec{r} \times (\vec{F}_1 + \vec{F}_2) = (-5.0\hat{j} + 5.0\hat{k}) \times (-3.0\hat{k} + 2.0\hat{j})$$

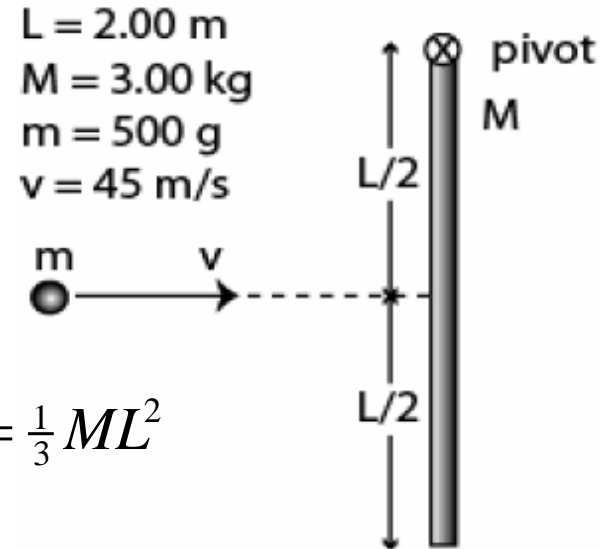
$$15(\hat{j} \times \hat{k}) + 10(\hat{k} \times \hat{j}) = 15\hat{i} - 10\hat{i} = 5.0\hat{i} \text{ N}\cdot\text{m}$$

Q19.

A thin uniform rod of mass $M = 3.0 \text{ kg}$ and length $L = 2.0 \text{ m}$ is suspended vertically from a frictionless pivot at its upper end. An object of mass $m = 500 \text{ g}$, traveling horizontally with a speed $v = 45 \text{ m/s}$ strikes the rod at its center of mass and sticks there (See Fig 6). What is the angular velocity of the system just after the collision?

- A) 5.0 rad/s
- B) 2.1 rad/s
- C) 4.3 rad/s
- D) 3.7 rad/s
- E) 0.57 rad/s

Fig# 6



$$I_{(\text{rod about pivot})} = I_{\text{com}} + M \left(\frac{L}{2} \right)^2 = \frac{1}{12} ML^2 + \frac{1}{4} ML^2 = \frac{1}{3} ML^2$$

$$L_i = L_f$$

$$mv \left(\frac{L}{2} \right) = I_f \omega_f = \left(I_{\text{rod}} + I_p \right)_{\text{around pivot}} \omega = \left(\frac{1}{3} ML^2 + m \times \left(\frac{L}{2} \right)^2 \right) \omega$$

$$\omega = \left(\frac{mv \left(\frac{L}{2} \right)}{\left(\frac{1}{3} ML^2 + m \times \left(\frac{L}{2} \right)^2 \right)} \right) = \frac{22.5}{4.5} = 5.0 \text{ rad / s}$$

Q20.

A thin hoop rolls without sliding along the floor. The ratio of its translational kinetic energy of the center of mass to its rotational kinetic energy about an axis through its center of mass is:

- A) 1
- B) 2
- C) 3
- D) 4
- E) $\frac{1}{2}$

$$\omega = \frac{v_{com}}{R} \text{ (pure rolling)} \quad I_{com} \text{ (hoop)} = mR^2$$

$$\frac{K_t}{K_r} = \frac{\frac{1}{2}mv_{com}^2}{\frac{1}{2}I_{com}\omega^2} = \frac{mv_{com}^2}{(mR^2) \times \frac{v_{com}^2}{R^2}} = \frac{mv_{com}^2}{mv_{com}^2} = 1$$