Name: ID#: Serial #:

1. [5pts] Let $f:\mathbb{R}\longrightarrow\mathbb{R}$ be defined by $f(x)=2x^2+5$. Find each of the following.
$f(\{1,2\})$, $f([1,2])$, $f^{-1}(\{7\})$, $f^{-1}(\{3\})$, $f^{-1}([23,37])$.

**Solution**. $f\{(1,2)\}=\{f(1),f(2)\}=\{7,13\}$.
$f([1,2])=[7,13]$.
$f^{-1}(\{7\})=\{-1,1\}$.
$f^{-1}(\{3\})=\emptyset$.
$f^{-1}([23,37])=[-4,-3]\cup[3,4]$.

2. [5pts] Show that the set $3\mathbb{Z}$ of all multiples of 3 is countable.

**Solution**. Let $f:\mathbb{Z}\longrightarrow 3\mathbb{Z}$ be given $f(x)=3x$ for each integer $x$. Then $f$ is one-to-one because if $f(x)=f(x')$, then $3x=3x'$ i.e. $x=x'$. Also, $f$ is onto because if $y\in 3\mathbb{Z}$, then $y=3x$ for some integer $x$ and so $f(x)=y$. Hence $f$ is a bijection from the denumerable set $\mathbb{Z}$ onto $3\mathbb{Z}$, and therefore $3\mathbb{Z}$ is denumerable (and hence countable).
[Another way: Construct a one-to-one correspondence between the sequence $1,2,3,4,5,\ldots$ and $0,3,-3,6,-6,\ldots$.
A third way: Since $3\mathbb{Z}$ is clearly an infinite subset of $\mathbb{Z}$ and $\mathbb{Z}$ is denumerable, it follows that $3\mathbb{Z}$ is denumerable (and hence countable).]

3. [5pts] The sum of the digits of a positive integer $n$ is 12. Does this mean $n$ is even? Does it mean $n$ is a multiple of 3? Justify your answers.

**Solution**. No, $n$ is not necessarily even; for example, the sum of the digits of 57 is 12 but 57 is not even.
Yes, $n$ is a multiple of 3 because $n$ is divisible by 3 iff the sum of its digits (which is 12 in this case) is divisible by 3.

4. [5pts] Let $n$ be an integer. Determine $\gcd(n,n+1)$.

**Solution**. Let $d=\gcd(n,n+1)$, then $d$ divides $(n+1)-n$, i.e. $d$ divides 1. Since $d$ is positive (as a gcd), it follows that $d=1$.