King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics

Math 472 (181)
Numerical Analysis II

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Homework 1 (Taylor Polynomial)

Let $f(x) = \sin x$.

1) Find the Taylor polynomials $P_n(x)$, $n = 1, 3, 5, 7$, about $x_0 = 0$, in the interval $[-\pi, \pi]$.
2) Write a Matlab script to plot in **a single figure** $f(x)$ and $P_n(x)$, $n = 1, 3, 5, 7$. Submit the script and the graph.
3) Find an upper bound for $|f(x) - P_5(x)|$, $0 \leq x \leq 0.3$, and compare the bound to the actual error. Plot.
4) Approximate $\int_0^\pi f(x)\,dx$ using $\int_0^\pi P_5(x)\,dx$ and compute the relative error.

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Homework 2 (Lagrange Interpolating Polynomial)

1. Consider the function $f(x) = \sin(\ln x)$.
   a. Construct the second degree Lagrange interpolating polynomial P for f using the nodes $x_0 = 2.0, x_1 = 2.4, x_2 = 2.6$. Plot f and P in the interval $[2.0, 2.6]$.
   b. Find a bound for the absolute error on the interval $[2.0, 2.6]$.
   c. Compare the bound and the actual error. Plot.

2. "Discussion Questions" number 1, page 115
3. "Discussion Questions" number 2, page 115

DISCUSSION QUESTIONS

1. Suppose that we use the Lagrange polynomial to fit two given data sets that match exactly except for a small perturbation in one of the data points due to measurement error. Although the perturbation is small, the change in the Lagrange polynomial is large. Explain why this discrepancy occurs.
2. If we decide to increase the degree of the interpolating polynomial by adding nodes, is there an easy way to use a previous interpolating polynomial to obtain a higher-degree interpolating polynomial, or do we need to start over?

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Homework 3 (Divided Differences)

Consider the function $f(x) = \ln x$.

1. Write a Matlab code that constructs the divided difference table of f corresponding to the ordered nodes $\{1, 4, 5, 6\}$.
2. Let P be the cubic Newton's interpolating polynomial obtained from the table. Plot f and P. Calculate the relative error at $x = 2$.
3. Plot and compare the graphs of the quadratic Newton's interpolating polynomials corresponding to the following ordered nodes:

$$\{1, 4, 5\}, \qquad \{4,1,5\}, \qquad \{5,1,4\}.$$

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Homework 4 (Cubic splines)

Consider the function $f(x) = \cos(2\pi x)$ and the following data generated by this function.

x	**0.0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
y	1.000	0.3090	-0.8090	-0.8090	0.3090	1.0000

Use Matlab functions spline and polyfit to compare the following:

- the function f
- interpolating polynomial
- clamped spline
- not-a-knot spline

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Homework 5 (Numerical differentiation)

1) Let $h = x_{j+1} - x_j$ and $y_j = y(x_j)$. Explain why the following is wrong.

$$y'(x_j) = \frac{1}{h}\left(2y_{j+1} - y_j\right) + O(h).$$

2) Faraday's law characterizes the voltage drop across an inductor as

$$V = L\frac{di}{dt}$$

where, $V =$ voltage (V), $L =$ inductance (H), $i =$ current (A), $t =$ time (s). The following data is for an inductance of 4 H.

t	0	0.1	0.2	0.3	0.4	0.5
j	0	0.16	0.32	0.56	0.84	2.0

Using Matlab, plot in <u>one figure</u> the interpolating polynomials for the voltage using the following approximations.

a. Forward-difference
b. Centered-difference
c. Richardson extrapolation using forward-difference

Compare and comment on your results.

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Homework 6 (Numerical integration)

Use Gaussian quadrature with $n = 2$ to approximate the integral

$$\int_0^2 e^x x \cos x \; dx.$$

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Homework 7 (Numerical IVP)

Consider the IVP

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 2.0.$$

The exact solution is

$$y(t) = (t+1)^2 + e^t.$$

Produce Table 5.8 and Table 5.13 for this problem.

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Homework 8 (Approximations)

1) Repeat Example 1 of section 8.3 for the function $f(x) = xe^{2x}$. Produce Table 8.8 and Figure 8.12 for this function.

2) Derive the Padé approximation to e^{-x} of degree 4 with n = 2 and m = 2:

$$R_{2,2}(x) = \frac{12 - 6x + x^2}{12 + 6x + x^2}.$$

3) Plot the function e^{-x} and its 4th order Padé approximations:

$$R_{2,2}(x), \qquad R_{0,4}(x) = \frac{24}{24 + 24x + 12x^2 + 4x^3 + x^4}, \qquad R_{1,3}(x) = \frac{1 - \frac{1}{4}x}{1 + \frac{3}{4}x + \frac{1}{4}x^2 + \frac{1}{24}x^3}.$$

Show all the curves in one figure for the interval $[0, 6]$. Compare.

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