King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 201

FINAL EXAM

181

Sunday 16/12/2018

Net Time Allowed: 180 minutes

MASTER VERSION

1. The slope of the tangent line to the polar curve $r = 1 + \sin\theta$ at $\theta = \pi$ is

(a) -1

(b) $\frac{1}{2}$

(c) $\frac{\sqrt{2}}{2}$

(d) $-\frac{\sqrt{2}}{2}$

(e) 1

2. The area of the region enclosed by one loop of the curve $r = \sin 3\theta$ is

(a) $\frac{\pi}{12}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{10}$

(d) $\frac{\pi}{3}$

(e) $\frac{\pi}{6}$

3. Which vector is always orthogonal to $\mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$?

(a) $\mathbf{a}$

(b) $\mathbf{b}$

(c) $\mathbf{a} - \mathbf{b}$

(d) $\mathbf{b} - \text{proj}_{\mathbf{b}}\mathbf{a}$

(e) $\text{proj}_{\mathbf{b}}\mathbf{a}$

4. The equation of the plane which contains the line

$$L_1 : x = t - 1, \quad y = -2t + 1, \quad z = 2t,$$

and is parallel to the line

$$L_2 : \frac{x}{2} = \frac{y-2}{-3} = z - 2$$

is

(a) $4x + 3y + z + 1 = 0$

(b) $4x - 3y + z + 1 = 0$

(c) $4x - 3y - z - 1 = 0$

(d) $4x + 3y - z + 1 = 0$

(e) $4x + 3y + z - 1 = 0$

5. The distance between the intersection point of the lines

$$L_1 : x + 1 = 4t, \quad y - 3 = t, \quad z - 1 = 0$$

$$L_2 : x + 13 = 12s, \quad y - 1 = 6s, \quad z - 2 = 3s$$

and the $yz-$plane is

(a) 17

(b) 7

(c) 1

(d) 16

(e) 4

6. Suppose (a, b) is an extremal point of $f(x, y) = x^3 y$ under the constraint condition $x^2 + 2xy = 3$. Then $\dfrac{a}{b} =$

(a) 2

(b) -2

(c) 1

(d) 0

(e) -1

7. Let E be the region in $\mathbb{R}^3$ enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the paraboloid $z = x^2 + y^2$. Then $\iiint_E z^2 \, dV$ is

(a) $\frac{\pi}{20}$

(b) $\frac{\pi}{10}$

(c) $\frac{\pi}{30}$

(d) $\frac{\pi}{40}$

(e) $\frac{\pi}{50}$

8. An integral that represents the volume enclosed by the semi-sphere $z = \sqrt{4 - x^2 - y^2}$ and the paraboloid $z = x^2 + y^2 - 4$ is

(a) $2\pi \int_0^2 (r\sqrt{4 - r^2} + 4r - r^3) \, dr$

(b) $2\pi \int_0^2 (r^3 - 4r + r\sqrt{4 - r^2}) \, dr$

(c) $2\pi \int_0^2 (r^3 - 4r - r\sqrt{4 - r^2}) \, dr$

(d) $2\pi \int_0^2 (4 - r^2 + \sqrt{4 - r^2}) \, dr$

(e) $\pi \int_0^2 (r^3 - 4r - r\sqrt{4 - r^2}) \, dr$

9. The volume of the solid that lies above the cone

$$\sqrt{3}\,z = \sqrt{x^2 + y^2}$$

and below the sphere

$$x^2 + y^2 + z^2 - 4z = 0$$

is equal to

(a) 10π

(b) π

(c) $\dfrac{2\pi}{3}$

(d) 5π

(e) 4π

10. The length, width and the height of a rectangular box are measured as 15 cm, 10 cm and 5 cm, respectively, with an error in measurement of at most 0.1 cm in each. Using differentials, the maximum error in the calculated volume of the box is estimated to be

(a) $27.5\ cm^3$

(b) $20\ cm^3$

(c) $25.5\ cm^3$

(d) $17.5\ cm^3$

(e) $37.5\ cm^3$

11. If $P = \sqrt{u^2 + v^2 + w^2}$, $u = xe^y$, $v = ye^x$, $w = e^{xy}$, what is $\dfrac{\partial P}{\partial x}$ at $x = 0$ and $y = 2$?

(a) $\dfrac{6}{\sqrt{5}}$

(b) $\dfrac{2}{\sqrt{5}}$

(c) $\dfrac{3}{\sqrt{5}}$

(d) $\dfrac{1}{\sqrt{5}}$

(e) $\dfrac{4}{\sqrt{5}}$

12. By reversing the order of integration, the integral

$$\int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} f(x, y)\, dxdy$$

can be written as

(a) $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{\sin x} f(x, y)\, dydx$

(b) $\displaystyle\int_0^{\frac{\pi}{2}} \int_{\arcsin x}^{1} f(x, y)\, dydx$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \int_{\sin x}^{1} f(x, y)\, dydx$

(d) $\displaystyle\int_{\arcsin y}^{\frac{\pi}{2}} \int_0^{1} f(x, y)\, dydx$

(e) $\displaystyle\int_0^{1} \int_{\cos x}^{1} f(x, y)\, dydx$

13. The volume of the solid bounded by the paraboloids

$$z = 5x^2 + 5y^2 \quad \text{and } z = 6 - x^2 - y^2$$

is

(a) 3π

(b) 4π

(c) 7π

(d) 6π

(e) π

14. Let E be the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = 0, y = x$ and $z = 0$ in the first octant. The value of $\iiint_E z\,dV$ is

