EXAMPLE 4–8 Find the maximum deflection of a cantilever of length l loaded by a concentrated force F in the middle. Neglect shear.

Solution The problem is illustrated in Fig. 4–10, where the force F acts in the center and a fictitious force Q has been placed at the end where the deflection is desired.

The moments are

$$M_{AB} = F\left(x - \frac{l}{2}\right) + Q(x - l) \tag{1}$$

$$M_{BC} = Q(x - l) \tag{2}$$

The total strain energy is

$$U = \int_0^{l/2} \frac{M_{AB}^2\,dx}{2EI} + \int_{l/2}^{l} \frac{M_{BC}^2\,dx}{2EI} \tag{3}$$

Then, by Castigliano's theorem, the deflection is

$$y = \frac{\partial U}{\partial Q} = \frac{1}{2EI}\left[\int_0^{l/2} 2M_{AB}\left(\frac{\partial M_{AB}}{\partial Q}\right)dx + \int_{l/2}^{l} 2M_{BC}\left(\frac{\partial M_{BC}}{\partial Q}\right)dx\right] \tag{4}$$

Next, we have

$$\frac{\partial M_{AB}}{\partial Q} = \frac{\partial M_{BC}}{\partial Q} = x - l \tag{5}$$

Making the appropriate substitutions in Eq. (4) yields

$$y = \frac{1}{EI}\left\{\int_0^{l/2}\left[F\left(x - \frac{l}{2}\right) + Q(x - l)\right](x - l)dx + \int_{l/2}^{l}[Q(x - l)](x - l)dx\right\} \tag{6}$$

Since Q is fictitious, we substitute $Q = 0$ in Eq. (6) to get

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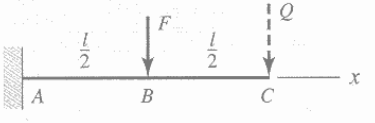

$$y = \frac{F}{EI}\int_0^{l/2}(x - l/2)(x - l)dx = \frac{5Fl^3}{48EI}$$