KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
Term 181
STAT 302 Exam 4

Name: _____ ID #: _____

Instructions:

- Justify your work
- State theorems and results you are using
- Show all details
- In hypothesis testing problems, write down the hypotheses, the rejection region, the decision and the conclusion.

Question	Marks	Marks Obtained
1	6	
2	3	
3	9	
4	7	
5	15	
Total	40	

1) The hourly wages in a particular industry have a mean of \$13.20 and a standard deviation of \$2.50. A company in this industry employs 25 workers paying them an average of \$12.20 per hour.

   a) Can this company be accused of paying below market rate? (5 marks)

   b) Find the p-value of the test. (1 mark)

2) Let $Y_{11}, \cdots, Y_{1n_1}$ and Let $Y_{21}, \cdots, Y_{2n_2}$ be two independent random samples from two normal populations with variances $\sigma_1^2 \; and \; \sigma_2^2$ respectively.
   Show that if we use $F = \frac{S_1^2}{S_1^2}$ for testing $H_0: \sigma_1^2 = \sigma_2^2$ vs $H_0: \sigma_1^2 \neq \sigma_2^2$, the test has rejection region $\left\{F > F_{n_2-1,\alpha/2}^{n_1-1} \; or \; F < \left(F_{n_1-1,\alpha/2}^{n_2-1}\right)^{-1}\right\}$. (3 marks)

3) A random sample of 500 measurements on the length of stay in hospitals had a mean of 5.4 days and a standard deviation of 3.1 days.

a) Is there evidence to support that the length of stay exceeds 5 days? Use $\alpha = 0.05$. (5 marks)

b) If the ministry of health is interested in detecting cases where the average stay is at least 5.5 days, what is the power of the test in a)? (2 marks)

c) How large should the sample size be if we require that

$\alpha = 0.01, \beta = 0.05$ and $\mu = 5.5$ days (2 marks)

4) Let $Y_1, \cdots, Y_n$ are i.i.d random variables from a population with probability density

$$f(y) = \begin{cases} \theta e^{-\theta y}, & y \geq 0 \\ 0, & otherwise \end{cases}$$

a) Find the likelihood ratio test of $H_0: \theta = \theta_0$ vs $H_a: \theta > \theta_0$, and express it in its simplest form (7 marks)

5) Consider a radar system that uses radio waves to detect aircraft. The system receives a signal and based on that signal it decides whether an aircraft is present or not. Let Y be the received signal. Suppose that we know that $Y = W$ if no aircraft is present, and $Y = 1 + W$ if an aircraft is present, where $Y \sim N\left(0, \frac{1}{9}\right)$. Let $Y = \theta + W$.

a) In terms of θ, write down the hypotheses
H_0: No aircraft is present, H_a: aircraft is present. (1 mark)

b) At the 5% significance level find the most powerful test of H_0 vs H_a. (5 marks)

c) Explain what does the probability of type II error mean in the above problem. (1 mark)

d) Find the probability of type II error. (3 marks)

e) If we observe $Y = 0.6$, is there enough evidence to reject H_0 at the 1% significance level? (2 marks)

f) If we would like the probability of missing a present aircraft to be less 5%, what is the smallest significance level that we can achieve? (3 marks)