KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
Term 181
STAT 302 Exam 5

Name: ______________________ ID #: ______________

1) Let $Y_1, \cdots, Y_n$ denote the random variables associated with a sample of size n from a density function $f_Y(y|\theta)$. Explain the Bayesian paradigm, with all its details and using proper mathematical/statistical notation, in estimating a function $t(\theta)$. *(5 marks)*

2) Show that the posterior density is proportional to the product of the conditional likelihood of the data and the prior density for the parameter θ. *(3 marks)*

3) Let $Y_1, \cdots, Y_n$ denote a random sample from a normal population with unknown mean θ and known variance σ^2. Assume that θ has normal prior with mean η and variance δ^2. The posterior distribution of θ is normal with mean $\eta^* = \frac{\delta^2 u + \sigma^2 \eta}{n\delta^2 + \sigma^2}$ and variance $\delta^{*2} = \frac{\sigma^2 \delta^2}{n\delta^2 + \sigma^2}$ with $u = \sum y_i$

a) Express the Bayes' estimator of θ in terms of $\bar{Y}$ and η, and discuss its behavior as a function of the sample size. *(2 marks)*

b) Find the Bayes' estimator of θ^2, in terms of $n, u, \eta, \delta^2, and\ \sigma^2$. *(2 marks)*

c) Given the following information $\bar{Y} = 100, n = 10, \eta = 50, \delta^2 = 220, \sigma^2 = 125$. Construct a 90% credible interval for θ. *(2 marks)*

4) Let Y be a single observation from a density $f_Y(y|\theta) = \frac{2y}{\theta^2} I_{(0,\theta)}(y)$. Assume θ has a Uniform $(0,1)$ prior.

a) Find the posterior distribution of θ. *(3 marks)*

b) Is the Uniform $(0,1)$ a conjugate prior? Explain. *(1 mark)*

c) If $Y = \frac{1}{2}$, test the hypothesis $H_0: \theta \leq {}^3/_4 \ vs. H_a: \theta > {}^3/_4$ *(2 marks)*