STAT 319: Probability & Statistics for Engineers & Scientists

Term 181, Quiz # 2

Name: ID #:

Q.No.1: - A quality control engineer is interested in the mean length of sheet insulation being cut automatically by machine. The length of insulation is normally distributed with mean length of 12 feet. It is known that the standard deviation in the cutting length is 0.16 feet.

(a) What is the probability that a cut sheet is between 11.6 and 12.2 feet?

Since $X \sim N\left(12, 0.16^2\right)$, so $P\left(11.6 < X < 12.2\right) = P\left(\frac{11.6-12}{0.16} < \frac{X-\mu}{\sigma} < \frac{12.2-12}{0.16}\right)$

$= P\left(-2.5 < Z < 1.25\right) = P\left(Z < 1.25\right) - P\left(Z < -2.5\right) = 0.89435 - 0.00621$

which is 0.88814

(b) If the shortest 2.5% and the lengthiest 2.5% cut sheets are scrapped for some application, what are the minimum and maximum lengths of a cut sheet that will be classified as scrapped?

① $P(X < P_{2.5}) = 0.025 \Rightarrow P\left(Z < \frac{P_{2.5}-\mu}{\sigma}\right) = 0.025$

$\Rightarrow \frac{P_{2.5}-\mu}{\sigma} = -1.96 \Rightarrow P_{2.5} = \mu - 1.96\sigma = \boxed{11.6864}$

② $P(X > P_{97.5}) = 0.025 \Rightarrow P\left(Z < \frac{P_{97.5}-\mu}{\sigma}\right) = 0.975$

$\Rightarrow \frac{P_{97.5}-\mu}{\sigma} = 1.96 \Rightarrow P_{97.5} = \mu + 1.96\sigma = \boxed{12.3136}$

Q.No.2: - A company claims that its chocolate chip cookies have, on the average, 16 chocolate chips in each cookie. Assume that a Poisson random variable with mean 16 is the appropriate model for the number of chips in a cookie. What can you say about the probability that in a sample of 3000 cookies, at least 117 will have 10 chips?

Let Y denotes the number of cookies with exactly 10 chips out of 3000 cookies.

$Y \sim Binomial\left(n = 3000, p = 0.0341\right)$

$P\left(Y \geq 117\right) = ???$

$\cong P\left(\frac{Y - np}{\sqrt{np\left(1-p\right)}} \geq \frac{117 - 0.5 - 103.2}{9.9404}\right)$ #Normal Approximation

$= P\left(Z \geq 1.337974\right) \cong P\left(Z \geq 1.34\right) = 1 - P\left(Z < 1.34\right)$

$= 1 - 0.9099 = 0.0901$

Q.No.3: - The average lifetime of a light bulb is 3000 hours with a standard deviation of 696 hours. A simple random sample of 36 bulbs is taken from this population.

(a) What is the probability that the average lifetime in the sample will be equal to or greater than 3219.24 hours?

We are supposed to find $P(\bar{X} \geq 3219.24)$

$$= P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \geq \frac{3219.24-3000}{696/\sqrt{36}}\right)$$

$$= P(Z \geq 1.89) = 1 - P(Z < 1.89)$$

$$= 1 - 0.9706 = 0.0294$$

(b) Find the 80th percentile of the sampling distribution of the sample mean $\bar{X}$.

We are supposed to find a from the equation $P(\bar{X} \leq a) = 0.8$

$$\Rightarrow P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{a-3000}{696/\sqrt{36}}\right) = 0.8$$

$$\Rightarrow P\left(Z \leq \frac{a-3000}{696/\sqrt{36}}\right) = 0.8 \qquad (1)$$

Also, from the Z-table,

$$P(Z \leq 0.84) = 0.8 \qquad (2)$$

Equating (1) and (2) we get

$$\frac{a-3000}{696/\sqrt{36}} = 0.84 \qquad \Rightarrow a = 3097.44$$

Some Useful Formulas:

- $f(x) = \lambda e^{-\lambda x}$, where $F(x) = 1 - e^{-\lambda x}$; $x > 0$ and $\mu = \frac{1}{\lambda}$ & $\sigma = \frac{1}{\lambda}$
- $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$ & $Z = \frac{X-\mu}{\sigma}$ or $Z = \frac{(\bar{X}-\mu)}{\frac{\sigma}{\sqrt{n}}}$