King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics

Math 472 (181): Numerical Analysis II

Final Exam

Instructor: Khaled Furati
Duration: 150 minutes

Student Name: ……

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1. Determine the Padé approximation $r(x) = p_1(x)/q_1(x)$ of degree 2 for e^x. Compare the approximated value at $x = 1$ with the actual value.

2. Consider the initial-value problem
$$y' = 1 + (t - y)^2, \qquad 2 \le t \le 3,$$
$$y(2) = 1,$$
with exact solution $y(t) = t + (1 - t)^{-1}$.
   a) Use Euler's method with $h = 0.5$ to approximate the solution.
   b) Use the Midpoint method with $h = 0.5$ to approximate the solution.
   c) Compare the actual error of both methods at each step.

3. The boundary-value problem
$$y'' = y' + 2y + \cos x\,, \qquad 0 < x < \frac{\pi}{2},$$
$$y(0) = -0.3, \qquad y\left(\frac{\pi}{2}\right) = -0.1$$
has the solution $y = -\frac{1}{10}(\sin x + 3\cos x)$.
   a) Use the linear finite-difference method with $h = \pi/4$ to approximate the solution.
   b) Compare the results to the actual solution.

4. Consider the problem
$$-x^2y'' - 2xy' + 2y = -4x^2, \qquad 0 \le x \le 1,$$
$$y(0) = y(1) = 0$$
with exact solution $y(x) = x^2 - x$.
   a) Use piecewise-linear Raleigh-Ritz method with $h = 0.25$ to approximate the solution.
   b) Compare the results with the actual solution.

$$\phi_i(x) = \begin{cases} 0, & \text{if } 0 \le x \le x_{i-1}, \\ \dfrac{1}{h_{i-1}}(x - x_{i-1}), & \text{if } x_{i-1} < x \le x_i, \\ \dfrac{1}{h_i}(x_{i+1} - x), & \text{if } x_i < x \le x_{i+1}, \\ 0, & \text{if } x_{i+1} < x \le 1, \end{cases}$$

Q	1	2	3	4	Total
Max	10	20	20	20	70
Points					