Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies II
Dr. Mohammad H. Omar
FINAL Exam Term 181 FORM A
Saturday Dec 22, 2018
7.00pm-9.30pm

Name_____ ID#:_____ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on, with any cheating material or cheating actions during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 150 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	5		
2	3+6=9		
3	6		
4	5+3+3+5=16		
5	8		
6	6+6+3+3=18		
7	3+3=6		
8	12		
9	5		
10	5		
Total	90		

Extra blank page

1. (5 points) A 20 year endowment contract of benefit face amount 100000 is issued to (40) and surrendered at age 53. The cash value at that time is 60000. If the extended term insurance option is selected, find the **amount of pure endowment** payable at age 60 using the attached Appendix A life table with 6% interest.

2. (3+6=9 points) The **annual** coupon bearing bond **yield rates** in the following table.

Maturity (in years)	1	2	3	4
Annual Yield Rates for coupon bearing bonds	2.0%	4.0%	6.0%	8.0%

a) Based on the above information, you are also given that $z_2 = 4.040\,808\,3\%$ and $z_3 = 6.169259\,8\%$. Find the term interest rate, z_4, applicable to the last year.

b) Find all possible forward rates for forward securities with maturities of one, two, three, and four years by completing the missing $f_{n,k}$ values in Table below. (Be sure to show at **least one** sample calculation)

n	$f_{n,1}$	$f_{n,2}$	$f_{n,3}$
1.0	$6.122\,448\,9\%$		$10.6858\,54\%$
2.0	$10.5576\,84\%$	$13.0406\,24\%$	-
3.0		-	

3. (6 points) Consider a universal life contract issued to (30) with benefit face amount 100000. The contract receives an annual premium of $G_t = 5000$, earns interest at $i^{(12)} = 0.03$, assesses expense charges at 50% of premium plus 10 per month for the first month of the year and 10 per month in the remaining months, and estimates monthly mortality rates at 1/12 the corresponding annual rate given by the attached Appendix A life table. The account value roll forward process under a **universal life contract** is often done on a **monthly basis.**

   Calculate the **account values** at the end of each of the **first three months**.

4. (5+3+3+5=16 points) At time t a variable **Universal Life** contract has the following investment portfolio.

	# units	**Value/unit**	**Market Value**
Fund A	100	20	10 000 000
Fund B	200	10	5 000 000

   From time t to time $t+1$ Fund A increases by 25% and Fund B decreases by 15%.

   a) Find the **unit value** of Fund A and Fund B at time $t+1$.
   b) Find the **market value** of Fund A and Fund B at time $t+1$.
   c) A premium payment of 2000 is made at time t, allocated equally to Fund A and Fund B. Find the **account value** of the contract at time $t+1$.

5. (8 points) Consider a discrete three year **term insurance** of benefit face amount 50000 issued to (40). The interest rate is $i = 0.06$, mortality is given by the attached Appendix A life table, expenses are 5% of gross premium plus 10 per contract, and there are no lapses. The gross annual premium is $G = 95.00$, the reserves are ${}_1V = 5.66$ and ${}_2V = 6.17$, and the pre-contract expense is 15.00. Determine the **profit vector** for this contract.

6. (6+6+3+3=18 points) We assume a fully discrete five year **term insurance** of 1 000 000 benefit face amount, gross annual premium of 19 250.00, pre contract expenses of 5000, annual per policy expenses of 240 (payable at the beginning of the year), an interest rate of $i = 0.06$ on invested assets, and a risk discount rate of $r = 0.10$. The Table below gives values of the terminal reserves and the mortality factors. There are no surrenders.

Policy Year t	1	2	3	4	5
q_{x+t-1}	0.015	0.017	0.019	0.021	0.024
p_{x+t-1}	0.985	0.983	0.981	0.979	0.976
${}_tp_x$	0.98500	0.96826	0.94986	0.92991	0.90759
${}_tV^G$	2500	4000	5000	4000	0

For the policy described above, calculate

a) the profit vector and
b) the profit signature
c) the net present value and
d) the profit margin.

7. (3+3=6 points) Brian begins work at age 30 exact at GNP Life on January 1, 2015, with salary of \$40000. It is now December 31, 2024. Brian's current salary is \$70000, and the average salary over January 1, 2022 to December 31, 2024 is \$69800.
   The defined benefit pension plan of GNP Life guarantees an annual retirement benefit of 2% of final 3-year average salary at normal retirement age 60 for each year of service.
   Calculate the **annual accrued retirement benefit** of Brian provided by the pension, using

   a) the **projected unit credit** method, assuming the company offers 3% annual salary raises on January 1 of each year.
   b) the **traditional unit credit** method.

8. (12 points) An employer establishes a Defined Contibution (DC) pension plan. The contribution rate is set using the following assumptions:
   i) Contributions are payable monthly in arrears at a fixed percentage of the salary at that time
   ii) Contibutions earn investment returns of 9% effective annually.
   iii) Salary increases at a rate of 0.35% per month.
   iv) The retirement age is 63.
   v) Upon retirement, the employee will use the proceeds to buy a life annuity due payable monthly. The annuity is priced at an assumed interest rate of 5% per year. Under an appropriate assumption, $\ddot{a}_{63} = 13.213$.

   In additon, assume **uniform distribution of death** (UDD) over each year of age, that the first contribution is made 1 month from now, and the entrant receives no other retirement income. Calculate the **smallest contribution rate** required to meet a target replacement ratio of 70% for an entrant aged 23.

9. (5 points) Two lives (x) and (y) have independent and identically distributed future lifetimes. Given the values $P_x = P_y = 0.10$, $P_{\overline{xy}} = 0.05$ and $d = 0.06$, find the value of P_{xy}.

10. (5 points) Find the value of $q_x^{(1)}$, given $q_x'^{(1)} = 0.15$, $q_x'^{(2)} = 0.10$ and decrements are **uniformly distributed** over $(x, x+1)$ in the **multiple decrement** context.

END OF TEST PAPER