KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
STAT416 : Stochastic Processes for Actuaries (181)
Second Exam **Thursday November 22, 2018**
Name: **ID:**

Time Allowed : 120 minutes.

Question Number	Full Mark	Marks Obtained
One	16	
Two	15	
Three	14	
Four	12	
Five	8	
Six	14	
Seven	18	
Eight	8	
Total	105	

Some Formulas:

(1.) $\sum_{j=m}^{\infty} r^j = \frac{r^m}{1-r}$, $|r| < 1$

(2.) For $|x| < 1$, $(1-x)^{-\alpha} = \sum_{n=0}^{\infty} \frac{\alpha(\alpha+1)(\alpha+2)\ldots(\alpha+n-1)}{n!} x^n$, with $\frac{\alpha(\alpha+1)(\alpha+2)\ldots(\alpha+n-1)}{n!} = 1$, when $n = 0$

(3.) $\boldsymbol{\pi}_{n_1,n_2,\ldots,n_k} = \prod_{j=1}^{k} \left(\frac{\lambda_j}{\mu_j}\right)^{n_j} \left(1 - \frac{\lambda_j}{\mu_j}\right)$

(4.) For $M/M/1$: $E(L) = \frac{\lambda}{\mu-\lambda}$, $L = \lambda W$, $E(L_q) = \frac{\lambda^2}{\mu(\mu-\lambda)}$

(5.) For $M/G/1$: $E(W_q) = \frac{\lambda E(S^2)}{2(1-\lambda E(S))}$, $L_q = \lambda W_q$, $W = W_q + S$

Question.1 (**2+4+2+4+4=16-Points**)

Answer the following:

(a) For a continuous-time Markov chain, define the meaning of $P_{ij}(t) = P\{X(t+s) = j|X(s) = i\}$.

(b) For any pair of states i and j, let $q_{ij} = \nu_i P_{ij}$. Explain the meaning of ν_i and q_{ij}

(c) If $\pi_n = 0.65$ for an $M/G/1$ queueing system with G distributed as uniform $(0, 2)$. what is meaning of π_n

(d) For an $M/M/1$ queueing system, define a_n and d_n

(e) For an $M/M/k$ queueing system, we get $E(L) = 5$, and $E(W_q) = \frac{2}{3}$. Explain the meaning of these two numbers.

Question.2 **(2+13=15-Points)**

Passengers arrive at a train station according to a Poisson process with rate λ and wait for a train to arrive. Independently, trains arrive at the same station according to a Poisson process with rate μ. Suppose that each time a train arrives, all the passengers waiting at the station will board the train. The train then immediately leaves the station. If there are no passengers waiting at the station, the train will not wait until passengers arrive. Suppose that at time 0, there is no passenger waiting for a train at the station. For $t > 0$. Let $X(t) = 1$ if there is at least one passenger waiting in the train station for a train, and let $X(t) = 0$, otherwise

(a) Write down the rate matrix

(b) Using Kolomogrov forward equations, find the probability that at time t also there is no passenger waiting for a train.

Question.3 **(14-Points)**

Consider a linear Growth model with immigration, where $\lambda_n = n\lambda + \theta, n \geq 0$, and $\mu_n = n\mu, n \geq 1$. Find the limiting probability vector for this system.

Question.4 **(3+2+7=12-Points)**

Consider a shoe shine establishment consisting of two chairschair 1 and chair 2. A customer upon arrival goes initially to chair 1 where his shoes are cleaned and polish is applied. After this is done the customer moves on to chair 2 where the polish is buffed. The service times at the two chairs are assumed to be independent random variables that are exponentially distributed with respective rates μ_1 and μ_2. Suppose that potential customers arrive in accordance with a Poisson process having rate λ, and that a potential customer will enter the system only if both chairs are empty.

(a) Write down the state space for this system.

(b) Is this a birth and death process? Explain.

(c) Determine the proportion of time the process is in each of the states.

Question 5. **(6+2=8-Points)**

The manager of a market can hire either Mary or Alice. Mary, who gives service at an exponential rate of 20 customers per hour, can be hired at a rate of $3 per hour. Alice, who gives service at an exponential rate of 30 customers per hour, can be hired at a rate of $C per hour. The manager estimates that, on the average, each customers time is worth$1 per hour and should be accounted for in the model. Assume customers arrive at a Poisson rate of 10 per hour.

(a) What is the average cost per hour if Mary is hired? If Alice is hired?

(b) Find C if the average cost per hour is the same for Mary and Alice.

Question 6. (**10+4=14-Points**)

A supermarket has two exponential checkout counters, each operating at rate μ. Arrivals are Poisson at rate λ. The counters operate in the following way:

(i) One queue feeds both counters.

(ii) One counter is operated by a permanent checker and the other by a stock clerk who instantaneously begins checking whenever there are two or more customers in the system. The clerk returns to stocking whenever he completes a service, and there are fewer than two customers in the system.

(a) Find π_n, proportion of time there are n in the system.

(b) At what rate does the number in the system go from 0 to 1? From 2 to 1?

Question 7. (**11+4+3=18-Points**)

Consider a network of three stations with a single server at each station. Customers arrive at stations 1, 2, 3 in accordance with Poisson processes having respective rates 6, 8, and 12. The service times at the three stations are exponential with respective rates 20, 40, and 80. A customer completing service at station 1 is equally likely to (i) go to station 2, (ii) go to station 3, or (iii) leave the system . A customer departing service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to either go to station 2 or leave the system.

(a) What is the limiting probability of having one, two, and three customers at station 1, station 2, and station 3, respectively?

(b) What is the average number of customers in the system (consisting of all three stations)?

(c) What is the average time a customer spends in the system?

Question 8. (**8-Points**)

There are two workers competing for a job. Ali claims an average service time which is faster than Ahmads, but Ahmad claims to be more consistent, if not as fast. The arrivals occur according to a Poisson process at a rate of $\lambda = 2$ per hour. Alis statistics are an average service time of 24 minutes with a standard deviation of 20 minutes. Ahmads service statistics are an average service time of 25 minutes, but a standard deviation of only 2 minutes. If the average length of the queue is the criterion for hiring, which worker should be hired?