KFUPM – Department of Mathematics & Statistics

MATH202 – Term 181

Quiz 2A (Duration: 25 Minutes)

Name: Solution ID #: ______ Section: 03

Q1 (3.5 Points) Solve the DE $(x+1)y' + (x+2)y = 2xe^{-x}$. Give the largest interval I over which the solution is defined.

Solution: the DE can be written as:

$$y' + \left(\frac{x+2}{x+1}\right) y = \frac{2x}{x+1} e^{-x}$$

a linear equation with

$$P(x) = \frac{x+2}{x+1} = \frac{x+1+1}{x+1}$$

$$= 1 + \frac{1}{x+1}$$

I.F. : $e^{\int P(x)\,dx} = e^{\int \left(1+\frac{1}{x+1}\right) dx}$

$$= e^{x + \ln|x+1|} = (x+1)e^{x}$$

Multiplying both sides of the DE by I.F. gives

$$d\left((x+1)e^{x} y\right) = 2x$$

$$(x+1)e^{x} y = x^2 + c$$

or

$$y = \frac{x^2 + C}{e^{x}(x+1)}$$

The interval

$$I = (-1, \infty)$$

Q2 (3.5 Points) Solve the initial value problem $(2xy + x^2 - 2)y' + (x+y)^2 = 0,\ y(1) = 1.$

Solution: The DE can be written as

$$(x+y)^2\,dx + (2xy + x^2 - 2)\,dy = 0$$

Let $M = (x+y)^2$ and $N = 2xy + x^2 - 2$.

We have

$$M_y = 2x + 2y = N_x.$$

The DE is an exact equation

$$F(x,y) = \int M\,dx + g(y)$$

$$= \int (x+y)^2\,dx + g(y)$$

$$= \frac{1}{3}x^3 + x^2 y + y^2 x + g(y)$$

$$\frac{\partial f}{\partial y} = N$$

$$\Rightarrow x^2 + 2xy + g'(y) = 2xy + x^2 - 2$$

$$\Rightarrow g'(y) = -2 \Rightarrow g(y) = -2y$$

So, $F(x,y) = \frac{1}{3}x^3 + x^2 y + y^2 x - 2y$

The general solution is

$$F(x,y) = C$$

$$\Rightarrow \frac{1}{3}x^3 + x^2 y + y^2 x - 2y = C$$

Imposing the initial condition: $y(1) = 1$ gives

$$\frac{1}{3} + 1 + 1 - 2 = C$$

$$\Rightarrow C = \frac{1}{3}$$

The solution is

$$\frac{1}{3}x^3 + x^2 y + xy^2 - 2y = \frac{1}{3}$$

Q3 (3 Points) Solve the differential equation using an appropriate substitution:

$$x^2\frac{dy}{dx} - xy = y^2$$

Solution: The equation can be written as

$$\frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x^2},$$

It is a Bernoulli's equation with $n=2$.

put $u = y^{1-n} = y^{-1} \Rightarrow y = u^{-1}$.

So,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -u^{-2}\frac{du}{dx}$$

Thus, we have

$$-u^{-2}\frac{du}{dx} - x^{-1}u^{-1} = x^{-2}u^{-2}$$

$$\Rightarrow \quad \frac{du}{dx} + x^{-1}u = -x^{-2}$$

a linear equation with $P(x) = \frac{1}{x}$

I.F. : $e^{\int p(x)\,dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$$\Rightarrow \quad d(xu) = -x^{-1}dx$$

$$xu = \int -\frac{1}{x}\,dx = -\ln|x| + c$$

$$\Rightarrow \quad u = -\frac{1}{x}\ln|x| + \frac{c}{x}$$

The solution is

$$y^{-1} = -\frac{\ln|x|}{x} + \frac{c}{x}$$

or

$$y = \frac{x}{c - \ln|x|}$$