KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
STAT416 : Stochastic Processes for Actuaries (181)
Third Exam **Thursday December 14, 2018**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	14	
Two	18	
Three	12	
Four	15	
Five	10	
Six	11	
Total	80	

Question.1 (**4+4+3+3=14-Points**)

Define the following:

(a) Standard Brownian motion.

(b) Brownian motion with a drift parameter.

(c) Brownian Bridge Process $\{X(t); t \geq 0\}$.

(d) Integrated Brownian motion.

Question.2 **(14+2+2=18-Points)**

Consider a network with four servers. Any arrival must go first to server 4, the arrival rate at that server is a Poisson rate of 4 customers per minute. The service rates at servers 1, 2, 3, and 4 are, respectively 25, 30, 15, and 20. An arrival upon completion of service at server 4 will always go to server 1 ($\pi_{41} = 1$). An arrival departing service at server 1 will equally likely go to server 2 or 3 ($\pi_{12} = \pi_{13} = 0.5$). An arrival departing service at server 2 will always go to server 1 ($\pi_{21} = 1$). An arrival departing service from server 3 will either go to server 1 with probability (0.6) or leave the system. ($\pi_{31} = 0.6, \pi_{33} = 0$).

(a) Find the probability of having (3,2,4,1) arrivals at servers (1,2,3,4).

(b) Find the average number of arrivals in the system.

(c) Find the average amount of time an arrival spent on the system.

Question.3 **(12-Points)**

Let $\{Z(t); t \geq 0\}$ denote a Brownian bridge process. show that: $Y(t) = (t+1)Z\left(\frac{t}{t+1}\right)$ is a standard Brownain motion process.

Question.4 **(6+4+5=15-Points)**

Suppose that the price of a stock is modeled as a standard Brownian motion.

(a) If the price at time $t = 4$ is \$3, where t is measured in months, find the probability that the price is at least \$ 4.25 by month 10.

(b) Find the distribution of $3X(12) - X(4)$?

(c) Calculate the probability that the stock price reaches to a price of \$4.75 at some time within the next 9 months.

Question 5. **(10-Points)**

Let $\{X(t); t \geq 0\}$ be the price of a stock at time t. Suppose that the stock price is modeled as a geometric Brownain motion given by $X(t) = e^{\mu t + \sigma B(t)}$, where $\{B(t); t \geq 0\}$ is a standard Brownian motion. Suppose that the parameter values are $\mu = 0.055$ and $\sigma = 0.07$. Given that $X(5) = 100$, find the probability that $X(10)$ is grater than 150.

Question 6. (**5+6=11-Points**)

Let $\{B(t); t \geq 0\}$ be a standard Brownian motion,then

(a) If $\{X(t); t \geq 0\}$ is a process with $X(t) = 1 + 0.4B(t)$. Calculate $P\left(X(5) > 1 | X(0) = 1\right)$

(b) If$\{X(t); t \geq 0\}$ is a process with $X(t) = 1 + 0.1t + 0.4B(t)$. Calculate $P\left(X(10) > 1 | X(0) = 1\right)$.