Homework Chapter 13

1.

    a. **r = 0.6603**

$$H_0: \rho = 0 \, vs. H_1: \rho \neq 0$$

d.f. = 8 - 2 = 6

Decision Rule:

        If $|t_{Stat}| > 2.4469$, reject H_0, otherwise do not reject H_0

$$t_{Stat} = \; = 0.6603/\sqrt{(1-0.6603^2)/(8-2)} = 2.153$$

Since $2.153 < 2.4469$ we fail to reject H_0 and conclude that the correlation coefficient is equal to 0.

    b.

      Y=14.4 + 0.675X

    c. $H_0: \beta_1 = 0 \, vs. H_1: \beta_1 \neq 0$

d.f. = 8 - 2 = 6

Decision Rule:

        If $|t_{Stat}| > 2.4469$, reject H_0, otherwise do not reject H_0

$$t_{Stat} = \frac{b_1}{s_{b_1}} = \frac{0.675}{0.3132} = 2.153$$

Since $2.153 < 2.4469$ we fail to reject H_0 and conclude that $\beta_1 = 0$.

    d. **r^2=0.436; Only 43.6% of Y explained by X.**

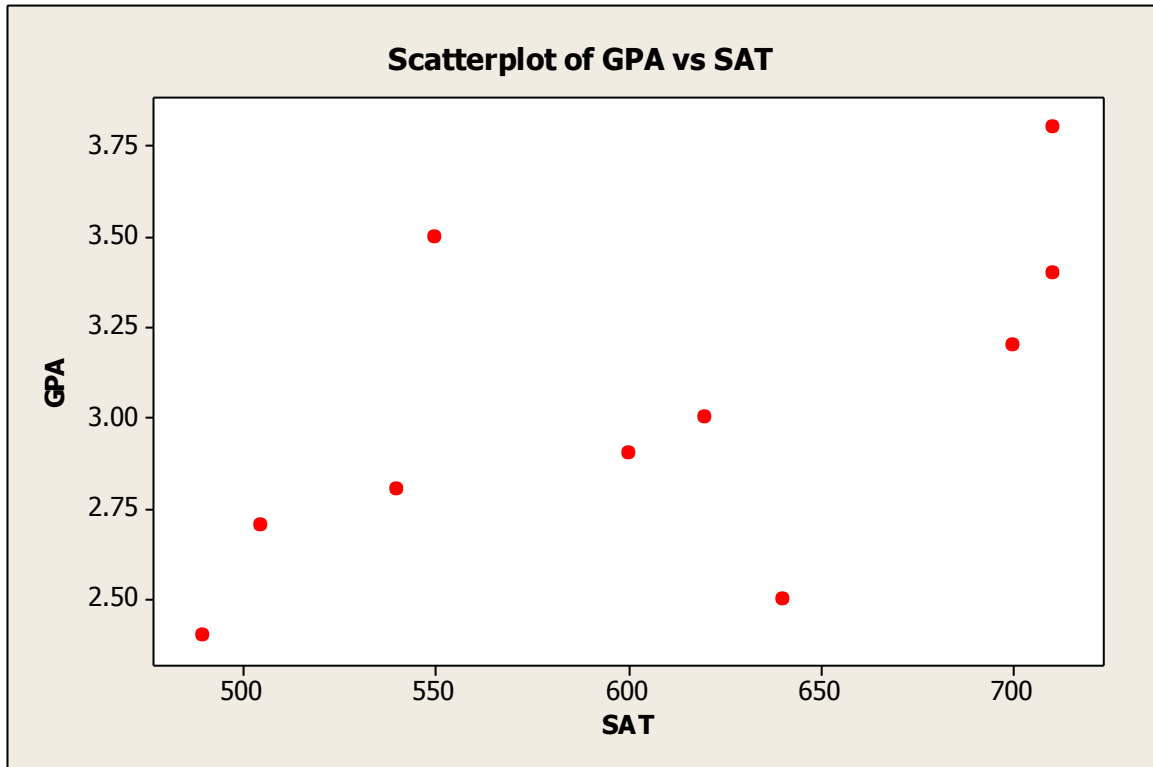   e. $b_1 \pm t_{\alpha/2}s_{b_1} = 0.675 \pm 1.9432 * 0.3132$.

    f.

$$\hat{y} \pm t_{\alpha/2}s_{XY}\sqrt{\frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum(x-\bar{x})^2}} = 81.9 \pm 2.4469 * 8.78912\sqrt{\frac{1}{8} + \frac{(100-96.25)^2}{787.5}}$$

$$\hat{y} = 14.4 + 0.675(100) = 81.9; s_{XY} = 8.78912;$$

$$\sum(x-\bar{x})^2 = \sum x^2 - \left(\sum x\right)^2/n = 74900 - (770)^2/8 = 787.5$$

2.

a.

