King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Final Exam
181
Saturday 22/12/2018

EXAM COVER

Number of versions: 4
Number of questions: 28
Number of Answers: 5

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101

Final Exam

181

Saturday 22/12/2018

Net Time Allowed: 180 minutes

MASTER VERSION

1. If $[\![x]\!]$ represents the largest integer that is less than or equal to x, then $\lim_{x\to 2}([\![x]\!] + [\![2-x]\!]) =$

(a) 1

(b) 2

(c) 0

(d) -1

(e) does not exist

2. If $f(x) = e^{\sin^2(\pi x)}$, then $f'(x) =$

(a) $\pi\, e^{\sin^2(\pi x)}\, \sin(2\pi x)$

(b) $\pi\, e^{\sin^2(\pi x)}\, \cos(2\pi x)$

(c) $e^{\sin^2(\pi x)}\, \cos^2(\pi x)$

(d) $\pi\, e^{\sin^2(\pi x)}$

(e) $e^{\sin^2(\pi x)}\, \sin(\pi x)$

3. If $y = x \cosh(x^2)$, then $y'(1) =$

(a) $\dfrac{3e}{2} - \dfrac{1}{2e}$

(b) $\dfrac{e}{2} - \dfrac{3}{2e}$

(c) $\dfrac{3e}{2} + \dfrac{1}{2e}$

(d) $\dfrac{e}{2} + \dfrac{3}{2e}$

(e) $\dfrac{3e}{2} + \dfrac{3}{2e}$

4. The sum of the absolute maximum and the absolute minimum values of the function $f(x) = x^4 - 2x^2 + 2$ in the interval $[-1, 2]$ is

(a) 11

(b) 13

(c) 15

(d) 9

(e) 7

5. Consider the function $f(x) = x + e^x$. The value of c that satisfies the conclusion of the **Mean Value Theorem** on the interval $[0, 1]$ is

(a) $\ln(e - 1)$

(b) $\ln(1 - e)$

(c) $1 - e$

(d) $e - 1$

(e) 0

6. If $f(x) = \ln(4 - x^2)$, then the graph of f is increasing on

(a) $(-2, 0)$

(b) $(-\infty, -2)$

(c) $(-2, 2)$

(d) $(-\infty, 0]$

(e) $[0, 2)$

7. By using the linear approximation of $f(x) = \sqrt{100 + x}$ at $a = 0$, $\sqrt{100.5}$ is approximately equal to

(a) $\frac{401}{40}$

(b) $\frac{3}{20}$

(c) $\frac{77}{40}$

(d) $\frac{71}{20}$

(e) $\frac{399}{40}$

8. The sum of all critical numbers of the function $f(x) = \frac{x^2 + 14}{\sqrt{4x + 1}}$ is

(a) 2

(b) $\frac{-7}{3}$

(c) $\frac{2}{3}$

(d) $\frac{-1}{12}$

(e) $\frac{-2}{3}$

9. $\lim_{x\to\infty}\left(\frac{x+2}{x-1}\right)^x =$

(a) e^3

(b) e^2

(c) e

(d) e^{-2}

(e) e^{-4}

10. If $1200\,cm^2$ of material is available to make a box with a square base and an open top, then the largest possible volume of the box is

(a) $4000\,cm^3$

(b) $2000\,cm^3$

(c) $1000\,cm^3$

(d) $500\,cm^3$

(e) $200\,cm^3$

11. The slope of the line tangent to the curve $2x + \tan^{-1}(xy) = x^2 y^3 + 2$ at the point $(1, 0)$ is

(a) -2

(b) 2

(c) 1

(d) -1

(e) 0

12. If $f'(x) = \dfrac{3x^3 - \sqrt{x} + 1}{x}$ and $f(1) = 0$, then $f(3) =$

(a) $28 - 2\sqrt{3} + \ln 3$

(b) $26 - 2\sqrt{3} + \ln 3$

(c) $9 - 2\sqrt{3}$

(d) $10 - 2\sqrt{3} - \dfrac{1}{9}$

(e) $8 - 2\sqrt{3} - \dfrac{1}{9}$

13. $\lim_{x\to 0} \dfrac{\cos(2x) - 1}{x^{10}} =$

(a) $-\infty$

(b) $\dfrac{-2^{10}}{10!}$

(c) $\dfrac{2^{10}}{10}$

(d) ∞

(e) 0

14. Let $f(x) = e^x\, g(x)$, with $g(1) = 1$, $g'(1) = 2$, and $g''(1) = 3$. Then $f''(1) =$

(a) $8\,e$

(b) e

(c) $3\,e$

(d) $e + 1$

(e) $e + 3$

15. The equation of the horizontal asymptote of the graph of $f(x) = \ln(1 + x^2) - \ln(1 + 2x^2)$ is

(a) $y = -\ln 2$

(b) $y = 0$

(c) $y = 2$

(d) $y = 1$

(e) $y = 2\ln 2$

16. If $f(x) = \ln\left(\frac{1+x}{1+\sqrt{x}}\right)$, then $f'(1) =$

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) 1

(e) 0

17. The graph of $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is concave up on

(a) $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

(b) $\left(0, \frac{3\pi}{4}\right)$

(c) $\left(\frac{7\pi}{4}, 2\pi\right)$

(d) $\left(0, \frac{\pi}{4}\right)$

(e) $\left(\frac{5\pi}{4}, 2\pi\right)$

18. If a and b are the values that make $f(x) = \begin{cases} ax^3 & x \leq 2 \\ x^2 + b & x > 2 \end{cases}$ differentiable, then $a + b =$

