Math201.15, Quiz # 2, Term 181

Name: Solutions

ID#: **Serial #:**

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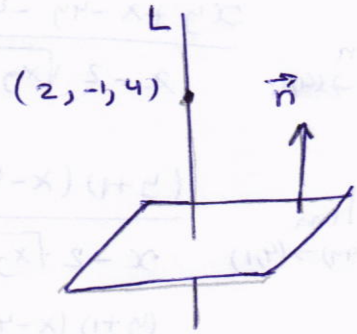
1. **[2 points]** Find parametric equations for the line that passes through $(2,-1,4)$ and is perpendicular to the plane $2x - y + 5z = 10$.
2. **[3 points]** Identify (name, center, axis) and sketch the surface

$$z^2 = \frac{x^2}{4} + y^2 - 2y.$$

3. **[2.5 points]** Let $f(x,y) = \frac{3}{\sqrt{y+x^2}}$.
   a. Find and sketch the domain of f.
   b. Find an equation for the level curve of f that passes through the point $(0,1)$. Sketch the level curve.
4. **[2.5 points]** Find the limit if it exists: $\lim\limits_{(x,y)\to(4,1)} \frac{xy+x-4y^2-4y}{x-2\sqrt{xy}}$.

Good luck,

Ibrahim Al-Rasasi



$\boxed{1}$ • a point on the line is $(2,-1,4)$

• a vector parallel to the line
  = normal vector of the plane
  = $\vec{n} = \langle 2,-1,5\rangle$

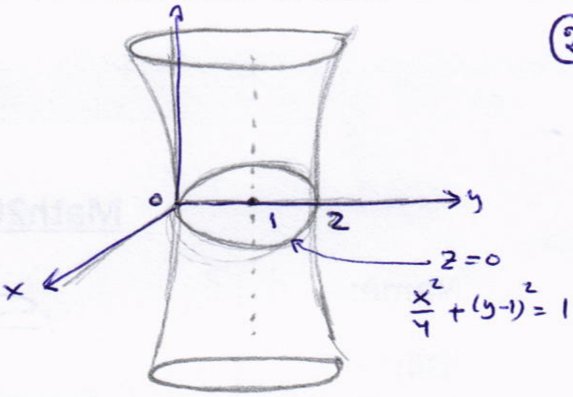
• parametric equations of the line are

$$x = 2 + 2t$$
$$y = -1 - t \quad , t \in \mathbb{R}$$
$$z = 4 + 5t$$

[2] $z^2 = \frac{x^2}{4} + y^2 - 2y \Rightarrow z^2 = \frac{x^2}{4} + (y-1)^2 - 1$

$$\Rightarrow \frac{x^2}{4} + (y-1)^2 - z^2 = 1 \quad 1$$

- a hyperboloid of one sheet 1
- center $(0,1,0)$
- axis : the line through $(0,1,0)$ and parallel to the z-axis.



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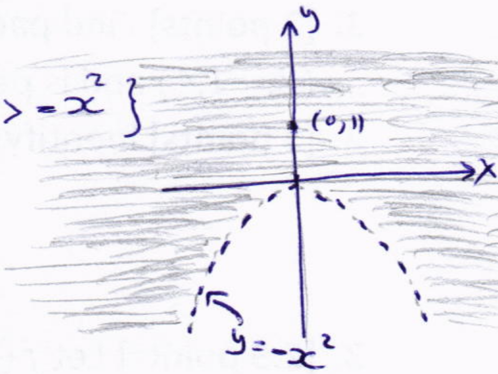
[3] $f(x,y) = \frac{3}{\sqrt{y+x^2}}$

a) Domain $= \{(x,y) \in \mathbb{R}^2 : y + x^2 > 0\} = \{(x,y) \in \mathbb{R}^2 : y > -x^2\}$



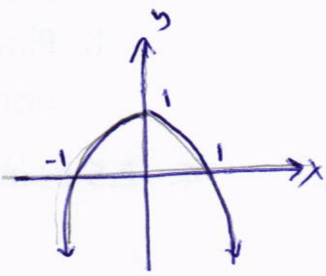
b) An equation for the level curve is

$$f(x,y) = c \Rightarrow \frac{3}{\sqrt{y+x^2}} = c$$

To find c, substitute $(0,1)$:

$$\frac{3}{\sqrt{1+0}} = c \Rightarrow c = 3$$

The equation is $\frac{3}{\sqrt{y+x^2}} = 3 \Rightarrow y + x^2 = 1 \Rightarrow y = -x^2 + 1$.



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[4]

$$\lim_{(x,y)\to(4,1)} \frac{xy + x - 4y^2 - 4y}{x - 2\sqrt{xy}}$$

Factor: $x(y+1) - 4y(y+1) = (y+1)(x-4y)$ 1

$$= \lim_{(x,y)\to(4,1)} \frac{(y+1)(x-4y)}{x-2\sqrt{xy}} \cdot \frac{x+2\sqrt{xy}}{x+2\sqrt{xy}}$$

$$= \lim_{(x,y)\to(4,1)} \frac{(y+1)(x-4y)(x+2\sqrt{xy})}{x^2 - 4xy} \quad 0.5$$

$$= \lim_{(x,y)\to(4,1)} \frac{(y+1)(x-4y)(x+2\sqrt{xy})}{x(x-4y)}$$

$$= \lim_{(x,y)\to(4,1)} \frac{(y+1)(x+2\sqrt{xy})}{x}$$

$$= \frac{2(4+4)}{4} = 4 \quad 0.5$$

Math201.10, Quiz # 2, Term 181

Name:

Solutions

ID#: **Serial #:**

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1. **[2.5 points]** Find the point of intersection, if it exists, between the following two lines:

$$L_1: x = 2 - t, \qquad y = 1 - 3t, \qquad z = 5 + 5t$$
$$L_2: x = 2s, \qquad y = 4 + s, \qquad z = -12 + 5s.$$

2. **[2.5 points]** Identify (name, vertex, axis) and sketch the surface

$$z^2 + 1 = 2x^2 + 3y^2 - 6y + 4.$$

3. **[2.5 points]** Let $f(x, y) = \ln(x^3 - y)$.
   a. Find and sketch the domain of f.
   b. Find an equation for the level curve of f that passes through the point $(1, -1)$. Sketch the level curve.

