KFUPM **Semester 181**

Dept. Math. &Stat. **A.Y:2018/2019**

TEST 2

(November 01, 2018, Duration: 60 mn)

Name : **ID :**

Exercise 1[10 pt]. For each of the following sets, decide whether it is a subspace of $\mathbb{R}^3$ or not.

$$E_1 = \{(x,y,z) \in \mathbb{R}^3 \mid 3x - 7y = z\}$$
$$E_2 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 - z^2 = 0\}$$
$$E_3 = \{(x,y,z) \in \mathbb{R}^3 \mid x + y - z = x + y + z = 0\}$$
$$E_4 = \{(x,y,z) \in \mathbb{R}^3 \mid z(x^2 + y^2) = 0\}$$

Exercise 2[10 pt]. Find all values of t such that

$$\{(1,0,t),(1,1,t),(t,0,1)\}$$

is a basis of $\mathbb{R}^3$

Exercise 3[10 pt]. Consider the set

$$E = \mathbb{R}_+^* \times \mathbb{R}$$

equipped with the operations:

$$+ : (a,b) + (a',b') = (aa', b+b')$$

$$(\forall \lambda \in \mathbb{R}) \forall (a,b) \in E \lambda.(a,b) = (a^{\lambda}, \lambda b).$$

Show that $(E, +.)$ is a real vector space.

Exercise 4[10 pt].

- Show that

$$S = \{\mathbf{e}_1 = (1,1,1), \mathbf{e}_2 = (1,1,2), \mathbf{e}_3 = (1,2,3)\}$$

is a basis of $\mathbb{R}^3$.

- For each vector $u = (a, b, c)$ of $\mathbb{R}^3$, find the vector coordinates of u with respect to the basis S.

Exercise 5[10 pt]. Find a basis and the dimension of the following real vector space

$$V = \{\, (x, y, z, t) \in \mathbb{R}^4 \;\mid\; x - 2y + 3z + t = 0\}.$$

Exercise 6[10 pt]. Let $B = (v_1, v_2, v_3)$ and $B' = (u_1, u_2, u_3)$, where

$$v_1 = (1,1,1), v_2 = (2,3,2), v_3 = (1,5,4), u_1 = (1,1,0), u_2 = (1,2,0), u_3 = (1,2,1).$$

1. Show that B and B' are basis of $\mathbb{R}^3$.
2. Find the transition matrix from B to B'.
3. Let $\boldsymbol{x} = av_1 + bv_2 + cv_3$, find the coordinates of $\boldsymbol{x}$ with respect to the basis B'.