STAT212 Chapter 12 Homework

Q1:

a) Determine the critical value of χ^2 with 1 degree of freedom in each of the following circumstances:

1) $\alpha = 0.05$
2) $\alpha = 0.025$
3) $\alpha = 0.01$

(1) For $df = 1$ and $\alpha = 0.05$, $\chi^2 = 3.8415$.

(2) For $df = 1$ and $\alpha = 0.025$, $\chi^2 = 5.0239$.

(3) For $df = 1$ and $\alpha = 0.01$, $\chi^2 = 6.6349$.

b) An experiment was conducted to study the choices made in mutual fund selection. Undergraduate and MBA students were presented with different S&P 500 index funds that were identical except for fees. Suppose 100 undergraduate students and 100 MBA students were selected. Partial results are shown as follows:

	STUDENT GROUP	
FUND	**Undergraduate**	**MBA**
Highest-cost fund	27	18
Not Highest-cost fund	73	82

Source: *Extracted from J. Choi, D. Laibson, and B. Madrian, "Why Does the Law of One Practice Fail? An Experiment on Mutual Funds," www.som.yale.edu/faculty/jjc83/fees.pdf.*

1) At the 0.05 level of significance, is there evidence of a difference between undergraduate and MBA students in the proportion who selected the highest-cost fund?
2) Determine the p-value in (1) and interpret its meaning.
3) Compare the results of (1) and (2) to those of Problem 10.36 on page 449 of your Berenson textbook

H_0: $\pi_1 = \pi_2$ H_1: $\pi_1 \neq \pi_2$

Decision rule: $df = 1$. If $\chi_{stat}^2 > 3.841$, reject H_0.

Test statistics:

$\chi_{stat}^2 = 2.3226$

Decision: Since $\chi_{STAT}^2 = 2.3226$ is smaller than the upper critical bound of 3.841, do not reject H_0. There is not enough evidence to conclude that there is a significant difference between undergraduate and M.B.A. students in the proportion who selected the highest cost fund.

(2) p-value is 0.1275. The probability of obtaining a test statistic of 2.3226 or larger when the null hypothesis is true is 0.1275.

(3) The results of (1) and (2) are exactly the same as those of Problem 10.36. The χ_{STAT}^2 in (1) and the Z_{STAT} in Problem 10.36 (a) satisfy the relationship that $\chi_{STAT}^2 = 2.3226 = (Z_{STAT})^2 = (1.5240)^2$ and the p-value in Problem 10.36 (b) is exactly the same as the p-value obtained in (b).

Q2:

a) Consider a contingency table with two rows and five columns.

1) Find the degrees of freedom.
2) Find the critical value for $\alpha = 0.05$.
3) Find the critical value for $\alpha = 0.01$.

(1) $df = (r - 1)(c - 1) = (2 - 1)(5 - 1) = 4$

(2) $\chi^2 = 9.488$ (3 $\chi^2 = 13.277$

b) More shoppers do the majority of their grocery shopping on Saturday than any other day of the week. However, is there a difference in the various age groups in the proportion of people who do the majority of their grocery shopping on Saturday? A study showed the results for the different age groups (extracted from "Major Shopping by Day," *Progressive Grocer Annual Report,* April 30, 2002). The data were reported as percentages, and no sample sizes were given:

	AGE		
MAJOR SHOPPING DAY	**Under 35**	**35-54**	**Over 54**
Saturday	24%	28%	12%
A day other than Saturday	76%	72%	88%

Assume that 200 shoppers for each age category were surveyed.

1) Is there evidence of a significant difference among the age groups with respect to major grocery shopping day? (Use $\alpha = 0.05$.)
2) Determine the *p*-value in (1) and interpret its meaning.
3) If appropriate, use the Marascuilo procedure and $\alpha = 0.05$ to determine which age groups are different. Discuss your results.
4) Discuss the managerial implications of (1) and (3). That is, how can grocery stores use this information to improve marketing and sales? Be specific.

(1) $H_0: \pi_1 = \pi_2 = \pi_3$ H_1: at least one proportion differs

where population 1 = under 35, 2 = 35-54, 3 = over 54

Decision rule: $df = (c - 1) = (3 - 1) = 2$. If $\chi^2_{STAT} > 5.9915$, reject H_0.

Test statistic:

$\chi^2_{stat} = 16.5254$

Decision: Since $\chi^2_{STAT} = 16.5254$ is greater than the upper critical bound of 5.9915, reject H_0. There is enough evidence to conclude that there is a significant relationship between age and major grocery shopping day.

(2) *p*-value = 0.0003. The probability of obtaining a sample that gives rise to a test statistic that is equal to or more than 16.5254 is 0.03% if the null hypothesis is true.

(3)

Pairwise Comparisons	**Critical Range**	$\lvert p_j - p_{j'} \rvert$
1 to 2	0.1073	0.04
2 to 3	0.0959	0.16*
1 to 3	0.0929	0.12*

There is a significance difference between the 35-54 and over 54 groups, and between the under 35 and over 54 groups.

(4) The stores can use this information to target their marketing on the specific group of shoppers on Saturday and the days other than Saturday.

