KFUPM/ Department of Mathematics & Statistics/ 181/ MATH 232/ Quiz 1

Name: **ID #:** **Serial #:**

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1. Let P and Q be statements. Determine whether the statement $((P \longrightarrow Q) \longrightarrow P) \longrightarrow P$ is a tautology, a contradiction, or neither.

Solution. This can be done using a truth table or using equivalence of compound statements (e.g. using $P \longrightarrow Q \equiv \sim P \vee Q$).

Another way is to proceed by cases, as follows.

Assume first that P is true. Then the statement $((P \longrightarrow Q) \longrightarrow P) \longrightarrow P$ is trivially true.

Assume next that P is false. Then $P \longrightarrow Q$ is vacuously true, so $(P \longrightarrow Q) \longrightarrow P$ is false. Hence the statement $((P \longrightarrow Q) \longrightarrow P) \longrightarrow P$ is vacuously true.

This means $((P \longrightarrow Q) \longrightarrow P) \longrightarrow P$ is a tautology.

2. Let $A = \emptyset$ and $B = \{1, 2\}$. Determine $A \times B$ and $\mathcal{P}(A) \times \mathcal{P}(B)$.

Solution. $A \times B = \emptyset$ (since $A = \emptyset$).

We have $\mathcal{P}(A) = \{\emptyset\}$ and $\mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}, B\}$.

Hence $\mathcal{P}(A) \times \mathcal{P}(B) = \{(\emptyset, \emptyset), (\emptyset, \{1\}), (\emptyset, \{2\}), (\emptyset, B)\}$