MATH 202.5 (Term 181)
Quiz 1 (Sects. 2.2 & 2.3) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Solve the DE: $\frac{dy}{dx} = y^2(y-1)x^2$.

2.)(5pts)Solve the IVP: $\begin{cases} x \ln x \frac{dy}{dx} + y = x, & x > 0 \\ y(e) = 0. \end{cases}$

1.) • ~~$y=0, y=1$ are solutions.~~

• $\int \frac{dy}{y^2(y-1)} = \int x^2 dx$, if $y \neq 0, y \neq 1$

$\frac{1}{y^2(y-1)} = -\frac{1}{y} - \frac{1}{y^2} + \frac{1}{y-1}$

$\Rightarrow -\ln|y| + \frac{1}{y} + \ln|y-1| = \frac{x^3}{3} + C$

$\frac{1}{y} + \ln\left|\frac{y-1}{y}\right| = \frac{x^3}{3} + C$

$\frac{y-1}{y} = C e^{-1/y} e^{x^3/3}$

y is an implicit solution

2.) $\frac{dy}{dx} + \frac{1}{x \ln x} y = \frac{1}{\ln x}$

$e^{\int \frac{dx}{x \ln x}} = e^{\ln(\ln x)} = \ln x, \ x > 0$

$\Rightarrow \frac{d}{dx}(y \ln x) = 1$

$y \ln x = x + C$

$y = \frac{x+C}{\ln x}$

$y(e) = 0 \Rightarrow e + C = 0, \ C = -e$

$y = \frac{x-e}{\ln x}, \ x \in (0, \infty)$

MATH 202.6 (Term 181)
Quiz 1 (Sects. 2.2 & 2.3) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Solve the DE: $\frac{dy}{dx} = y(y-1)^2x^3$.

2.)(5pts)Solve the IVP: $\begin{cases} (\sin x)\frac{dy}{dx} + (\cos x)y = 1, & x \in (0,\pi), \\ y(\frac{\pi}{2}) = 0. \end{cases}$

1.) • $y=0$, $y=1$ are solutions

• $\int \frac{dy}{y(y-1)^2} = \int x^3 dx$, $y\neq 0, y\neq 1$

$\frac{1}{y(y-1)^2} = \frac{1}{y} - \frac{1}{y-1} + \frac{1}{(y-1)^2}$

$\Rightarrow \ln|y| - \ln|y-1| - \frac{1}{y-1} = \frac{x^4}{4} + C$

$\ln\left|\frac{y}{y-1}\right| - \frac{1}{y-1} = \frac{x^4}{4} + C$

$\frac{y}{y-1} = C e^{\frac{1}{y-1}} e^{\frac{x^4}{4}}$

y is an implicit solution

2.) $\frac{dy}{dx} + \frac{\cos x}{\sin x} y = \frac{1}{\sin x}$

$e^{\int \frac{\cos x}{\sin x} dx} = e^{\ln \sin x} = \sin x$

$\frac{d}{dx}(y \sin x) = 1$

$y \sin x = x + C$

$y = \frac{x+C}{\sin x}$

$y(\frac{\pi}{2}) = 0 \Rightarrow \frac{\pi}{2} + C = 0, C = -\frac{\pi}{2}$

$y = \frac{x - \pi/2}{\sin x}$, $x \in (0, \pi)$

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