(a) -1

(b) 1

(c) $\frac{5}{3}$

(d) $\frac{1}{3}$

(e) 0

19. If $f'(x) = e^{(x-4)^3}\,(x^2+x+2)^3(x-2)^3(x-1)$, then f is decreasing on

(a) $(1,2)$

(b) $(1,4)$

(c) $(2,4)$

(d) $(4,\infty)$

(e) $(-\infty,2)$

20. The number of vertical asymptotes of the function $y = \dfrac{x^2-4}{(x^2+2x-8)(x^2+x+1)}$ is

(a) 1

(b) 2

(c) 3

(d) 4

(e) 0

21. If $f(x) = \dfrac{(x+1)^3(x-2)\cos x}{3^x(2x+1)}$, then $f'(0) =$

(a) $-1 + 2\ln 3$

(b) 0

(c) $1 + 3\ln 3$

(d) $1 - 3\ln 3$

(e) -2

22. Two cars leave an intersection. One car travels north at $30\,km/h$ and the other travels east at $40\,km/h$. How fast is the distance between them increasing at the end of $30\,mins$?

(a) $50\,km/h$

(b) $100\,km/h$

(c) $60\,km/h$

(d) $25\,km/h$

(e) $40\,km/h$

23. The number of inflection points of $f(x) = x\sqrt{6-x}$ is

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

24. Newton's Method is used to estimate the critical number of the function $g(x) = x^6 + 15x^2 + 30x + 90$. If we start with $x_1 = 0$, then $x_3 =$

(a) -0.9

(b) -0.7

(c) -0.5

(d) -1.1

(e) -1.3

25. If $\sinh(x\,f(x)) + \sinh(x) = \dfrac{3}{4}$ and $f(\ln(2)) = 0$ then $(\ln 2)\,f'(\ln 2) =$

(a) $-\dfrac{5}{4}$

(b) $-\dfrac{3}{4}$

(c) $-\dfrac{1}{4}$

(d) $-\dfrac{7}{4}$

(e) $-\dfrac{9}{4}$

26. Which one of the following statements is **TRUE**?

(a) If f' exists and is non-zero for all x, then $f(1) \neq f(0)$.

(b) $\lim\limits_{x\to 0} \dfrac{x}{e^x} = 1$

(c) If f and g are increasing on an interval I, then $f - g$ is increasing on I.

(d) If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

(e) If f has an absolute minimum at c, then $f'(c) = 0$

27. Let $a > 0$ and let $f(x) = x^2 + \frac{x}{a}$, $a \leq x \leq 2a$. The value of a, that makes the average rate of change of the function f on $[a, 2a]$ the smallest possible, is

(a) $\frac{1}{\sqrt{3}}$

(b) 1

(c) 2

(d) $\sqrt{2}$

(e) $\frac{1}{\sqrt{5}}$

28. If the volume of an expanding cube is increasing at the rate of $4\, m^3/min$, how fast is its surface area increasing when the surface area is $24\, m^2$?

(a) $8\, m^2/min$

(b) $4\, m^2/min$

(c) $3\, m^2/min$

(d) $2\, m^2/min$

(e) $5\, m^2/min$

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 001 **CODE 001**

Math 101
Final Exam
181
Saturday 22/12/2018
Net Time Allowed: 180 minutes

Name: ____

ID: ______________________ Sec: ______________________.

Check that this exam has 28 questions.

Important Instructions:

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3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The sum of the absolute maximum and the absolute minimum values of the function $f(x) = x^4 - 2x^2 + 2$ in the interval $[-1, 2]$ is

(a) 7

(b) 15

(c) 9

(d) 13

(e) 11

2. If $f(x) = e^{\sin^2(\pi x)}$, then $f'(x) =$

(a) $e^{\sin^2(\pi x)} \sin(\pi x)$

(b) $e^{\sin^2(\pi x)} \cos^2(\pi x)$

(c) $\pi e^{\sin^2(\pi x)} \cos(2\pi x)$

(d) $\pi e^{\sin^2(\pi x)} \sin(2\pi x)$

(e) $\pi e^{\sin^2(\pi x)}$

3. If $f(x) = \ln(4 - x^2)$, then the graph of f is increasing on

(a) $(-2, 2)$

(b) $(-2, 0)$

(c) $[0, 2)$

(d) $(-\infty, 0]$

(e) $(-\infty, -2)$

4. Consider the function $f(x) = x + e^x$. The value of c that satisfies the conclusion of the **Mean Value Theorem** on the interval $[0, 1]$ is

(a) $\ln(e - 1)$

(b) $e - 1$

(c) $\ln(1 - e)$

(d) 0

(e) $1 - e$

5. By using the linear approximation of $f(x) = \sqrt{100 + x}$ at $a = 0$, $\sqrt{100.5}$ is approximately equal to

(a) $\frac{71}{20}$

(b) $\frac{77}{40}$

(c) $\frac{3}{20}$

(d) $\frac{401}{40}$

(e) $\frac{399}{40}$

6. If $y = x \cosh(x^2)$, then $y'(1) =$

(a) $\frac{e}{2} - \frac{3}{2e}$

(b) $\frac{3e}{2} + \frac{1}{2e}$

(c) $\frac{e}{2} + \frac{3}{2e}$

(d) $\frac{3e}{2} + \frac{3}{2e}$

(e) $\frac{3e}{2} - \frac{1}{2e}$

7. If $[\![x]\!]$ represents the largest integer that is less than or equal to x, then $\lim_{x\to 2}([\![x]\!] + [\![2-x]\!]) =$

