KFUPM – Department of Mathematics & Statistics

MATH202 – Term 181

Quiz **1**A (Duration: 25 Minutes)

Name: Solution ID #:__________ Section: 03

Q1 (3.5 Points) Verify that the function $y = 1/(4 - x^2)$ is an explicit solution of the DE $y' = 2xy^2$. Give at least one interval I of definition of the solution.

LHS: $y' = \dfrac{+2x}{(4-x^2)^2} = 2xy^2$

RHS: $2xy^2$

So

LHS = RHS.

The given function is a solution of the DE.

y is not defined at $x = \pm 2$.

The interval of definition:

$I = (-\infty, -2)$

or $I = (-2, 2)$

or $I = (2, \infty)$

Q2 (2.5 Points) Determine a region of the xy-plane for which the DE $(1 - y^2)y' = x^2$ would have a unique solution whose graph passes through a point (x_0, y_0) in the region.

$y' = \dfrac{x^2}{1-y^2}$

define $f(x,y) = \dfrac{x^2}{1-y^2}$

$\dfrac{\partial f}{\partial y} = \dfrac{2x^2 y}{(1-y^2)^2}$

Both f & $\dfrac{\partial f}{\partial y}$ are not defined at $y = \pm 1$

The DE would have a unique solution whose graph passes through (x_0, y_0) in any region in $\mathbb{R}^2$ that does not contain the lines $y = \pm 1$.

Q3 (4 Points) Find an explicit solution of the Initial Value Problem $x^2y' = y - xy,\ y(-1) = -1$.

Determine the largest interval I of definition of the solution.

$\dfrac{dy}{dx} = \dfrac{y(1-x)}{x^2}$, $x \neq 0$

$\dfrac{dy}{y} = \dfrac{1-x}{x^2}\,dx$

$\dfrac{dy}{y} = \left(\dfrac{1}{x^2} - \dfrac{1}{x}\right)dx$

$\ln|y| = -\dfrac{1}{x} - \ln|x| + c_1$

$\ln|y| + \ln|x| = c_1 - \dfrac{1}{x}$

$\ln|xy| = c_1 - \dfrac{1}{x}$

$xy = ce^{-1/x}$, $c = \pm e^{c_1}$

The explicit solution is

$y = \dfrac{c}{xe^{1/x}}$

$y(-1) = -1 \Rightarrow -1 = \dfrac{c}{-1e^{-1}}$

or $c = e^{-1}$.

The Solution is $y = \dfrac{1}{xe^{1/x+1}}$

The interval is

$I = (-\infty, 0)$