4. **[2.5 points]** Find the limit if it exists: $\lim_{(x,y)\to(0,0)} \frac{(x+y)^3}{x^3+y^3}$.

Good luck,

Ibrahim Al-Rasasi

$\boxed{1}$ $x = x$, $y = y$, $z = z$ $\Rightarrow$ $2 - t = 2s$, $1 - 3t = 4 + s$, $5 + 5t = -12 + 5s$ $\Rightarrow$ $\begin{cases} -t - 2s = -2 & (1) \\ -3t - s = 3 & (2) \\ 5t - 5s = -17 & (3) \end{cases}$ $\Rightarrow$ $\begin{array}{r} 3t + 6s = 6 \\ -3t - s = 3 \\ \hline 5s = 9 \end{array}$ Add

0.5

$s = \frac{9}{5}$

$\overset{(1)}{\Longrightarrow} t = 2 - 2\left(\frac{9}{5}\right) = 2 - \frac{18}{5} = \frac{-8}{5}$

$t = -\frac{8}{5}$

0.5

Sub. $t = -\frac{8}{5}$ & $s = \frac{9}{5}$ in (3): $\cancel{-\frac{40}{5}}$ $\cancel{/4\bar{5}}$ $-8 - 9 = -17$ ✓

1

So the solution of the system is $s = \frac{9}{5}$, $t = -\frac{8}{5}$

Sub. $t = -\frac{8}{5}$ in L_1 : $x = 2 + \frac{8}{5} = \frac{18}{5}$; $y = 1 + \frac{24}{5} = \frac{29}{5}$; $z = 5 - 8 = -3$

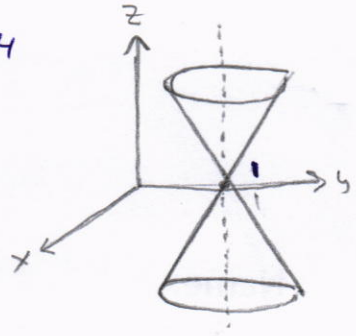
The point of intersection is $\left(\frac{18}{5}, \frac{29}{5}, -3\right)$.

0.5

2 $z^2+1 = 2x^2+3y^2-6y+4 \Rightarrow z^2+1 = 2x^2+3(y^2-2y+)-3+4$

$\Rightarrow z^2+1 = 2x^2+3(y-1)^2+1 \Rightarrow z^2 = 2x^2+3(y-1)^2$ ①

- an elliptic cone
- vertex is $(0,1,0)$
- axis: the line through $(0,1,0)$ & parallel to the z-axis.



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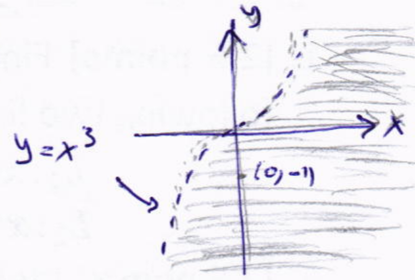
3 $f(x,y) = \ln(x^3-y)$.

(a) Domain $= \{(x,y)\in\mathbb{R}^2 : x^3-y>0\} = \{(x,y)\in\mathbb{R}^2 : y<x^3\}$



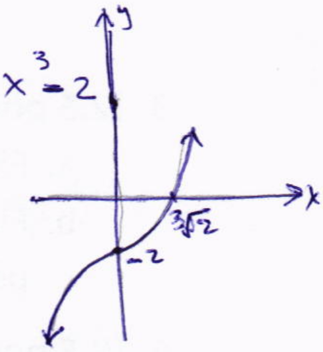
(b) An equation for the level curve is

$f(x,y)=c \Rightarrow \ln(x^3-y)=c$

To find c, substitute $(1,-1)$:

$\ln(1+1)=c \Rightarrow c=\ln 2$

The equation is $\ln(x^3-y)=\ln 2 \Rightarrow x^3-y=2 \Rightarrow y=x^3-2$



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4 $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x+y)^3}{x^3+y^3}$

- Try Paths
- along the x-axis : $y=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{(x+y)^3}{x^3+y^3} = \lim_{(x,y)\to(0,0)} \left.\frac{(x+y)^3}{x^3+y^3}\right|_{y=0} = \lim_{(x,y)\to(0,0)} \frac{x^3}{x^3} = \lim_{(x,y)\to(0,0)} 1 = 1$$

- along the line $y=x$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{(x+y)^3}{x^3+y^3} = \lim_{(x,y)\to(0,0)} \left.\frac{(x+y)^3}{x^3+y^3}\right|_{y=x} = \lim_{(x,y)\to(0,0)} \frac{8x^3}{2x^3} = \lim_{(x,y)\to(0,0)} 4 = 4$$

Since the limits along the above two paths are not equal, then

$$\lim_{(x,y)\to(0,0)} \frac{(x+y)^3}{x^3+y^3} \text{ DNE.}$$