(a) 1

(b) 2

(c) -1

(d) 0

(e) does not exist

8. If $f'(x) = \dfrac{3x^3 - \sqrt{x} + 1}{x}$ and $f(1) = 0$, then $f(3) =$

(a) $28 - 2\sqrt{3} + \ln 3$

(b) $10 - 2\sqrt{3} - \dfrac{1}{9}$

(c) $26 - 2\sqrt{3} + \ln 3$

(d) $8 - 2\sqrt{3} - \dfrac{1}{9}$

(e) $9 - 2\sqrt{3}$

9. The slope of the line tangent to the curve $2x + \tan^{-1}(xy) = x^2 y^3 + 2$ at the point $(1, 0)$ is

(a) 2

(b) -2

(c) 0

(d) 1

(e) -1

10. If $1200\, cm^2$ of material is available to make a box with a square base and an open top, then the largest possible volume of the box is

(a) $4000\, cm^3$

(b) $2000\, cm^3$

(c) $1000\, cm^3$

(d) $500\, cm^3$

(e) $200\, cm^3$

11. Let $f(x) = e^x\, g(x)$, with $g(1) = 1$, $g'(1) = 2$, and $g''(1) = 3$. Then $f''(1) =$

(a) $e + 3$

(b) $8\,e$

(c) $e + 1$

(d) $3\,e$

(e) e

12. $\displaystyle\lim_{x\to\infty} \left(\frac{x+2}{x-1}\right)^x =$

(a) e^{-2}

(b) e^{-4}

(c) e

(d) e^2

(e) e^3

13. $\lim_{x\to 0} \dfrac{\cos(2x)-1}{x^{10}} =$

(a) $\dfrac{2^{10}}{10}$

(b) $\dfrac{-2^{10}}{10!}$

(c) ∞

(d) $-\infty$

(e) 0

14. The sum of all critical numbers of the function $f(x) = \dfrac{x^2+14}{\sqrt{4x+1}}$ is

(a) 2

(b) $\dfrac{-2}{3}$

(c) $\dfrac{-7}{3}$

(d) $\dfrac{2}{3}$

(e) $\dfrac{-1}{12}$

15. The equation of the horizontal asymptote of the graph of $f(x) = \ln(1 + x^2) - \ln(1 + 2x^2)$ is

(a) $y = -\ln 2$

(b) $y = 2\ln 2$

(c) $y = 2$

(d) $y = 1$

(e) $y = 0$

16. The graph of $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is concave up on

(a) $\left(0, \frac{3\pi}{4}\right)$

(b) $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

(c) $\left(\frac{7\pi}{4}, 2\pi\right)$

(d) $\left(0, \frac{\pi}{4}\right)$

(e) $\left(\frac{5\pi}{4}, 2\pi\right)$

17. If a and b are the values that make $f(x) = \begin{cases} ax^3 & x \leq 2 \\ x^2 + b & x > 2 \end{cases}$ differentiable, then $a + b =$

(a) 1

(b) $\frac{1}{3}$

(c) -1

(d) $\frac{5}{3}$

(e) 0

18. If $f(x) = \ln\left(\frac{1+x}{1+\sqrt{x}}\right)$, then $f'(1) =$

(a) $\frac{1}{3}$

(b) 0

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

(e) 1

19. If $f'(x) = e^{(x-4)^3}\,(x^2+x+2)^3(x-2)^3(x-1)$, then f is decreasing on

(a) $(1, 4)$

(b) $(4, \infty)$

(c) $(1, 2)$

(d) $(2, 4)$

(e) $(-\infty, 2)$

20. If $f(x) = \dfrac{(x+1)^3(x-2)\ \cos\, x}{3^x(2x+1)}$, then $f'(0) =$

(a) $-1 + 2\ \ln\, 3$

(b) $1 + 3\ \ln\, 3$

(c) $1 - 3\ \ln\, 3$

(d) -2

(e) 0

21. The number of vertical asymptotes of the function $y = \dfrac{x^2 - 4}{(x^2 + 2x - 8)(x^2 + x + 1)}$ is

(a) 4

(b) 0

(c) 3

(d) 1

(e) 2

22. Two cars leave an intersection. One car travels north at $30\,km/h$ and the other travels east at $40\,km/h$. How fast is the distance between them increasing at the end of $30\,mins$?

(a) $50\,km/h$

(b) $40\,km/h$

(c) $60\,km/h$

(d) $25\,km/h$

(e) $100\,km/h$

23. The number of inflection points of $f(x) = x\sqrt{6-x}$ is

(a) 2

(b) 0

(c) 4

(d) 3

(e) 1

24. If the volume of an expanding cube is increasing at the rate of $4\,m^3/min$, how fast is its surface area increasing when the surface area is $24\,m^2$?

(a) $8\,m^2/min$

(b) $4\,m^2/min$

(c) $5\,m^2/min$

(d) $3\,m^2/min$

(e) $2\,m^2/min$

25. If $\sinh(x\,f(x)) + \sinh(x) = \frac{3}{4}$ and $f(\ln(2)) = 0$ then $(\ln 2)\, f'(\ln 2) =$

(a) $-\frac{3}{4}$

(b) $-\frac{5}{4}$

(c) $-\frac{7}{4}$

(d) $-\frac{1}{4}$

(e) $-\frac{9}{4}$

26. Newton's Method is used to estimate the critical number of the function $g(x) = x^6 + 15x^2 + 30x + 90$. If we start with $x_1 = 0$, then $x_3 =$

(a) -0.9

(b) -1.1

(c) -1.3

(d) -0.7

(e) -0.5

27. Which one of the following statements is **TRUE**?

(a) If f has an absolute minimum at c, then $f'(c) = 0$

(b) If f and g are increasing on an interval I, then $f - g$ is increasing on I.

