KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 225: TEST 3, SEMESTER (181), DECEMBER 03, 2018

Time: 17:30 to 18:30

Name : ....

ID : .... Section : ....

Exercise	Points
1	____ / 12
2	____ / 12
3	____ / 12
4	____ / 12
5	____ / 12
Total	____ / 60

Exercise 1. Determine whether the following functions are linear Transformations.

(*a*) $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ x-y \end{bmatrix}.$$

(*b*) $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 \\ y \end{bmatrix}.$$

Exercise 2. Let $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be the linear Transformation defined by

$$L(\mathbf{x}) = \begin{bmatrix} x_1 - x_2 \\ 3x_2 - x_3 \end{bmatrix},$$

where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

(1) Let S be the subspace of $\mathbb{R}^3$ spanned by $\mathbf{e}_1$ and $\mathbf{e}_3$; then find a basis of $L(S)$.

(2) Find $\ker(L)$ and $\mathrm{Im}(L)$.

Exercise 3. Let $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be the linear transformation defined by $L(\mathbf{x}) = (x+2y)\mathbf{b}_1 + (-2x+3y+z)\mathbf{b}_2$ for each $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, where

$$\mathbf{b}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{ and } \mathbf{b}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(1) Show that $(\mathbf{b}_1, \mathbf{b}_2)$ is a basis of $\mathbb{R}^2$.
(2) Find the matrix A representing L with respect to the ordered bases $\mathcal{B}_1 = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ and $\mathcal{B}_2 = (\mathbf{b}_1, \mathbf{b}_2)$.
(3) Find the matrix B representing L with respect to the canonical basis $\mathcal{B}_1 = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ of $\mathbb{R}^3$ and the canonical basis $\mathcal{B} = (\mathbf{f}_1, \mathbf{f}_2)$ of $\mathbb{R}^2$.

Exercise 4. Let $S = \{(2x - y, 2x - y, x - 3y)^T : x, y \in \mathbb{R}\}$.

(1) Find an orthonormal basis of the vector space S (use Gram-Schmidt).
(2) Find the closest vector $\mathbf{p}$ of S to $\mathbf{w} = (2, 1, 2)^T$.

Exercise 5. Let

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & -1 & 1 \end{pmatrix}$$

(1) Find the reduced row echelon form of A.

(2) Find bases for $\mathbf{NS}(A)$ and $\mathbf{CS}(A)$.

(3) Use (2) to find bases for $\mathbf{NS}(A^T)$ and $\mathbf{CS}(A^T)$.