It look like that we have linear relationship between GPA and SAT.

b. r= 0.62369

$$H_0 : \rho = 0 \ vs. H_1 : \rho \neq 0$$

d.f. = 10 - 2 = 8

Decision Rule:

If $\left| t_{Stat} \right| > 2.3060$, reject H$_0$, otherwise do not reject H$_0$

$$t_{Stat} = \ = 0.62369/\sqrt{(1 - 0.62369^2)/(10 - 2)} = 2.26$$

Since $2.26 < 2.306$ we fail to reject H$_0$ and conclude that the correlation coefficient is equal to 0.

c.   GPA = 0.977 + 0.00337 SAT

d. $H_0 : \beta_1 = 0 \ vs. H_1 : \beta_1 \neq 0$

d.f. = 10 - 2 = 8

Decision Rule:

If $\left| t_{Stat} \right| > 3.3554$, reject H$_0$, otherwise do not reject H$_0$

$$t_{Stat} = \frac{b_1}{s_{b_1}} = \frac{0.003368}{0.001492} = 2.26$$

Since 2.26< 3.3554 we fail to reject H_0 and conclude that $\beta_1 = 0$.

e. $H_0 : \beta_1 = 0 \, vs. \, H_1 : \beta_1 \neq 0$

        d.f. = 1, 10 - 2 = 8, $\alpha/2 = 0.05$

        If $F_{Stst} > 5.32$, reject H_0, otherwise do not reject H_0

    $F_{Stat} = 5.10$

Since 5.10< 5.32 we fail to reject H_0 and conclude that $\beta_1 = 0$.

f. Since we fail to reject H_0 we may commit II error.

g. r^2 = 38.9%; Only about 39% of GPA explained SAT.

h. $b_1 \pm t_{\alpha/2} s_{b_1} = 0.003368 \pm 1.383 * 0.001492$.

i.

$$\hat{y} \pm t_{\alpha/2} s_{XY} \sqrt{1 + \frac{1}{n} + \frac{\left(x_p - \bar{x}\right)^2}{\sum\left(x - \bar{x}\right)^2}} = 3.7335 \pm 2.306 * 0.37438 \sqrt{1 + \frac{1}{10} + \frac{(650 - 606.5)^2}{63002.5}}$$

$$= (2.249, \, 4.084)$$

$$\hat{y} = 0.977 + 0.003379(650) = 3.7335; \, s_{XY} = 0.37438;$$

$$\sum\left(x - \bar{x}\right)^2 = \sum x^2 - \left(\sum x\right)^2 / n = 3741425 - (6065)^2 / 10 = 63002.5$$

3.

    a. The value of y will decrease by 10.12 if we increase x by one unit; the value of y equal to 2140.23 ix x=0.

    b. $H_0 : \beta_1 = 0 \, vs. \, H_1 : \beta_1 \neq 0$

    d.f. = 10 - 2 = 8, $\alpha = 0.01$

    Decision Rule:

        If $\left|t_{Stat}\right| > 3.3554$, reject H_0, otherwise do not reject H_0

    $t_{Stat} = \dfrac{b_1}{s_{b_1}} = \dfrac{-10.12}{3.12} = -3.2436$

    Since $3.2436 < 3.3554$ we fail to reject H_0 and conclude that $\beta_1 = 0$, there is no relationship between x and y.

    c. $H_0 : \beta_1 = 0 \, vs. \, H_1 : \beta_1 > 2.5$

    d.f. = 10 - 2 = 8, $\alpha = 0.05$

    Decision Rule:

        If $t_{Stat} > 1.8595$, reject H_0, otherwise do not reject H_0

$$t_{Stat} = \frac{b_1 - 2.5}{s_{b_1}} = \frac{-10.12 - 2.5}{3.12} = -4.04487$$

Since -4.04487 < 1.8595 we fail to reject H_0 and conclude that $\beta_1 = 0$.

d. $b_1 \pm t_{\alpha/2} s_{b_1} = -10.12 \pm 2.306 * 3.12$