(c) If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

(d) If f' exists and is non-zero for all x, then $f(1) \neq f(0)$.

(e) $\lim_{x \to 0} \frac{x}{e^x} = 1$

28. Let $a > 0$ and let $f(x) = x^2 + \frac{x}{a}$, $a \leq x \leq 2a$. The value of a, that makes the average rate of change of the function f on $[a, 2a]$ the smallest possible, is

(a) $\frac{1}{\sqrt{5}}$

(b) 1

(c) $\sqrt{2}$

(d) $\frac{1}{\sqrt{3}}$

(e) 2

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 002 **CODE 002**

Math 101
Final Exam
181
Saturday 22/12/2018
Net Time Allowed: 180 minutes

Name: ______________________________

ID: ______________ Sec: ______________.

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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $y = x \cosh(x^2)$, then $y'(1) =$

(a) $\dfrac{3e}{2} + \dfrac{1}{2e}$

(b) $\dfrac{e}{2} - \dfrac{3}{2e}$

(c) $\dfrac{3e}{2} - \dfrac{1}{2e}$

(d) $\dfrac{3e}{2} + \dfrac{3}{2e}$

(e) $\dfrac{e}{2} + \dfrac{3}{2e}$

2. If $f(x) = \ln(4 - x^2)$, then the graph of f is increasing on

(a) $(-2, 2)$

(b) $(-2, 0)$

(c) $(-\infty, 0]$

(d) $[0, 2)$

(e) $(-\infty, -2)$

3. By using the linear approximation of $f(x) = \sqrt{100 + x}$ at $a = 0$, $\sqrt{100.5}$ is approximately equal to

(a) $\frac{77}{40}$

(b) $\frac{399}{40}$

(c) $\frac{3}{20}$

(d) $\frac{71}{20}$

(e) $\frac{401}{40}$

4. If $f(x) = e^{\sin^2(\pi x)}$, then $f'(x) =$

(a) $\pi\, e^{\sin^2(\pi x)} \cos(2\pi x)$

(b) $e^{\sin^2(\pi x)} \sin(\pi x)$

(c) $\pi\, e^{\sin^2(\pi x)} \sin(2\pi x)$

(d) $e^{\sin^2(\pi x)} \cos^2(\pi x)$

(e) $\pi\, e^{\sin^2(\pi x)}$

5. The sum of the absolute maximum and the absolute minimum values of the function $f(x) = x^4 - 2x^2 + 2$ in the interval $[-1, 2]$ is

(a) 7

(b) 15

(c) 13

(d) 9

(e) 11

6. If $[\![x]\!]$ represents the largest integer that is less than or equal to x, then $\lim\limits_{x \to 2}([\![x]\!] + [\![2 - x]\!]) =$

(a) 2

(b) 1

(c) -1

(d) 0

(e) does not exist

7. Consider the function $f(x) = x + e^x$. The value of c that satisfies the conclusion of the **Mean Value Theorem** on the interval $[0, 1]$ is

(a) $1 - e$

(b) $\ln(e - 1)$

(c) 0

(d) $\ln(1 - e)$

(e) $e - 1$

8. $\lim_{x\to\infty} \left(\frac{x+2}{x-1}\right)^x =$

(a) e

(b) e^2

(c) e^{-4}

(d) e^3

(e) e^{-2}

9. The slope of the line tangent to the curve $2x + \tan^{-1}(xy) = x^2\, y^3 + 2$ at the point $(1, 0)$ is

(a) 2

(b) -1

(c) -2

(d) 0

(e) 1

10. If $f'(x) = \dfrac{3x^3 - \sqrt{x} + 1}{x}$ and $f(1) = 0$, then $f(3) =$

(a) $10 - 2\sqrt{3} - \dfrac{1}{9}$

(b) $26 - 2\sqrt{3} + \ln 3$

(c) $8 - 2\sqrt{3} - \dfrac{1}{9}$

(d) $9 - 2\sqrt{3}$

(e) $28 - 2\sqrt{3} + \ln 3$

11. The sum of all critical numbers of the function $f(x) = \dfrac{x^2 + 14}{\sqrt{4x+1}}$ is

(a) 2

(b) $\dfrac{-1}{12}$

(c) $\dfrac{-7}{3}$

(d) $\dfrac{-2}{3}$

(e) $\dfrac{2}{3}$

12. $\displaystyle\lim_{x\to 0} \frac{\cos(2x) - 1}{x^{10}} =$

(a) $-\infty$

(b) ∞

(c) 0

(d) $\dfrac{2^{10}}{10}$

(e) $\dfrac{-2^{10}}{10!}$

13. If $1200\, cm^2$ of material is available to make a box with a square base and an open top, then the largest possible volume of the box is

(a) $2000\, cm^3$

(b) $500\, cm^3$

(c) $200\, cm^3$

(d) $1000\, cm^3$

(e) $4000\, cm^3$

14. Let $f(x) = e^x\, g(x)$, with $g(1) = 1$, $g'(1) = 2$, and $g''(1) = 3$. Then $f''(1) =$

(a) e

(b) $8\, e$

(c) $3\, e$

(d) $e + 1$

(e) $e + 3$

15. If $f(x) = \dfrac{(x+1)^3(x-2)\cos x}{3^x(2x+1)}$, then $f'(0) =$

(a) $1 - 3\ln 3$

(b) $1 + 3\ln 3$

(c) $-1 + 2\ln 3$

(d) 0

(e) -2

16. If a and b are the values that make $f(x) = \begin{cases} ax^3 & x \le 2 \\ x^2 + b & x > 2 \end{cases}$ differentiable, then $a + b =$

