King Fahd University of Petroleum and Minerals - Department of Mathematics and Statistics

MATH101 - QUIZ 2

Term 181

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Give all details

1. Let $g(x) = \frac{1+xf(x)}{\sqrt{x}}$. If $f(1) = 3$ and $f'(1) = 2$ then $g'(1) = 3$.

Solution

We have

$$g'(x) = \frac{(1+xf(x))' \times \sqrt{x} - (1+xf(x)) \times \sqrt{x}'}{\sqrt{x}^2}$$
$$= \frac{\Big(f(x) + xf'(x)\Big) \times \sqrt{x} - (1+xf(x)) \times \frac{1}{2\sqrt{x}}}{x}$$

Then

$$g'(1) = \frac{\Big(f(1) + f'(1)\Big) \times \sqrt{1} - \frac{(1+f(1))}{2\sqrt{1}}}{1} = f(1)+f'(1) - \frac{(1+f(1))}{2} = 3+2-\frac{1+3}{2} = 3.$$

2. Equations of the tangent lines to the curve $g(x) = \frac{1-x}{1+x}$ that are parallel to the line $x+2y=2$ are $y = \frac{1-x}{2}$ and $y = \frac{-7-x}{2}$

Solution

The slope of the tangent line at $(a, g(a))$, which is given by $g'(a)$, is equal to $-1/2$ car the tangent line is parallel to the line $x + 2y = 2 \Leftrightarrow y = 1 - \frac{x}{2}$. We start with

$$g'(x) = \frac{(1-x)' \times (1+x) - (1-x) \times (1+x)'}{(1+x)^2} = \frac{-(1+x)-(1-x)}{(1+x)^2} = \frac{-2}{(1+x)^2}.$$

Then

$$g'(x) = -\frac{1}{2} \Rightarrow \frac{-2}{(1+x)^2} = -\frac{1}{2} \Rightarrow (1+x)^2 = 4 \Rightarrow 1+x = \pm 2 \Rightarrow x = 1 \text{ and } x = -3.$$

To find the equations, we need to calculate $g(1) = \frac{1-1}{1+1} = 0$ and $g(-3) = \frac{1-(-3)}{-3+1} = -2$. We know that the equation of the tangent line at $(a, g(a))$ is given $y = g'(a)(x-a) + g(a)$. Then, the equations are

$$y = -\frac{1}{2}(x-1) = \frac{1-x}{2}$$

and

$$y = -\frac{1}{2}(x+3) - 2 = \frac{-x-7}{2}.$$

3. The graph of $f(x) = \frac{\sec x}{1+\tan x}$ has a horizontal tangent line at the point $\frac{\pi}{4}$.

Solution

We have

$$\begin{aligned} f'(x) =& \frac{\sec' x \times (1+\tan x) - \sec x \times (1+\tan x)'}{(1+\tan x)^2} \\ =& \frac{\sec x \tan x(1+\tan x) - \sec x(1+\tan^2 x)}{(1+\tan x)^2} \\ =& \frac{\sec x \tan x + \sec x \tan^2 x - \sec x - \sec x \tan^2 x}{(1+\tan x)^2} \\ =& \frac{\sec x(\tan x - 1)}{(1+\tan x)^2} \end{aligned}$$

Then, $f'(x) = 0$ is equivalent to $\tan x - 1 = 0$, which has $\pi/4$ as a solution.