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STAT 319: Probability & Statistics for Engineers & Scientists
Term 181, Quiz # 3

Name: ID #:

Q.No.1: - A company that manufactures computer chips finds that 8% of all chips manufactured as defective. Management is concerned that untrained employees are partially responsible for the high defect rate. In an effort to decrease the percentage of defective chips, management decides to provide additional training to those employees hired within the last year.

(a) After training was implemented, a sample of 450 chips revealed only 27 defectives. Was the additional training effective in lowering defect rate? Test at 1% significance level.

$H_0: p = 0.08$ vs $H_1: p < 0.08$

where p is the proportion of defective chips

Test Statistic $Z = \dfrac{\hat{p} - 0.08}{\sqrt{\frac{(0.08)(0.92)}{450}}}$

At the 1% significant level, we reject H_0 if $Z < -2.33$

Observed Test statistic $Z_0 = \dfrac{\frac{27}{450} - 0.08}{\left(\frac{(0.08)(0.92)}{450}\right)^{1/2}}$

$= -1.564$

Since $Z_0 > -2.33$, we do not reject H_0

and conclude that there is no evidence that Training reduces the proportion of defective chips

(b) What should be the sample size if the management wants the error in estimating the proportion of defectives to be within ± 0.04? Use $\alpha = 0.05$ and assume that there is no prior estimate of proportion available.

$$n \geq \left(\frac{Z_{\alpha/2}}{e}\right)^2 \left[p(1-p)\right] = \left(\frac{1.96}{0.04}\right)^2 [0.25] = 600.25 \cong 601$$

Q.No.2: - Light bulbs of a certain type are advertised as having an average lifetime of 750 hours. A customer will purchase the light bulbs unless it can be conclusively demonstrated that the true average lifetime is smaller than what is advertised. A random sample of 26 bulbs was selected, and gave a mean of 738.44 hours and a standard deviation of 38.20 hours. Test an appropriate hypothesis and state your conclusions.

$H_0: \mu \geq 750;$ $\quad\quad H_1: \mu < 750$

Since σ is unknown, n is small and we assume normal population

$\Rightarrow$ use t-test

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{738.44 - 750}{38.2/\sqrt{26}} = \frac{-11.56}{2.2191} = -1.543$$

$p - value = P[T_{25} < -1.543] = P[T_{25} > 1.543]$ #symmetry

But $P[T_{25} > 1.543]$ is between 0.05 and 0.1 #from t-table

$\Rightarrow 0.05 < p - value < 0.1$

Decision Rule: Reject H_0 if $p - value < \alpha$

Assume $\alpha = 0.05$

Decision: Since $p - value \not< \alpha$ so we fail to reject H_0.

Conclusion: The data does not provide the sufficient evidence that the average lifetime is less than 750 hours so the customers will purchase the light bulbs.

Q.No.3: - A quality control engineer is interested in the mean length of sheet insulation being cut automatically by machine. The desired length of the insulation is 12 feet. It is known that the standard deviation in the cutting length is 0.15 feet. A sample of 70 cut sheets yields a mean length of 12.14 feet.

(a) Obtain a 99% confidence interval for the mean length cut by machine.

$x \to$ length of sheet, $n = 70$, $\sigma = 0.15$

$\bar{x} = 12.14$

99% C-I for μ

$\bar{x} \pm Z_{0.005} \cdot \frac{\sigma}{\sqrt{n}}$

$12.14 \pm 2.575\left(\frac{0.15}{\sqrt{70}}\right)$

$(12.09, 12.19)$

$\alpha = 0.01$, $\frac{\alpha}{2} = 0.005$

$Z_{\frac{\alpha}{2}} = 2.575$

(b) Using the confidence interval in part (i), can we say the machine is working properly? Justify your answer?

No, because $H_0: \mu = 12$ does not belong to $[12.09, 12.19]$