(a) 0

(b) -1

(c) 1

(d) $\dfrac{5}{3}$

(e) $\dfrac{1}{3}$

17. The equation of the horizontal asymptote of the graph of $f(x) = \ln(1 + x^2) - \ln(1 + 2x^2)$ is

(a) $y = 1$

(b) $y = 2\ln 2$

(c) $y = -\ln 2$

(d) $y = 2$

(e) $y = 0$

18. If $f'(x) = e^{(x-4)^3}(x^2 + x + 2)^3(x - 2)^3(x - 1)$, then f is decreasing on

(a) $(-\infty, 2)$

(b) $(4, \infty)$

(c) $(2, 4)$

(d) $(1, 4)$

(e) $(1, 2)$

19. The graph of $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is concave up on

(a) $\left(0, \frac{\pi}{4}\right)$

(b) $\left(\frac{5\pi}{4}, 2\pi\right)$

(c) $\left(0, \frac{3\pi}{4}\right)$

(d) $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

(e) $\left(\frac{7\pi}{4}, 2\pi\right)$

20. The number of vertical asymptotes of the function $y = \dfrac{x^2 - 4}{(x^2 + 2x - 8)(x^2 + x + 1)}$ is

(a) 2

(b) 0

(c) 4

(d) 1

(e) 3

21. If $f(x) = \ln\left(\dfrac{1+x}{1+\sqrt{x}}\right)$, then $f'(1) =$

(a) 1

(b) 0

(c) $\dfrac{1}{4}$

(d) $\dfrac{1}{3}$

(e) $\dfrac{1}{2}$

22. Which one of the following statements is **TRUE**?

(a) If f has an absolute minimum at c, then $f'(c) = 0$

(b) If f' exists and is non-zero for all x, then $f(1) \neq f(0)$.

(c) $\lim\limits_{x \to 0} \dfrac{x}{e^x} = 1$

(d) If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

(e) If f and g are increasing on an interval I, then $f - g$ is increasing on I.

23. If the volume of an expanding cube is increasing at the rate of $4\,m^3/min$, how fast is its surface area increasing when the surface area is $24\,m^2$?

(a) $2\,m^2/min$

(b) $5\,m^2/min$

(c) $8\,m^2/min$

(d) $3\,m^2/min$

(e) $4\,m^2/min$

24. If $\sinh(x\,f(x)) + \sinh(x) = \frac{3}{4}$ and $f(\ln(2)) = 0$ then $(\ln 2)\,f'(\ln 2) =$

(a) $-\frac{7}{4}$

(b) $-\frac{3}{4}$

(c) $-\frac{9}{4}$

(d) $-\frac{5}{4}$

(e) $-\frac{1}{4}$

25. Newton's Method is used to estimate the critical number of the function $g(x) = x^6 + 15x^2 + 30x + 90$. If we start with $x_1 = 0$, then $x_3 =$

(a) -1.1

(b) -1.3

(c) -0.5

(d) -0.9

(e) -0.7

26. Let $a > 0$ and let $f(x) = x^2 + \frac{x}{a}$, $a \leq x \leq 2a$. The value of a, that makes the average rate of change of the function f on $[a, 2a]$ the smallest possible, is

(a) $\sqrt{2}$

(b) 1

(c) 2

(d) $\frac{1}{\sqrt{5}}$

(e) $\frac{1}{\sqrt{3}}$

27. Two cars leave an intersection. One car travels north at $30\,km/h$ and the other travels east at $40\,km/h$. How fast is the distance between them increasing at the end of $30\,mins$?

(a) $40\,km/h$

(b) $100\,km/h$

(c) $50\,km/h$

(d) $25\,km/h$

(e) $60\,km/h$

28. The number of inflection points of $f(x) = x\sqrt{6-x}$ is

(a) 0

(b) 2

(c) 3

(d) 1

(e) 4

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 003 **CODE 003**

Math 101
Final Exam
181
Saturday 22/12/2018
Net Time Allowed: 180 minutes

Name: ____

ID: ______________________ Sec: ______________________.

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Consider the function $f(x) = x + e^x$. The value of c that satisfies the conclusion of the **Mean Value Theorem** on the interval $[0, 1]$ is

(a) $1 - e$

(b) $e - 1$

(c) $\ln(1 - e)$

(d) 0

(e) $\ln(e - 1)$

2. If $[\![x]\!]$ represents the largest integer that is less than or equal to x, then $\lim\limits_{x \to 2}([\![x]\!] + [\![2 - x]\!]) =$

(a) 0

(b) 1

(c) 2

(d) does not exist

(e) -1

3. If $y = x\cosh(x^2)$, then $y'(1) =$

(a) $\frac{3e}{2} - \frac{1}{2e}$

(b) $\frac{e}{2} + \frac{3}{2e}$

(c) $\frac{3e}{2} + \frac{1}{2e}$

(d) $\frac{e}{2} - \frac{3}{2e}$

(e) $\frac{3e}{2} + \frac{3}{2e}$

4. By using the linear approximation of $f(x) = \sqrt{100 + x}$ at $a = 0$, $\sqrt{100.5}$ is approximately equal to

(a) $\frac{399}{40}$

(b) $\frac{77}{40}$

(c) $\frac{401}{40}$

(d) $\frac{71}{20}$

(e) $\frac{3}{20}$

5. If $f(x) = \ln(4 - x^2)$, then the graph of f is increasing on

(a) $[0, 2)$

(b) $(-\infty, -2)$

(c) $(-2, 2)$

(d) $(-\infty, 0]$

(e) $(-2, 0)$

6. The sum of the absolute maximum and the absolute minimum values of the function $f(x) = x^4 - 2x^2 + 2$ in the interval $[-1, 2]$ is

