KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
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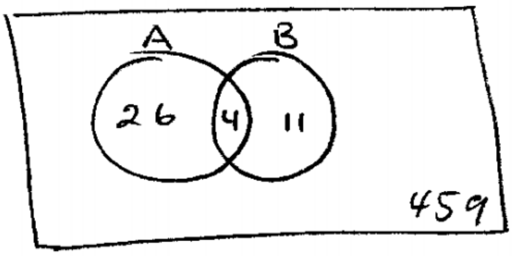
STAT 319: Probability & Statistics for Engineers & Scientists
Term 181, Quiz # 1

Name: ID #:

Q.No.1: - A box of 500 rivets contains good rivets as well as rivets with defects summarized below:

i. 30 rivets with type A defect
ii. 15 rivets with type B defect
iii. 4 rivets with type A and type B defects

A rivet is chosen at random, what is the probability that it is not defective?



$$P(\text{No defect}) = \frac{500-41}{500} = \frac{459}{500} = 0.918$$

Q.No.2: - Suppose that a box contains 10% defective microchips. A purchaser decides to select 5 microchips one after another without replacement. Assume that the box has 30 microchips.
(a) What is the probability that two microchips in the sample will be defective?

$$P(2 \text{ defective microchips}) = \frac{\binom{3}{2}\binom{27}{3}}{\binom{30}{5}}$$

$$= 0.0616$$

(b) What is the probability that the first two microchips in the sample will be defective and the last three will be good?

$$P(DDGGG) = \frac{3}{30} \cdot \frac{2}{29} \cdot \frac{27}{28} \cdot \frac{26}{27} \cdot \frac{25}{26} = 0.0062$$

Q.No.3: - A survey of those using a particular statistical software system indicated that 10% were dissatisfied. Half of those dissatisfied purchased the system from company A. It is also known that 20% of those surveyed purchased from company A.

(a) What is probability of a satisfied customer purchasing the system from company A?

Solution: We have $P(D) = 0.10$, $P(A|D) = 0.50$, $P(A) = 0.20$.

$$P(A|S) = \frac{P(SA)}{P(S)},$$

Since $P(A) = P(AD) + P(AS)$,
$= P(D)\,P(A|D) + P(AS)$,

or, $0.20 = P(D)\,P(A|D) + P(AS)$,

or, $0.20 = 0.10\,(0.50) + P(AS)$,

or, $P(AS) = 0.15$,

Also $P(S) = 1 - P(D) = 1 - 0.10 = 0.90$

$$\therefore P(A|S) = \frac{0.15}{0.90} \approx 0.17$$

(b) Given that the software package was purchased from company A, what is the probability that a particular user is dissatisfied?

$$P(D|A) = \frac{P(DA)}{P(A)} = \frac{0.05}{0.20} \approx 0.25$$

Q.No.4: - A company claims that its chocolate chip cookies have, on the average, 16 chocolate chips in each cookie. Assume that a Poisson random variable with mean 16 is the appropriate model for the number of chips in a cookie. What is the probability that there will be 10 chips in a cookie?

Let X denotes the number of chocolate chips in a cookie.

$X \sim Poisson\,(16 \text{ per cookie})$

$$P(X = x) = \frac{e^{-16}16^{x}}{x!};\ x = 0,1,2,......$$

$$P(X = 10) = ???$$

$$= \frac{e^{-16}16^{10}}{10!} = 0.0341$$

Q.No.5: - Specifications call for the thickness of aluminum sheets that are to be made into cans be between 8 and 11 thousandth of an inch. Let $f(x)$ be the probability density function of X.

$$f(x) = \begin{cases} cx & 6 \le x \le 12 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of c.

$$\int_6^{12} f(x) = 1 \Rightarrow \int_6^{12} c\,x\,dx = \frac{c}{2}x^2\Big]_6^{12} = 1 \Rightarrow$$

$$\frac{c}{2}(144-36) = 1 \Rightarrow 54C = 1 \Rightarrow \boxed{C = \frac{1}{54}}$$

(b) What is the probability that a sheet doesn't meet the specification?

$$P(\text{Meet specs}) = P(8 < X < 11) = \int_8^{11} \frac{x}{54}\,dx$$

$$= \frac{x^2}{108}\Big]_8^{11} = \frac{121-64}{108} = 0.528$$

$$P(\text{Does NOT meet specs}) = 1 - 0.528 = \boxed{0.472} = p$$

(c) If the aluminum sheets are selected one by one, what is the probability that that 1st sheet that doesn't meet the specification is the 4th sheet?

X: # trials to get the 1st sheet not meet spec.

$$X : G(0.472) \Rightarrow f(x) = (0.472)(0.528)^{x-1}, \quad x = 1, 2, \ldots$$

$$P(4^{th}) = P(X=4) = f(4) = (0.472)(0.528)^3$$

$$= \boxed{0.069}$$