KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
Term 181
STAT 302 Final Exam

Name: _____ ID #: _____

Observe the following rules, otherwise you will lose marks

- Show all details.
- State all assumptions needed
- If there is a rule you are using, write down that rule.
- Answers without justification are not accepted.
- For testing problems
  - Use α=5%, unless otherwise is stated
  - Always write down
    - the hypotheses
    - the test statistic
    - the rejection region
    - the observed test statistic
    - the decision, and
    - the conclusion

Question	Maximum Marks	Marks Obtained
1	37	
2	6	
3	12	
Total	55	

1) Let $Y_1, \cdots, Y_{100}$ be a random sample from the exponential distribution $f(y) = \frac{1}{\beta} e^{-\frac{y}{\beta}}, y > 0$.

a) Find a sufficient statistic for β. *(2 marks)*

b) Find a UMVU estimator for β. *(2 marks)*

c) Find the maximum likelihood estimator of $P(Y > 1)$. *(4 marks)*

d) Find the density of $Y_{(100)}$. *(2 marks)*

e) Does $Y_{(100)}$have an exponential density? *(1 mark)*

f) If m is the median of Y, find $P(Y_{(100)} > m)$. *(4 marks)*

g) If $\beta = 2$, Find $P(\bar{Y} > 2.1)$. *(4 marks)*

h) Is $Y_{(1)}$ an unbiased estimator of β? *(3 marks)*

i) Is $Y_{(1)}$ a consistent estimator of β? *(2 marks)*

j) Construct an unbiased estimator of β based on $Y_{(1)}$, call it $\hat{\beta}_1$. *(1 marks)*

k) Find the efficiency of $\hat{\beta}_1$ relative to $\bar{Y}$. *(2 marks)*

l) Construct a 90% upper one-sided confidence interval for β based on $Y_{(1)}$. *(5 marks)*

m) At the 5% level, find the rejection region of the most powerful test of $H_0: \beta = 1$ vs $H_a: \beta = 2$, based on $Y_{(1)}$. *(5 marks)*

2) Aptitude tests should produce a large amount of variation so that an administrator can distinguish between persons with low aptitude and persons with high aptitude. The standard test used a certain industry has been producing scores with a standard deviation of 10 points. A new test is given to 20 prospective employees and produces a sample standard deviation of 12 points. Are scores from the new test significantly more variable than scores from the standard test? Use $\alpha = 0.01$

(6 marks)

3) The following table shows the time intervals, in seconds, between successive SUV's in free flowing traffic on a highway.

Time	(0,20]	(20,40]	(40,60]	(60,90]	(90,120]	(120,180]
Frequency	41	19	16	13	9	2

Can these time be modeled by an exponential distribution? *(12 marks)*