(a) 15

(b) 7

(c) 11

(d) 13

(e) 9

7. If $f(x) = e^{\sin^2(\pi x)}$, then $f'(x) =$

(a) $e^{\sin^2(\pi x)} \cos^2(\pi x)$

(b) $\pi e^{\sin^2(\pi x)} \sin(2\pi x)$

(c) $\pi e^{\sin^2(\pi x)} \cos(2\pi x)$

(d) $\pi e^{\sin^2(\pi x)}$

(e) $e^{\sin^2(\pi x)} \sin(\pi x)$

8. The sum of all critical numbers of the function $f(x) = \dfrac{x^2 + 14}{\sqrt{4x+1}}$ is

(a) $\dfrac{-1}{12}$

(b) $\dfrac{2}{3}$

(c) $\dfrac{-2}{3}$

(d) 2

(e) $\dfrac{-7}{3}$

9. Let $f(x) = e^x\, g(x)$, with $g(1) = 1$, $g'(1) = 2$, and $g''(1) = 3$. Then $f''(1) =$

(a) e

(b) $3\,e$

(c) $e + 1$

(d) $e + 3$

(e) $8\,e$

10. If $f'(x) = \dfrac{3x^3 - \sqrt{x} + 1}{x}$ and $f(1) = 0$, then $f(3) =$

(a) $8 - 2\sqrt{3} - \dfrac{1}{9}$

(b) $9 - 2\sqrt{3}$

(c) $26 - 2\sqrt{3} + \ln 3$

(d) $28 - 2\sqrt{3} + \ln 3$

(e) $10 - 2\sqrt{3} - \dfrac{1}{9}$

11. If $1200\,cm^2$ of material is available to make a box with a square base and an open top, then the largest possible volume of the box is

(a) $4000\,cm^3$

(b) $500\,cm^3$

(c) $200\,cm^3$

(d) $2000\,cm^3$

(e) $1000\,cm^3$

12. The slope of the line tangent to the curve $2x + \tan^{-1}(xy) = x^2\,y^3 + 2$ at the point $(1, 0)$ is

(a) 2

(b) -2

(c) 1

(d) -1

(e) 0

13. $\lim_{x\to 0} \frac{\cos(2x) - 1}{x^{10}} =$

(a) 0

(b) $\frac{2^{10}}{10}$

(c) ∞

(d) $\frac{-2^{10}}{10!}$

(e) $-\infty$

14. $\lim_{x\to \infty} \left(\frac{x+2}{x-1}\right)^x =$

(a) e^2

(b) e^3

(c) e^{-4}

(d) e

(e) e^{-2}

15. The equation of the horizontal asymptote of the graph of $f(x) = \ln(1+x^2) - \ln(1+2x^2)$ is

(a) $y = -\ln 2$

(b) $y = 2$

(c) $y = 1$

(d) $y = 2\ln 2$

(e) $y = 0$

16. If a and b are the values that make $f(x) = \begin{cases} ax^3 & x \leq 2 \\ x^2 + b & x > 2 \end{cases}$ differentiable, then $a + b =$

(a) -1

(b) 1

(c) $\frac{5}{3}$

(d) 0

(e) $\frac{1}{3}$

17. If $f(x) = \dfrac{(x+1)^3(x-2)\cos x}{3^x(2x+1)}$, then $f'(0) =$

(a) 0

(b) $-1 + 2\ln 3$

(c) -2

(d) $1 - 3\ln 3$

(e) $1 + 3\ln 3$

18. The number of vertical asymptotes of the function $y = \dfrac{x^2-4}{(x^2+2x-8)(x^2+x+1)}$ is

(a) 3

(b) 1

(c) 4

(d) 2

(e) 0

19. If $f'(x) = e^{(x-4)^3}\,(x^2+x+2)^3(x-2)^3(x-1)$, then f is decreasing on

(a) $(1, 4)$

(b) $(4, \infty)$

(c) $(2, 4)$

(d) $(-\infty, 2)$

(e) $(1, 2)$

20. If $f(x) = \ln\left(\dfrac{1+x}{1+\sqrt{x}}\right)$, then $f'(1) =$

(a) 1

(b) 0

(c) $\dfrac{1}{4}$

(d) $\dfrac{1}{2}$

(e) $\dfrac{1}{3}$

21. The graph of $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is concave up on

(a) $\left(\frac{5\pi}{4}, 2\pi\right)$

(b) $\left(0, \frac{3\pi}{4}\right)$

(c) $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

(d) $\left(0, \frac{\pi}{4}\right)$

(e) $\left(\frac{7\pi}{4}, 2\pi\right)$

22. Let $a > 0$ and let $f(x) = x^2 + \frac{x}{a}$, $a \le x \le 2a$. The value of a, that makes the average rate of change of the function f on $[a, 2a]$ the smallest possible, is

(a) $\frac{1}{\sqrt{5}}$

(b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{2}$

(d) 1

(e) 2

23. Newton's Method is used to estimate the critical number of the function $g(x) = x^6 + 15x^2 + 30x + 90$. If we start with $x_1 = 0$, then $x_3 =$

(a) -0.7

(b) -1.3

(c) -0.9

(d) -0.5

(e) -1.1

24. The number of inflection points of $f(x) = x\sqrt{6-x}$ is

(a) 0

(b) 2

(c) 3

(d) 4

(e) 1

25. Which one of the following statements is **TRUE**?

(a) If f has an absolute minimum at c, then $f'(c) = 0$

(b) If f' exists and is non-zero for all x, then $f(1) \neq f(0)$.

