Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies II

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Major 1 Exam   Term 181   FORM A
Wednesday Oct 10 2018
6.00pm-7.20pm

Name____ ID#:__________ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 80 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	2+4+6=12		
2	2+4+4=10		
3	5		
4	6+6=12		
5	5		
6	1+5=6		
Total	50		

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1. (**2+4+6=12 points**) A certain animal species can be classified as thriving (State 0), endangered (State 1), or extinct (State 2). Moving among states is goverened by a non-homogeneous Markov process defined by the following transition probability matrices:

$$\mathbf{P}^{(0)} = \begin{bmatrix} 0.85 & 0.15 & 0 \\ 0 & 0.70 & 0.30 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{P}^{(1)} = \begin{bmatrix} 0.90 & 0.10 & 0 \\ 0.10 & 0.70 & 0.20 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{P}^{(2)} = \begin{bmatrix} 0.95 & 0.05 & 0 \\ 0.20 & 0.70 & 0.10 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{P}^{(k)} = \begin{bmatrix} 0.95 & 0.05 & 0 \\ 0.50 & 0.50 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $k = 3, 4, 5, \cdots$.

a) If the species is *thriving* at time n, is it possible for it to be *extinct* at time $n+1$? Why?

b) If the species is *endangered* at time t, how more or less likely is it to become *thriving* within one time interval compared to remaining endangered?

c) If the species is *endangered* at $t = 0$, what is the probability that it will ever become *extinct*?

2. (**2+4+4=10 points**) A four-state Markov process, with states denoted as 0, 1, 2, and 3, begins in state 0 at time 0 for a person age x. The process can transition only from State 0 to one of States 1, 2, or 3. The forces of transition are $\lambda_{01}(x+t) = \mu^{01}_{x+t} = 0.30$, $\lambda_{02}(x+t) = \mu^{02}_{x+t} = 0.50$, and $\lambda_{03}(x+t) = \mu^{03}_{x+t} = 0.70$, for all $t \geq 0$.

a) Is this process homogeneous or non-homogeneous? Provide **reason why**.

b) Solve the **Kolmogorov** Forward Equation for ${}_r p^{00}_x$ (or ${}_r p^{(x)}_{00}$ in MQR5 notation).

c) Calculate the **value** of $P[X(1) = 2|X(0) = 0]$.

3. (**5 points**) Consider a triple decrement model allowing for mortality (Decrement 1), disability (Decrement 2), and withdrawal (Decrement 3). Mortality and disability are uniformly distributed over each year of age in their associated single-decrement tables, but withdrawals can occur **only at the end** of a year of age. You are given:

(i) $q_x'^{(1)} = 0.01$,
(ii) $q_x'^{(2)} = 0.05$, and
(iii) $q_x'^{(3)} = 0.10$,

Calculate $q_x^{(3)}$. (Hint: For simplicity, you may assume $l_x^{(\tau)} = 10000$)

4. (**6+6=12 points**) For a double decrement table, you are given:

(i) $q_x^{(1)} = 0.02$
(ii) $q_x^{(2)} = 0.06$

Calculate $1000 q_x'^{(1)}$ when each decrement is **uniformly distributed** over $(x, x+1)$ in
a) its associated **single decrement** context
b) its associated **double decrement** context.

5. (**5 points**) XYZ Paper Mill purchases a 5-year special insurance paying a benefit in the event its machine breaks down. If the cause is "minor" (1), only a repair is needed. If the cause is "major" (2), the machine must be replaced. You are given:

(i)	The benefit for cause (1) is 2000 payable at the **moment** of breakdown.
(ii)	The benefit for cause (2) is 500 000 payable at the **moment** of breakdown.
(iii)	Once a benefit is paid, the insurance contract is terminated.
(iv)	$\delta = 0.04$
(v)	$\mu_t^{(1)} = 0.100$ and $\mu_t^{(2)} = 0.004$, $t > 0$

Calculate the **actuarial present value** of this insurance.

6. (**1+5=6 points**) For a fully **discrete whole** life insurance of 10000 on (x), you are given

(i)	${}_{10}AS = 1600$ is the asset share at the end of year 10.
(ii)	$G = 200$ is the gross premium.
(iii)	${}_{11}AS = 1627.625$ is the asset share at the end of year 11.
(iv)	$r_{10} = 0.04$ is the fraction of gross premium paid at time 10 for expenses.
(v)	$e_{10} = 70$ is the amount of per policy expense paid at time 10.
(vi)	Death and withdrawal are the only decrements.
(vii)	$q_{x+10}^{(death)} = 0.02$.
(viii)	$q_{x+10}^{(withdrawal)} = 0.18$.
(ix)	$i = 0.05$.

Calculate ${}_{11}CV$, the **cash value** at the end of year 11.
a) 1302
b) 1502
c) 1628
d) 1700
e) 1720

Final answer (1 point) and Work Shown (5 points):

Hence the answer is ( )

END OF TEST PAPER