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STAT 319: Probability & Statistics for Engineers & Scientists
Term 181, Quiz # 5

Name: ID #:

Q.No.1: - A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed with standard deviation 2 volts, and the manufacturer wishes to test $H_0: \mu \geq \mu_0$ against $H_1: \mu < \mu_0$. A sample of size 16 is selected and the sample mean is found to be 20.41 volts. After testing the above mentioned hypotheses, the manufacturer got p-value = 0.119. Find the value of μ_0.

p-value = $P(Z < Z_c) = 0.119$
From the table, $P(Z < -1.18) = 0.119$
This implies that $Z_c = -1.18$

$\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = -1.18$

$\frac{20.41-\mu_0}{2/\sqrt{16}} = -1.18$

$-\mu_0 = \frac{2}{4}(-1.18) - 20.41 = -21$

$\mu_0 = 21$

Q.No.2: - To estimate the average millage per gallon an experiment has been conducted with a car brand. A sample of 40 cars are chosen and average millage is found to be 35 miles with a standard deviation of 7 miles. A 99% confidence interval around the true mean is given as $(32.115\,, 37.844)$. In order to decrease the length of this confidence interval to 2 miles, how many additional cars should be sampled?

$$(2.575)\frac{7}{\sqrt{n}} = 1$$
$$n = 324.9$$

Thus, 325 - 40 = 285 cars.

Q.No.3: -

(a) Suppose that A and B are independent events with $P(A) = 0.4, P(B) = 0.7$. Find $P(\bar{B}|\bar{A})$.

$$P(\bar{B}|\bar{A}) = P(\bar{B}) = 1 - P(B) = 0.3$$

(b) Suppose that A and B are mutually exclusive events with $P(B) > 0$. Find $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0$$

(c) The probability mass function of a discrete random variable X is defined as

$$P(x) = kx, \quad x = 1, 2, 3,$$

Find the value of k.

$$1 = \sum P(x) = k + 2k + 3k \quad \rightarrow 6k = 1 \quad \rightarrow k = \frac{1}{6}$$

(d) If a random variable X has a Poisson distribution with parameter $\lambda = 4$. Find $E(X^2)$.

$$\mu = \sigma^2 = \lambda = 4 \ \ \& \ \ \sigma^2 = 4 = E(X^2) - \mu^2 \quad \rightarrow \quad E(X^2) = \sigma^2 + \mu^2 = 4 + 16 = 20$$

Q.No.4: - The salaries of engineers in a certain specialty are approximately normally distributed. If 25 percent of these engineers earn less than \$180,000 and 30 percent earn more than \$320,000. What percent of the engineers earn between \$280,000 and \$330,000?

$P(X<180000)=0.25 \Rightarrow P\left(Z<\frac{180000-\mu}{\sigma}\right)=0.25$

From the Z-table $P(Z<-0.67)\cong 0.25 \Rightarrow \frac{180000-\mu}{\sigma}=-0.67 \Rightarrow \mu=180000+0.67\sigma$

$P(X>320000)=0.3 \Rightarrow P\left(Z>\frac{320000-\mu}{\sigma}\right)=0.3 \Rightarrow P\left(Z<\frac{320000-\mu}{\sigma}\right)=0.7$

From the Z-table $P(Z<0.52)\cong 0.7 \Rightarrow \frac{320000-\mu}{\sigma}=0.52$

$\Rightarrow 320000-180000-0.67\sigma=0.52\sigma \Rightarrow 140000=1.19\sigma \Rightarrow \sigma=117647.1$

Also $\mu=180000+0.67\sigma \Rightarrow \mu=180000+0.67(117647.1) \Rightarrow \mu=258823.5$

$P(280000<X<330000)=P\left(\frac{280000-258823.5}{117647.1}<Z<\frac{330000-258823.5}{117647.1}\right)=P(0.18<Z<0.605)$

$\cong P(0.18<Z<0.61)=P(Z<0.61)-P(Z<0.18)=0.7291-0.5714=0.1577$

Some useful formulas

$P(\overline{A})=1-P(A), \quad P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$P(A \mid B)=\frac{P(A\cap B)}{P(B)}, P(B)\neq 0$ and $P(A\cap B)=P(A)\times P(B \mid A)=P(A \mid B)\times P(B)$

$\mu=E(X)=\sum x\, f(x); \quad E(X^2)=\sum x^2 f(x)$ and $\sigma^2=E(X^2)-\mu^2$

Poisson: $f(x)=\frac{e^{-\lambda}\lambda^x}{x!};\ x=0,1,2,......; \quad \mu=\lambda, \quad \sigma^2=\lambda$

$X\sim N(\mu,\sigma^2); \quad Z=\frac{X-\mu}{\sigma}\sim N(0,1); \quad Z=\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\sim N(0,1)$

Confidence Interval	Test Statistic
$\overline{X}\pm Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$ and $n\geq\left(\frac{\sigma Z_{\frac{\alpha}{2}}}{e}\right)^2$	$Z=\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$
$\overline{X}\pm T_{\frac{\alpha}{2},v}\frac{s}{\sqrt{n}}$	$T=\frac{\overline{X}-\mu}{s/\sqrt{n}}, \quad v=n-1$

With Best Wishes