(c) If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

(d) $\lim_{x \to 0} \frac{x}{e^x} = 1$

(e) If f and g are increasing on an interval I, then $f - g$ is increasing on I.

26. Two cars leave an intersection. One car travels north at $30\, km/h$ and the other travels east at $40\, km/h$. How fast is the distance between them increasing at the end of $30\, mins$?

(a) $100\, km/h$

(b) $50\, km/h$

(c) $60\, km/h$

(d) $25\, km/h$

(e) $40\, km/h$

27. If the volume of an expanding cube is increasing at the rate of $4\, m^3/min$, how fast is its surface area increasing when the surface area is $24\, m^2$?

(a) $3\, m^2/min$

(b) $4\, m^2/min$

(c) $2\, m^2/min$

(d) $8\, m^2/min$

(e) $5\, m^2/min$

28. If $\sinh(x\, f(x)) + \sinh(x) = \frac{3}{4}$ and $f(\ln(2)) = 0$ then $(\ln 2)\, f'(\ln 2) =$

(a) $-\frac{7}{4}$

(b) $-\frac{1}{4}$

(c) $-\frac{5}{4}$

(d) $-\frac{9}{4}$

(e) $-\frac{3}{4}$

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 004 **CODE 004**

Math 101
Final Exam
181
Saturday 22/12/2018
Net Time Allowed: 180 minutes

Name: ______________________________

ID: ____________________ Sec: ____________________.

Check that this exam has 28 questions.

Important Instructions:

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3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $f(x) = e^{\sin^2(\pi x)}$, then $f'(x) =$

(a) $\pi\, e^{\sin^2(\pi x)} \cos(2\pi x)$

(b) $e^{\sin^2(\pi x)} \sin(\pi x)$

(c) $e^{\sin^2(\pi x)} \cos^2(\pi x)$

(d) $\pi\, e^{\sin^2(\pi x)}$

(e) $\pi\, e^{\sin^2(\pi x)} \sin(2\pi x)$

2. If $[\![x]\!]$ represents the largest integer that is less than or equal to x, then $\lim_{x\to 2}([\![x]\!] + [\![2-x]\!]) =$

(a) 1

(b) 0

(c) 2

(d) does not exist

(e) -1

3. If $y = x\cosh(x^2)$, then $y'(1) =$

(a) $\dfrac{3e}{2} + \dfrac{1}{2e}$

(b) $\dfrac{3e}{2} + \dfrac{3}{2e}$

(c) $\dfrac{3e}{2} - \dfrac{1}{2e}$

(d) $\dfrac{e}{2} + \dfrac{3}{2e}$

(e) $\dfrac{e}{2} - \dfrac{3}{2e}$

4. Consider the function $f(x) = x + e^x$. The value of c that satisfies the conclusion of the **Mean Value Theorem** on the interval $[0, 1]$ is

(a) $1 - e$

(b) 0

(c) $\ln(e - 1)$

(d) $\ln(1 - e)$

(e) $e - 1$

5. By using the linear approximation of $f(x) = \sqrt{100 + x}$ at $a = 0$, $\sqrt{100.5}$ is approximately equal to

(a) $\dfrac{77}{40}$

(b) $\dfrac{399}{40}$

(c) $\dfrac{3}{20}$

(d) $\dfrac{71}{20}$

(e) $\dfrac{401}{40}$

6. The sum of the absolute maximum and the absolute minimum values of the function $f(x) = x^4 - 2x^2 + 2$ in the interval $[-1, 2]$ is

(a) 9

(b) 11

(c) 13

(d) 7

(e) 15

7. If $f(x) = \ln(4 - x^2)$, then the graph of f is increasing on

(a) $(-2, 0)$

(b) $(-2, 2)$

(c) $(-\infty, 0]$

(d) $[0, 2)$

(e) $(-\infty, -2)$

8. $\lim\limits_{x\to\infty} \left(\dfrac{x+2}{x-1}\right)^x =$

(a) e^{-2}

(b) e^2

(c) e^{-4}

(d) e

(e) e^3

9. The sum of all critical numbers of the function $f(x) = \dfrac{x^2+14}{\sqrt{4x+1}}$ is

(a) $\dfrac{-7}{3}$

(b) $\dfrac{2}{3}$

(c) $\dfrac{-1}{12}$

(d) 2

(e) $\dfrac{-2}{3}$

10. If $1200\,cm^2$ of material is available to make a box with a square base and an open top, then the largest possible volume of the box is

