King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 372 Major Exam II
14/11/ 2018 (Term181)
Time Allowed: 120 Minutes

Write all steps clear.

Problem 1(15points)

Use Newton's divided-difference formula to construct interpolating polynomial for the following data. Then approximate $f(2)$.

$$f(-1)=-2,\ f(0)=1,\ f(1)=0,\ f(3)=10.$$

Problem 2 (15 points)

Given the function $f(x)=e^x$ on [0 , 2] and nodes 0, 1, 2. State all the conditions that need to be satisfied so that S will be a cubic spline interpolant for *f* and find the system of equations.(Do not solve the system)

Problem 3. (10+ 10+10points)

a) Derive the formula

$$f''(x_0)=\frac{1}{h^2}[f(x_0-h)-2f(x_0)+f(x_0+h)]-\frac{h^2}{12}f^{(4)}(\xi)$$

for some ξ, where $x_0-h<\xi<x_0+h$.

(Using third Taylor polynomial about x_0).

b) Use the Composite Simpson's rule with $n=6$ to approximate $\int_0^2 \sin(x)e^{-x^2}\,dx.$

c) Determine the values of ***n* and *h*** required to approximate

d) $\int_0^2 xe^x\,dx.$ to within 10^{-2}. Use composite Trapezoidal rule.

Problem 4. (10+ 10+15+5 points)

a) Show that the initial-value problem has a unique solution

$$y'=\frac{1+y}{t+1},\ \ 0\le t\le 1,\ \ y(0)=1$$

b) Use Euler's method to approximate the solution in part(a) with $n=2$.
c) Use Runge-Kutta method to approximate the solution in part(a) with $n=2$.
d) Write your comments for parts (b) and (c).

The end

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right] - \frac{(b-a)f''(\mu)}{12}h^2$$

for some number μ in(a, b).

Runge-Kutta Order 4 Method

$$\begin{aligned}
w_0 &= \alpha \\
k_1 &= hf(t_i, w_i) \\
k_2 &= hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_1\right) \\
k_3 &= hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_2\right) \\
k_4 &= hf(t_{i+1}, w_i + k_3) \\
w_{i+1} &= w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)
\end{aligned}$$

for each $i = 0, 1, \ldots, N-1$. This method has local truncation error $O(h^4)$, provided the solution $y(t)$ has five continuous derivatives.

Euler's Method

Euler's method constructs $w_i \approx y(t_i)$, for each $i = 1, 2, \ldots, N$, by deleting the remainder term. Thus Euler's method is

$$\begin{aligned}
w_0 &= \alpha \\
w_{i+1} &= w_i + hf(t_i, w_i), \quad \text{for each } i = 0, 1, \ldots, N-1
\end{aligned}$$