MATH 202.5 (Term 181)

Quiz 2 (Sects. 2.4 & 2.5) Duration: 20min

Name: **ID number:**

1.)(5pts)Solve the exact DE: $(ye^{-3yx} + \cos^2 x)dx + (xe^{-3xy} + 2\ln y)dy = 0.$

2.)(5pts)Solve the DE: $\frac{1}{2}\frac{dy}{dx} + \frac{x}{x^2+1}y = x\sqrt{y}, \quad y > 0.$

1.) $M = ye^{-3yx} + \cos^2 x$

$N = xe^{-3xy} + 2\ln y$

$M_y = e^{-3yx} - 3xye^{-3xy}$

$N_x = e^{-3xy} - 3xye^{-3xy}$

$M_y = N_x$ DE exact

$$\begin{cases} \frac{\partial f}{\partial x} = ye^{-3yx} + \cos^2 x & (1) \\ \frac{\partial f}{\partial y} = xe^{-3xy} + 2\ln y & (2) \end{cases}$$

$(1) \Rightarrow f(x,y) = \int (ye^{-3yx} + \cos^2 x)\,dx$

$= -\frac{1}{3}e^{-3xy} + \int \left(\frac{\cos 2x + 1}{2}\right) dx$

$= -\frac{1}{3}e^{-3xy} + \frac{\sin 2x}{4} + \frac{x}{2} + g(y)$

$(2) \Rightarrow xe^{-3xy} + g'(y) = xe^{-3xy} + 2\ln y$

$g'(y) = 2\ln y$

$g(y) = 2\int \ln y\, dy$

integration by parts

$u = \ln y \to u' = \frac{1}{y}$

$v' = 1 \to v = y$

$g(y) = 2(y\ln y - y)$

$$\boxed{-\frac{1}{3}e^{-3xy} + \frac{\sin 2x}{4} + \frac{x}{2} + 2(y\ln y - y) = C}$$

implicit solution

2.) This is Bernoulli's DE, $\alpha = \frac{1}{2}$

$u = y^{1-\frac{1}{2}} = y^{\frac{1}{2}} \Leftrightarrow y = u^2$

$\frac{dy}{dx} = 2u\frac{du}{dx}$

Thus, $u\frac{du}{dx} + \frac{x}{x^2+1}u^2 = xu$, $u \neq 0$

$\frac{du}{dx} + \frac{x}{x^2+1}u = x$

$e^{\int \frac{x}{x^2+1}dx} = e^{\frac{1}{2}\ln(x^2+1)} = \sqrt{x^2+1}$

$\frac{d}{dx}(u\sqrt{x^2+1}) = x\sqrt{x^2+1}$

$u\sqrt{x^2+1} = \int x\sqrt{x^2+1}\,dx$

$= \frac{1}{3}(x^2+1)^{3/2} + C$

$u = \frac{1}{3}(x^2+1) + \frac{C}{\sqrt{x^2+1}}$

$\downarrow$

$\sqrt{y} = \underline{\qquad}$

$y = \left(\frac{1}{3}(x^2+1) + \frac{C}{\sqrt{x^2+1}}\right)^2,$

$x \in (-\infty, \infty)$

MATH 202.6 (Term 181)

Quiz 2 (Sects. 2.4 & 2.5) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Solve the exact DE: $(y^2e^x + \sin^2 x)dx + (2ye^x + \ln y)dy = 0.$

2.)(5pts)Solve the DE: $\frac{3}{2}\frac{dy}{dx} + 2xe^{x^2}y = \frac{e^{-e^{x^2}}}{(x^2+1)\sqrt{y}}, \quad y > 0.$

1.) $M = y^2e^x + \sin^2 x$

$N = 2ye^x + \ln y$

$M_y = 2ye^x, \; N_x = 2ye^x$

$M_y = N_x \Rightarrow$ Exact DE

$$\begin{cases} \frac{\partial f}{\partial x} = y^2e^x + \sin^2 x & (1) \\ \frac{\partial f}{\partial y} = 2ye^x + \ln y & (2) \end{cases}$$

$(1) \Rightarrow f(x,y) = \int (y^2e^x + \sin^2 x)\,dx$

$= y^2e^x + \int \frac{1-\cos 2x}{2}\,dx$

$= y^2e^x + \frac{x}{2} - \frac{\sin 2x}{4} + g(y)$

$(2) \Rightarrow 2ye^x + g'(y) = 2ye^x + \ln y$

$g'(y) = \ln y, \; g(y) = \int \ln y \,dy$

integration by parts

$g(y) = y\ln y - y$

$\Rightarrow \boxed{y^2e^x + \frac{x}{2} - \frac{\sin 2x}{4} + y\ln y - y = C}$

implicit solution

2.) This is a Bernoulli DE, $\alpha = -\frac{1}{2}$

$u = y^{1-(-\frac{1}{2})} = y^{3/2} \Leftrightarrow y = u^{\frac{2}{3}}$

$\frac{dy}{dx} = \frac{2}{3}u^{-1/3}\frac{du}{dx}$

$u^{-\frac{1}{3}}\frac{du}{dx} + 2xe^{x^2}u^{2/3} = \frac{e^{-e^{x^2}}}{x^2+1}u^{-1/3}$

$\frac{du}{dx} + 2xe^{x^2}u = \frac{e^{-e^{x^2}}}{x^2+1}$

$e^{\int 2xe^{x^2}dx} = e^{e^{x^2}}$

$\Rightarrow \frac{d}{dx}\left(ue^{e^{x^2}}\right) = \frac{1}{x^2+1}$

$ue^{e^{x^2}} = \int \frac{dx}{x^2+1} = \tan^{-1}x + C$

$u = e^{-e^{x^2}}(\tan^{-1}x + C)$

$\Updownarrow$

$y^{3/2} = \underline{\hspace{3cm}}$

$$\boxed{y = \left(e^{-e^{x^2}}(\tan^{-1}x + C)\right)^{\frac{2}{3}} \quad x \in (-\infty, \infty)}$$