KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
Term 181
STAT 302 Exam 1

Name: _____ ID #: _____

1) The population of soft drink cans filled by a particular machine is known to be normally distributed with a mean equal to 12 ounces and a standard deviation equal to 0.25 ounces. For a sample of size 25 cans,
   a) Find the probability that the sample mean exceeds 12.05 ounces. *(2 marks)*

   b) What sample size is needed in order to guarantee that the standard deviation is less than 0.2 with probability 0.99? *(2 marks)*

   c) If in a sample of size 10 the standard deviation of fill is 0.20, what is the probability that the sample mean is less than 12.05 ounces? *(2 marks)*

2) Find the probability density function, in its simplest form, of the median of a sample of size 9 from a uniform distribution on the interval [a, b]. *(2 marks)*

3) Let $Y_1, \cdots Y_n$ be a random sample from a $N(0,1)$ distribution. For all the following parts show details and give reasons.

   i) What is the distribution of $(Y_1 - Y_2)/\sqrt{2}$? *(2 marks)*

   ii) What is the distribution of $\frac{(Y_1 + Y_2)^2}{(Y_1 - Y_2)^2}$? *(2 marks)*

iii) What is the distribution of $\frac{(Y_1+Y_2)}{\sqrt{(Y_1-Y_2)^2}}$? *(2 marks)*

iv) Let $\bar{Y}_k = \frac{1}{k}\sum_1^k Y_i$ and $\bar{Y}_{n-k} = \frac{1}{n-k}\sum_{k+1}^n Y_i$, find the distribution of

$$k(\bar{Y}_k)^2 + (n-k)\,(\bar{Y}_{n-k})^2$$ *(2 marks)*

4) Let Y_i be a random variable distributed $N(i, i^2)$. Assume Y_1, Y_2, Y_3 are independent. Using only Y_1, Y_2, Y_3

a) Give an example of a statistic that has a chi-square distribution. Justify your answer. *(2 marks)*

b) Give an example of a statistics that has an F distribution with 1 and 2 degrees of freedom. Justify your answer. *(2 marks)*