Q3:
Wang Corporation is interested in determining whether a relationship exists between the commuting time of its employees and the level of stress related problems observed on the job. A study of 123 assembly-line workers reveals the following:

	STRESS LEVEL			
COMMUTING TIME	**High**	**Moderate**	**Low**	**Total**
Under 15 min.	10	6	19	35
15-45 min.	15	9	29	53
Over 45 min.	20	7	8	35
Total	45	22	56	123

a) At the 0.01 level of significance, is there evidence of a significant relationship between commuting time and stress level'?
b) What is your answer to (a) if you use the 0.05 level of significance?

(a) H_0: There is no relationship between the commuting time of company employees and the level of stress-related problems observed on the job.
H_1: There is a relationship between the commuting time of company employees and the level of stress-related problems observed on the job.
Decision rule: If $\chi^2_{STAT} > 13.277$, reject H_0.

Test statistic: $\chi_{stat}^2 = 9.831$

Decision: Since $\chi^2_{STAT} = 9.831$ is less than the critical bound of 13.277, do not reject H_0. There is not enough evidence to conclude there is any relationship between the commuting time of company employees and the level of stress-related problems observed on the job.

(b) Decision rule: If $\chi^2_{STAT} > 9.488$, reject H_0.

Decision: Since the $\chi_{stat}^2 = 9.831$ is greater than the critical bound of 9.488, reject H . There is enough evidence at the 0.05 level to conclude there is a relationship between the commuting time of company employees and the level of stress-related problems observed on the job.

Q4:
The CEO of Raja health care facility would like to assess the effects of recent implementation of Six Sigma management on customer satisfaction. A random sample of 100 patients is selected from a list of patients who were at the facility the past week and also a year ago:

	SATISFIED NOW		
SATISFIED LAST YEAR	**Yes**	**No**	**Total**
Yes	40	17	57
No	23	20	43
Total	63	37	100

a) At the 0.05 level of significance, is there evidence that satisfaction was lower last year, prior to introduction of Six Sigma management?
b) Compute the *p*-value in (a) and interpret its meaning.

(a) $H_0: \pi_1 \geq \pi_2$ $\quad H_1: \pi_1 < \pi_2$ where 1 = last year, 2 = now
Decision rule: If $Z_{STAT} < -1.645$, reject H_0.

Test statistic: $Z_{STAT} = \dfrac{B - C}{\sqrt{B+C}} = \dfrac{5 - 20}{\sqrt{5+20}} = -3$

Decision: Since $Z_{STAT} = -3 < -1.645$, reject H_0. There is enough evidence to conclude that satisfaction was lower last year prior to introduction of Six Sigma management.

(b) *p*-value = 0.0014. The probability of obtaining a data set which gives rise to a test statistic smaller than – 3 is 0.14% if the satisfaction was not lower last year prior to introduction of Six Sigma management.

Q5:

a) Determine the lower- and upper-tail critical values of χ^2 for each of the following two-tail tests:

1) $\alpha = 0.01$, $n = 24$
2) $\alpha = 0.05$, $n = 20$
3) $\alpha = 0.10$, $n = 16$

(1) For $df = 23$ and $\alpha = 0.01$, $\chi^2_{\alpha/2} = 9.2604$ and $\chi^2_{1-\alpha/2} = 44.1814$.

(2) For $df = 19$ and $\alpha = 0.05$, $\chi^2_{\alpha/2} = 8.9065$ and $\chi^2_{1-\alpha/2} = 32.8523$.

(3) For $df = 15$ and $\alpha = 0.10$, $\chi^2_{\alpha/2} = 7.2609$ and $\chi^2_{1-\alpha/2} = 24.9958$.

b) A manufacturer of doorknobs has a production process that is designed to provide a doorknob with a target diameter of 2.5 inches. In the past, the standard deviation of the diameter has been 0.035 inch. In an effort to reduce the variation in the process, various studies have resulted in a redesigned process. A sample of 25 doorknobs produced under the new process indicates a sample standard deviation of 0.025 inch.

1) At the 0.05 level of significance, is there evidence that the population standard deviation is less than 0.035 inch in the new process?
2) What assumption do you need to make in order to perform this test?
3) Compute the p-value in part (a) and interpret its meaning.

(1) H_0: $\sigma \geq$ 0.035 inch. The standard deviation of the diameter of doorknobs is greater than or equal to 0.035 inch in the redesigned production process.

H_1: σ < 0.035 inch. The standard deviation of the diameter of doorknobs is less than 0.035 inch in the redesigned production process.

Decision rule: $df = 24$. If $\chi^2_{STAT} < 13.848$, reject H_0.

Test statistic: $\chi^2_{STAT} = (n-1)S^2/\sigma^2 = 24(0.025^2)/0.035^2 = 12.245$

Decision: Since the test statistic of $\chi^2_{STAT} = 12.245$ is less than the critical boundary of 13.848, reject H_0. There is sufficient evidence to conclude that the standard deviation of the diameter of doorknobs is less than 0.035 inch in the redesigned production process.

(2) You must assume that the data in the population are normally distributed to be able to use the chi-square test of a population variance or standard deviation.

(3) p-value = (1 – 0.9770) = 0.0230. The probability of obtaining a test statistic equal to or more extreme than the result obtained from this sample data is 0.0230 if the population standard deviation is indeed no less than 0.035 inch.