MATH 202.5 (Term 181)

Quiz 4 (Sects. 4.5, 4.6 & 4.7) **Duration: 20min**

Name: **ID number:**

1.))(5pts)Solve the DE: $y'' - \frac{1}{4}y = \frac{1}{e^{-x}+1}$.

2.)(5pts)Solve the DE: $x^3y''' - 2x^2y'' + 4y = 0$.

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1) First, we solve the homogeneous equation $y'' - \frac{1}{4}y = 0$.

Its auxiliary equation is $m^2 - \frac{1}{4} = 0$,

$m = -\frac{1}{2}, \; m = \frac{1}{2}$

$\Rightarrow y_c = c_1 e^{-\frac{x}{2}} + c_2 e^{\frac{x}{2}}$.

Now, we use variation of parameters to find y_p. That is,

$y_p = u_1 e^{-x/2} + u_2 e^{x/2}$, where

$u_1' = \frac{-e^{x/2}}{(e^{-x}+1)W}, \quad u_2' = \frac{e^{-x/2}}{(e^{-x}+1)W}$,

$$W = \begin{vmatrix} e^{-x/2} & e^{x/2} \\ -\frac{1}{2}e^{-x/2} & \frac{1}{2}e^{x/2} \end{vmatrix} = 1$$

$u_1 = -\int \frac{e^{\frac{3x}{2}}}{e^{x}+1}dx, \quad v = e^{x/2}, \; dv = \frac{1}{2}e^{x/2}dx$

$= -2\int \frac{v^2}{v^2+1}dv = \int\left(1 - \frac{1}{v^2+1}\right)dv$

$= -2(v - \tan^{-1}v)$

$= -2(e^{x/2} - \tan^{-1}e^{x/2})$

$u_2 = \int \frac{e^{x/2}}{e^{x}+1}dx = 2\int \frac{dv}{v^2+1} = 2\tan^{-1}v$

$= 2\tan^{-1}e^{x/2}$

$\Rightarrow y_p = -2e^{-x/2}(e^{x/2} - \tan^{-1}e^{x/2}) + 2e^{x/2}\tan^{-1}e^{x/2}$

$= -2 + 2(e^{-x/2} + e^{x/2})\tan^{-1}e^{x/2}$

$y = c_1 e^{-x/2} + c_2 e^{x/2} + y_p, \; x \in (-\infty, \infty)$

2) This is Cauchy equation.

$y = x^m$

$\Rightarrow m(m-1)(m-2) - 2m(m-1) + 4 = 0$

$m^3 - 5m^2 + 4m + 4 = 0$

$(m-2)(m^2 - 3m - 2) = 0$

$m = 2, \; m = \frac{3+\sqrt{17}}{2}, \; m = \frac{3-\sqrt{17}}{2}$

$\Rightarrow y = c_1 x^2 + c_2 x^{\frac{3+\sqrt{17}}{2}} + c_3 x^{\frac{3-\sqrt{17}}{2}}, \; x \in (0, \infty)$

MATH 202.6 (Term 181)

Quiz 4 (Sects. 4.5, 4.6 & 4.7) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Solve the DE: $y'' + 3y' + 2y = \cos e^x$

2.)(5pts)Solve the DE: $x^3y''' + 3x^2y'' + xy' - y = 0.$

1.) First, we solve the homogeneous equation $y''+3y'+2y=0$. Its auxiliary equation is $m^2+3m+2=0$

$m=-1,\ m=-2$

$\Rightarrow\ y_c = C_1 e^{-x} + C_2 e^{-2x}.$

Now, we use variation of parameters to find y_p. That is,

$y_p = u_1 e^{-x} + u_2 e^{-2x}$, where

$u_1' = -\dfrac{e^{-2x}\cos e^x}{W},\quad u_2' = \dfrac{e^{-x}\cos e^x}{W},$

$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -e^{-3x}$

$u_1 = \int e^x \cos e^x\,dx$, $\quad v = e^x$, $dv = e^x\,dx$

$= \int \cos v\,dv = \sin v = \sin e^x$

$u_2 = -\int e^{2x}\cos e^x\,dx$

$= \underbrace{-\int v\cos v\,dv}_{\text{integration by parts}}$

$= -v\sin v - \cos v$

$= -e^x\sin e^x - \cos e^x$

$\Rightarrow y_p = e^{-x}\sin e^x - e^{-2x}(e^x\sin e^x + \cos e^x)$

$= -e^{-2x}\cos e^x$

$y = C_1e^{-x} + C_2e^{-2x} - e^{-2x}\cos e^x,\quad x\in(-\infty,\infty)$

2.) This is a Cauchy equation.

$y = x^m$

$\Rightarrow m(m-1)(m-2) + 3m(m-1) + m - 1 = 0$

$m^3 - 1 = 0$

$(m-1)(m^2+m+1) = 0$

$m = 1,\quad m = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

$y = C_1x + C_2x^{-\frac{1}{2}}\cos\left(\frac{\sqrt{3}}{2}\ln x\right) + C_3x^{-\frac{1}{2}}\sin\left(\frac{\sqrt{3}}{2}\ln x\right),$

$x\in(0,\infty)$