(a) $500\,cm^3$

(b) $1000\,cm^3$

(c) $200\,cm^3$

(d) $2000\,cm^3$

(e) $4000\,cm^3$

11. If $f'(x) = \dfrac{3x^3 - \sqrt{x} + 1}{x}$ and $f(1) = 0$, then $f(3) =$

(a) $28 - 2\sqrt{3} + \ln 3$

(b) $10 - 2\sqrt{3} - \dfrac{1}{9}$

(c) $9 - 2\sqrt{3}$

(d) $8 - 2\sqrt{3} - \dfrac{1}{9}$

(e) $26 - 2\sqrt{3} + \ln 3$

12. The slope of the line tangent to the curve $2x + \tan^{-1}(xy) = x^2 y^3 + 2$ at the point $(1, 0)$ is

(a) 1

(b) 0

(c) -1

(d) 2

(e) -2

13. Let $f(x) = e^x\, g(x)$, with $g(1) = 1$, $g'(1) = 2$, and $g''(1) = 3$. Then $f''(1) =$

(a) $e + 3$

(b) e

(c) $e + 1$

(d) $8\,e$

(e) $3\,e$

14. $\lim\limits_{x \to 0} \dfrac{\cos(2x) - 1}{x^{10}} =$

(a) $-\infty$

(b) 0

(c) $\dfrac{2^{10}}{10}$

(d) ∞

(e) $\dfrac{-2^{10}}{10!}$

15. If $f(x) = \ln\left(\dfrac{1+x}{1+\sqrt{x}}\right)$, then $f'(1) =$

(a) 0

(b) $\dfrac{1}{4}$

(c) $\dfrac{1}{2}$

(d) $\dfrac{1}{3}$

(e) 1

16. The number of vertical asymptotes of the function $y = \dfrac{x^2-4}{(x^2+2x-8)(x^2+x+1)}$ is

(a) 0

(b) 3

(c) 2

(d) 1

(e) 4

17. If a and b are the values that make $f(x) = \begin{cases} ax^3 & x \leq 2 \\ x^2 + b & x > 2 \end{cases}$ differentiable, then $a + b =$

(a) 1

(b) $\frac{5}{3}$

(c) 0

(d) -1

(e) $\frac{1}{3}$

18. If $f(x) = \dfrac{(x+1)^3(x-2)\cos x}{3^x(2x+1)}$, then $f'(0) =$

(a) $1 + 3\ln 3$

(b) $1 - 3\ln 3$

(c) $-1 + 2\ln 3$

(d) -2

(e) 0

19. The equation of the horizontal asymptote of the graph of $f(x) = \ln(1 + x^2) - \ln(1 + 2x^2)$ is

(a) $y = -\ln 2$

(b) $y = 2$

(c) $y = 2\ln 2$

(d) $y = 0$

(e) $y = 1$

20. If $f'(x) = e^{(x-4)^3}(x^2 + x + 2)^3(x - 2)^3(x - 1)$, then f is decreasing on

(a) $(2, 4)$

(b) $(1, 4)$

(c) $(-\infty, 2)$

(d) $(1, 2)$

(e) $(4, \infty)$

21. The graph of $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is concave up on

(a) $\left(\frac{5\pi}{4}, 2\pi\right)$

(b) $\left(\frac{7\pi}{4}, 2\pi\right)$

(c) $\left(0, \frac{3\pi}{4}\right)$

(d) $\left(0, \frac{\pi}{4}\right)$

(e) $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

22. If the volume of an expanding cube is increasing at the rate of $4\,m^3/min$, how fast is its surface area increasing when the surface area is $24\,m^2$?

(a) $3\,m^2/min$

(b) $2\,m^2/min$

(c) $4\,m^2/min$

(d) $8\,m^2/min$

(e) $5\,m^2/min$

23. If $\sinh(x\,f(x)) + \sinh(x) = \dfrac{3}{4}$ and $f(\ln(2)) = 0$ then $(\ln 2)\, f'(\ln 2) =$

(a) $-\dfrac{9}{4}$

(b) $-\dfrac{1}{4}$

(c) $-\dfrac{5}{4}$

(d) $-\dfrac{3}{4}$

(e) $-\dfrac{7}{4}$

24. Two cars leave an intersection. One car travels north at $30\,km/h$ and the other travels east at $40\,km/h$. How fast is the distance between them increasing at the end of $30\,mins$?

(a) $40\,km/h$

(b) $50\,km/h$

(c) $25\,km/h$

(d) $60\,km/h$

(e) $100\,km/h$

25. The number of inflection points of $f(x) = x\sqrt{6-x}$ is

(a) 2

(b) 1

(c) 3

(d) 4

(e) 0

26. Newton's Method is used to estimate the critical number of the function $g(x) = x^6 + 15x^2 + 30x + 90$. If we start with $x_1 = 0$, then $x_3 =$

(a) -1.3

(b) -0.9

(c) -0.7

(d) -1.1

(e) -0.5

27. Let $a > 0$ and let $f(x) = x^2 + \frac{x}{a}$, $a \le x \le 2a$. The value of a, that makes the average rate of change of the function f on $[a, 2a]$ the smallest possible, is

(a) $\frac{1}{\sqrt{3}}$

(b) 1

(c) $\sqrt{2}$

(d) 2

(e) $\frac{1}{\sqrt{5}}$

28. Which one of the following statements is **TRUE**?

(a) If f' exists and is non-zero for all x, then $f(1) \neq f(0)$.

(b) If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

(c) $\lim_{x \to 0} \frac{x}{e^x} = 1$

(d) If f has an absolute minimum at c, then $f'(c) = 0$

(e) If f and g are increasing on an interval I, then $f - g$ is increasing on I.

Answer KEY

Q	MM	V1	V2	V3	V4
1	a	e	c	e	e
2	a	d	b	b	a
3	a	b	e	a	c
4	a	a	c	c	c
5	a	d	e	e	e
6	a	e	b	c	b
7	a	a	b	b	a
8	a	a	d	d	e
9	a	b	c	e	d
10	a	a	e	d	e
11	a	b	a	a	a
12	a	e	a	b	e
13	a	d	e	e	d
14	a	a	b	b	a
15	a	a	c	a	b
16	a	b	b	a	d
17	a	c	c	b	d
18	a	d	e	b	c
19	a	c	d	e	a
20	a	a	d	c	d
21	a	d	c	c	e
22	a	a	b	b	d
23	a	b	c	c	c
24	a	a	d	a	b
25	a	b	d	b	e
26	a	a	e	b	b
27	a	d	c	d	a
28	a	d	a	c	a