Name:..ID:..................................... Section:.......

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Show all your work
(no mark will be given otherwise)

GOOD LUCK

1. [6 pts] If a fair yellow die and a fair red die are rolled, find the probability that the sum is greater than or equal to 9, given that a 5 shows on the yellow die.

2. [8 pts] If $P(E|F) = \frac{1}{2}$, $P(E \cup F) = \frac{9}{10}$ and $P(E \cap F) = \frac{2}{5}$, determine if E and F are dependent or independent.

3. [8 pts] A random variable X has a distribution given by $f(1) = 0.3$, $f(2) = 0.1$, $f(3) = k$. Find a numerical values for the mean and the variance.

4. [6 pts] A fair die is rolled 10 times. What is the probability that at least eight 2's appear?

5. [5 pts] If X is a normal random variable with $\mu = 16$ and $\sigma = 2$, determine the Z value that corresponds to $X = 18$.

6. [6 pts] If X is normally distributed with $\mu = 21$ and $\sigma = 5$, find $P(20 \leq X \leq 23)$.

7. [10 pts] Suppose X has a binomial distribution with $n = 100$ and $p = \frac{1}{5}$. Using the normal approximation of the binomial distribution, find $P(X \geq 21)$.

8. [10 pts] The monthly income of a group of $10,000$ employees is normally distributed with $\mu = 10,000$ SR and $\sigma = 2,000$ SR. What is the number of employees who have monthly incomes less than 5000 SR.

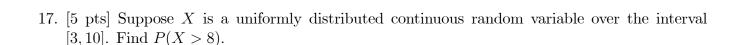9. [10 pts] Let the density of a random variable X be

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & \text{if } -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

10. Find (a) the mean μ , (c) the standard deviation σ , (d) the cumulative function $F(x)$ and (e) the probability $P(X > 2)$

11. [5 pts] An initial investment of \$4500 grows to \$5872 in four years. What was the nominal rate of interest, compounded monthly that was earned by the money?

12. [5 pts] An initial investment of \$3600 grows at an annual rate of 8.5% compounded monthly.In how many years the investment amounts to \$4700 ?

13. [5 pts] A bank pays 4.5% annual interest compounded quarterly. Find he deposit that must be made now in order that the account contains $2500 at the end of four years.

14. [8 pts] Suppose $1500 is initially placed in a savings account that earns interest at the rate of 8% compounded semiannually. Thereafter, $300 is deposited in the account at the end of every six months for four years. what will be the value of the account at the end of these four years ?

15. [5 pts] If a pair of dice are rolled, what is the probability that the sum of the number of dots appearing is 7?

16. [5 pts] A biased coin is tossed 10 times. If the probability of heads appearing on any toss is $\frac{1}{5}$, what is then the probability that exactly 4 heads appear ?

17. [5 pts] Suppose X is a uniformly distributed continuous random variable over the interval $[3, 10]$. Find $P(X > 8)$.

18. [5 pts] The life (in hours) of light bulbs of a certain brand is normally distributed with mean 1300 and standard deviation 110. Find the percentage of such bulbs which will burn more than 1350 hours.

19. [8 pts] If $P(E) = \frac{1}{3}$, $P(F) = \frac{2}{5}$ and $P(E \cup F) = \frac{8}{15}$ find $P(E|F)$.

20. [5 pts] Find the dual only of

$$\text{Maximize} \quad Z = 4x_1 + 6x_2 + 7x_3 \qquad \text{subject to} \quad \begin{cases} 3x_1 + 2x_2 + 5x_3 \leq 8 \\ x_1 + 4x_2 + 3x_3 \leq 9 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

21. [15 pts] Use the dual and the simplex method to

$$\text{Minimize} \quad Z = x_1 + 7x_2 \qquad \text{subject to} \quad \begin{cases} 3x_1 + 5x_2 \geq 14 \\ 2x_1 + 14x_2 \geq 16 \\ 5x_1 + 13x_2 \geq 3 \\ x_1, x_2 \geq 0 \end{cases}$$