(a) $\frac{1}{8}$

(b) 0

(c) $\frac{1}{4}$

(d) 2

(e) $\frac{1}{16}$

15. If D is the region in the xy-plane bounded by $y = x$, $y = -x$ and $y = \sqrt{4 - x^2}$, then $\iint_D y\, dA$ is equal to

(a) $\frac{8\sqrt{2}}{3}$

(b) $8\sqrt{2}$

(c) $\sqrt{2}$

(d) $\frac{4\sqrt{2}}{3}$

(e) $\frac{\sqrt{2}}{3}$

16. The directional derivative of the function $f(x, y, z) = x^2y^2z^6$ at the point $(1, 1, 1)$ in the direction of the vector $\langle 2, 1, -2\rangle$ is

(a) -2

(b) 0

(c) 2

(d) -6

(e) 4

17. Using spherical coordinates, the integral

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} y \; dz \, dy \, dx$$

can be rewritten as

(a) $\displaystyle\int_0^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^{3\csc\varphi} \rho^3 \sin^2\varphi \sin\theta \; d\rho \, d\varphi \, d\theta$

(b) $\displaystyle\int_0^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^{3\sec\varphi} \rho^3 \sin^2\varphi \sin\theta \; d\rho \, d\varphi \, d\theta$

(c) $\displaystyle\int_0^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^{3} \rho^2 \sin^3\varphi \sin\theta \; d\rho \, d\varphi \, d\theta$

(d) $\displaystyle\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{3} \rho^2 \sin^2\varphi \sin\theta \; d\rho \, d\varphi \, d\theta$

(e) $\displaystyle\int_0^{2\pi} \int_0^{\pi} \int_0^{3} \rho^3 \sin^2\varphi \sin\theta \; d\rho \, d\varphi \, d\theta$

18. The function $f(x, y) = 3x + 12y - x^3 - y^3$ has

(a) two saddle points

(b) exactly one saddle point

(c) two local minima

(d) two local maxima

(e) one local maximum, one local minimum, and no saddle points

19. The double integral

$$\int_0^4 \int_{1-y/4}^1 \cos(x^2)\, dx\, dy =$$

(a) $2\sin(1)$

(b) π

(c) $\sin(2)$

(d) $3\cos(1)$

(e) $2-\sin(1)$

20. The sum of the maximum and the minimum values of the function $f(x,y) = x^2 - 2x + 2y^2$ inside the region $x^2 + y^2 \leqslant 4$ is

(a) 8

(b) 0

(c) 7

(d) 12

(e) -1

21. Which of the following limits exist ?

(1) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4+y^4}{x^2+y^2}$

(2) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

(3) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+y}{\sqrt{x^2+y^2}}$

(a) (1) only

(b) (1) and (2) only

(c) (2) only

(d) (3) only

(e) (1) and (3) only

22. Which of the following lines is contained in the domain of $f(x,y) = \arcsin(y-x-2)$?

(a) $y = x + 2$

(b) $y = x - 4$

(c) $y = x + 4$

(d) $y = x$

(e) $y = -x - 2$

23. If D is the region bounded by $y = x$, $y = x^3$, $x \geqslant 0$, then $\iint_D (x^2 + 2y)\, dA$ is equal to

(a) $\frac{23}{84}$

(b) $\frac{33}{68}$

(c) $\frac{11}{84}$

(d) $\frac{3}{28}$

(e) $\frac{1}{4}$

24. Given that f is a differentiable function with $f(2,5) = 2$, $f_x(2,5) = 3$, and $f_y(2,5) = -2$, use a linear approximation to estimate $f(2.2, 4.9)$

(a) 2.8

(b) 2.5

(c) 2.6

(d) 2.7

(e) 2.9

25. If $f(x,y)$ is differentiable whose gradient at the point $P=(5,1)$ is $\nabla f(5,1)=\langle 1,-2\rangle$, then $\frac{d}{dt}|_{t=0}f(5+\frac{3}{5}t, 1+\frac{4}{5}t)$ is equal to

(a) -1

(b) $-\frac{3}{5}$

(c) $\frac{7}{10}$

(d) $-\frac{4}{5}$

(e) $\frac{11}{10}$

26. The double integral
$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} x^2(x^2+y^2)^2\,dy\,dx$$
is equal to

(a) $\frac{\pi}{16}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$

(d) $\frac{3\pi}{8}$

(e) $\frac{\pi}{4}$

27. An equation of the tangent plane to the surface $\sin(xz) - 4\cos(yz) = 4$ at the point $(\pi, \pi, 1)$ is

(a) $x + \pi z = 2\pi$

(b) $2x - \pi z = \pi$

(c) $x + 4y + \pi z = 6\pi$

(d) $x - \pi z = 0$

(e) $x + 4y - \pi z = 4\pi$

28. If $xy = ze^{xz}$, then $\dfrac{\partial z}{\partial y}(1, e, 1) - \dfrac{\partial z}{\partial x}(1, e, 1)$ is equal to

(a) $\dfrac{1}{2e}$

(b) $\dfrac{2+e}{e}$

(c) $\dfrac{e-1}{2e}$

(d) $\dfrac{e+3}{2e}$

(e) $\dfrac{2-e}{2e}$