Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS381: Actuarial Contingencies I
Dr. Mohammad H. Omar
Final Exam Term 181 FORM A
Saturday December 15 2018
7.00pm-9.30pm

Name_____ ID#:_ _ _ _ _ _ _ _ _ Serial #:_ _ _

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 150 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	3+3+4=10		
2	4+3=7		
3	4+3+4=11		
4	3+12×0.5+2=11		
5	4+4+2=10		
6	3+3+3+2=11		
7	3+2+3+3=11		
8	6		
9	4+4=8		
Total	85		

Extra blank page

1. (3+3+4=10 points) If ${}_kp_xq_{x+k} = c(0.90)^{k+1}$ and $q_{x+k} = 0.10 \qquad k = 0, 1, 2, ...$

where $c = 0.10/0.90$ and $i = 0.03$, calculate
a) $A_{x:\overline{5}|}$
b) $P_{x:\overline{5}|}$ and
c) $Var(L)$.

2. (4+3=7 points) A 5 year endowment insurance with a benefit amount of 10000 is issued to (50). On the basis of the illustrative Life Table with interest at the effective annual rate of 6%. Calculate a level annual benefit **premium** payable in semiannual installments if benefit (or sum insured or proceeds) amount are paid

   a) at the **end** of the policy year of death.
   b) at the **moment** of death (semicontinous).

For both parts, assume **uniform distribution of death** in each year of age.

3. (4+3+4=11 points) On the basis of the Illustrative Life Table, uniform distribution of death (UDD), and interest of 6%, calculate values for the benefit reserves in the following table.

a) **Fully Discrete**	b) **Semicontinuous**	c) **Fully Continuous**
${}_{10}V^{1}_{35:\overline{30}\rceil}$	${}_{10}V\left(\bar{A}_{35}\right)$	${}_{10}\bar{V}\left(\bar{A}_{35:\overline{30}\rceil}\right)$

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4. $(3 + 12 \times 0.5 + 2 = 11$ marks) A 3 year term life insurance of 5000 is issued on a fully discrete basis to each member of a group of l_{40} persons at age 40.

   a) Trace the cash-flow expected for this group on the basis of the illustrative Life Table with interest at 6% and, as a by-product, obtain the benefit reserves.

   To help you, part of the work is already shown below. **Complete** the work by filling in the blanks with the correct numbers. (show an example calculation work for each column)

Yr h	Expected Benefit Premiums at Start of Year	Expected Fund at **Start** of Year	Expected Interest	Expected Death Claims	Expected Fund at Year **End**	Expected Number of Survivors at Year **End**	$5000\times {}_hV^{1}_{40:\overline{3}\rceil}$
1	1308962.554		78537.75322		92407.30692		
2		1397729.362	83863.76169			92595.6943	1.0471
3		1398387.475		1482311.5	0	92299.232	
example calculation							

(b) How would the benefit reserves for year 3 change if the contract was a 3 year endowment insurance? (Hint: if necessary, you may use the endowment premium $\pi = 1486.484251$)

5. (4+4+2=10 points) For a three-year **term insurance** with benefit of 1000 to (40), the following are known:

(i) the premium is 2.810994317 and the interest rate is $i = 0.06$,

(ii) the reserves are ${}_1V^{1}_{\overline{40:3|}} = 0.1990$, ${}_2V^{1}_{\overline{40:3|}} = 0.2094$,

(iii) mortality are given in the table below

k	0	1	2	3
l_{40+k}	93131.64	92872.6214	92595.6943	92299.232
d_{40+k}	259.0186	276.9271	296.4623	317.7619

Consider an insured who has survived to the end of the second policy year. For this insured, evaluate

a. $Var\left[{}_2L|K(40) \geq 2\right]$ directly
b. $Var\left[{}_2L|K(40) \geq 2\right]$ by means of the Hattendorf theorem.
c. $Var\left[{}_3L|K(40) \geq 3\right]$

6. (3+3+3+2=11 points) A 3 year endowment policy for a face amount of 6 has the death benefit payable at the end of the year of death and a benefit premium of 1.88 payable annually. Using an interest rate of 20%, the following benefit reserves are generated:

End of year	1	2	3
Benefit reserve	1.32	3.12	6.00

Calculate

a. q_x

b. q_{x+1}

c. The **variance** of the loss at policy issue, ${}_0L$

d. The **conditional variance**, given that the insured has survived through the first year, of the loss at the end of the first year, ${}_1L$

7. (3+2+3+3=11 points) On the basis of De Moivre's law with $l_x = 100 - x$ and the interest rate of 6%, calculate

a. $\bar{A}_{25}$
b. $\bar{P}\left(\bar{A}_{25}\right)$
c. ${}_t\bar{V}\left(\bar{A}_{25}\right)$ and $Var[{}_tL|T(x > t]$ $\quad$ $t = 0$ and 60.

8. (6 points) A policy is issued at age 20 with the following graded scale of death benefits payable at the **moment of death**.

x	20	21	22	23	24	25-40	41 and over
Death Benefits b_{K+1}	1000	2000	4000	6000	8000	10000	50000

Under the **uniform distribution of deaths** assumption over each year of age, calculate the **actuarial present value** on the basis of the Illustrative Life Table and $i = 0.06$. [Hint: Draw the timeline for benefit amounts].

9. (4+4=8 points) If $q_{40} = 0.02$ and $q_{41} = 0.025$, calculate the probability that (40) will die between $40\frac{1}{2}$ and $41\frac{1}{2}$ under

   (a) The assumption that deaths are uniformly distributed within each year of age

   (b) The constant force assumption for each year of age

END OF TEST PAPER

BONUS QUESTIONS

Name_____ ID#:_____ Serial #:___

The test is 30 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
B1	5		
B2	5		
Total	10		

B1. (5 points) A person currently age 60 wishes to purchase a unit 20 year deferred, 2 year temporary annuity immediate, with net single premium (NSP) given by ${}_{20|}a_{60:\overline{2}|}$. The rate of interest is $i = 0.06$. The mortality rates are as follows

x	< 80	80	81
Current q_x	q_x	0.0479	0.0539
New q_x	q_x	$0.99^{20}(0.0479)$	$0.99^{21}\,(0.0539)$

Let X denote the **current** NSP with no mortality improvement projection. Let Y denote the **new** NSP with 1% annual mortality improvement projection only for the payout period of the annuity, but no mortality improvement during the deferred period. Find the value of $\dfrac{Y}{X}$.

B2. (5 points) A benefit of 50 is paid at the moment of failure t. The PDF of $T(x)$, the random variable for failure time, is given by $f_T(t) = \begin{cases} \dfrac{t}{5000} & \text{for} \quad 0 < t \leq 100 \\ 0 & \text{otherwise} \end{cases}$

Find the Actuarial Present Value (APV) of the benefit using force of interest $\delta = 0.10$.