SHOW ALL YOUR WORK. NO CREDITS FOR ANSWERES WITHOUT JUSTIFICATIONS

(1) (5+7 points) Evaluate the limit if it exists:

a. $\lim\limits_{x \to 0} \dfrac{e^x - 1}{\tan x}$

b. $\lim\limits_{x \to \infty} (1 + e^{-x})^x$

(2) (8 points) A square box with open top is to have a volume of 4000cm³. Find the dimensions of the box that minimize the amount of material used.

(20 points) Given the function

$$y = f(x) = \dfrac{x-4}{x^2} \quad \text{with} \quad f'(x) = \dfrac{8-x}{x^3} \quad \text{and} \quad f''(x) = \dfrac{2(x-12)}{x^4}.$$

a. Find the asymptotes if any exist, and if it does not exist say so.

Horizontal:

Vertical:

Slant:

b. Find the critical numbers.

c. Find intervals where the function is increasing and those where it is decreasing.

d. (Find the local maximum and minimum of the function.

e. Find the absolute maximum and absolute minimum values of the function if any exists.

f. Discuss the concavity of the function and find the infection points.

g. Sketch the graph of the function. Clearly indicate the **critical numbers**, **extrema** and **inflection points** on the graph.