King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics

Math 372 Final Exam

Semester 2018 (181)

Time Allowed: 180 Minutes

Name:________________________________ID#:____________ Sec.

- Mobiles are not allowed in this exam.
- Write all steps clear.

Question #	Answer
1/7	
2/8	
3/10	
4/15	
5/10	
6/10	
7/10	
8/10	
9/10	
10/10	
11/10	
12/10	
Total	
	Grand Total out of 120

Q:1(7 points) Use Secant method to solve the equation $x = cosx$ in the interval $[0, \pi/2]$

with initial guess $x_o = 1$ and $x_1 = 0.5$ (find x_3)

Q:2(8 points) Approximate value of $\int_0^1 xe^{sinx}\, dx$ $using\ composite\ Simpson's$ rule with $n = 6$

Q:3 (10 points) Given $f(x) = \sin x$ on $[0.5,1]$

a) Analyze the round-off errors for the formula

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)] + \frac{h^2}{3} f^{(3)}(z)$$

b) If the values of f are given in 4 decimal places, find the **optimal *h***.

Q:4 (5+10 points)

a) Show that the initial-value problem has a unique solution

$$y' = 2y\ cost, \qquad 0 \le t \le 2,\ \ y(0) = 2$$

b) Use Runge-Kutta of order four to approximate the solution in part(a) (find w_2 $with\ h = 0.5$)

Q:5 (10 points) Use Gaussian elimination with scaled partial pivoting and four-digit rounding to solve the system

$$\begin{aligned} 2.11x - 4.21y + 0.921z &= 2.01, \\ 4.01x + 10.2y - 1.12z &= -3.09, \\ 1.09x + 0.987y + .832z &= 4.21 \end{aligned}$$

Q:6 (10 points) Using the data points, (0, 0), (1, -1), (3, 3) second order interpolating polynomial using Newton's divided difference interpolation

Q.7 (10 points) The following data are exponentially related $y = be^{ax}$

$(1, 5.1), (1.25, 5.79), (1.5, 6.53), (1.75, 7.45), (2, 8.46).$

By least squares approximation find a and b.

Q:8(10 points) The boundary-value problem

$$y'' = -(x+1)y' + 2y + (1-x^2)e^{-x}, \quad 0 \le x \le 1, \qquad y(0) = -1, y(1) = 0, h = 0.2$$

By using Linear Finite-Difference method write the problem in matrix form.

Q:9 (10 points) Solve by using simplex method

Maximize $z = 5x_1 + 4x_2$

$$x_1 + x_2 \le 20$$
$$2x_1 + x_2 \le 35$$
$$-3x_1 + x_2 \le 12$$

and $x_1, x_2 \ge 0$

Q:10(10 points)

Write a MATLAB code that does the following:

a) Determine the linear least square polynomial for a set of data of the form

x	1	2	3	4	5
y	1.3	3.5	4.2	5	7

b) Plot the set of data and its linear fit in the same figure window.

Q:11 (10 points) Write a MATLAB code to approximate the solution of the initial value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \qquad y(0) = 0.5$$

using Euler method with $h = 0.2$.

Also include the commands to plot the numerical solution and the exact solution

$$y = 1 + t^2 + 2t - 0.5e^t$$

Q:12(10 points) Write a MATLAB code to approximate

$$\int_3^6 x^3 \ln x \, dx$$

By using Composite Simpson's rule.

The End

$$y''(x_i) = p(x_i)y'(x_i) + q(x_i)y(x_i) + r(x_i).$$

$$A\mathbf{w} = \mathbf{b}, \quad \text{where} \tag{11.19}$$

$$A = \begin{bmatrix} 2+h^2q(x_1) & -1+\frac{h}{2}p(x_1) & 0 & \cdots & \cdots & 0 \\ -1-\frac{h}{2}p(x_2) & 2+h^2q(x_2) & -1+\frac{h}{2}p(x_2) & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & \ddots & -1+\frac{h}{2}p(x_{N-1}) \\ 0 & \cdots & \cdots & 0 & -1-\frac{h}{2}p(x_N) & 2+h^2q(x_N) \end{bmatrix},$$

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-1} \\ w_N \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} -h^2r(x_1) + \left(1+\frac{h}{2}p(x_1)\right)w_0 \\ -h^2r(x_2) \\ \vdots \\ -h^2r(x_{N-1}) \\ -h^2r(x_N) + \left(1-\frac{h}{2}p(x_N)\right)w_{N+1} \end{bmatrix}.$$

$y_i = a_1x_i + a_0$

$$a_0 = \frac{\sum_{i=1}^{m} x_i^2 \sum_{i=1}^{m} y_i - \sum_{i=1}^{m} x_iy_i \sum_{i=1}^{m} x_i}{m\left(\sum_{i=1}^{m} x_i^2\right) - \left(\sum_{i=1}^{m} x_i\right)^2}$$

$$a_1 = \frac{m\sum_{i=1}^{m} x_iy_i - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} y_i}{m\left(\sum_{i=1}^{m} x_i^2\right) - \left(\sum_{i=1}^{m} x_i\right)^2}.$$

Runge-Kutta Order Four

$$\begin{aligned} w_0 &= \alpha, \\ k_1 &= hf(t_i, w_i), \\ k_2 &= hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_1\right), \\ k_3 &= hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_2\right), \\ k_4 &= hf(t_{i+1}, w_i + k_3), \\ w_{i+1} &= w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \end{aligned}$$

for each $i = 0, 1, \ldots, N-1$. This method has local truncation error $O(h^4)$, provided the