King Fahd University of Petroleum & Minerals

Second Major Examination

Faculty: Science	**Department: Mathematics**
Semester: 181	**Course Name: Financial Mathematics**
Instructor: Abedalhay Elmughrabi	**Course No: AS 201**
Exam Date: December 3rd, 2018	**Exam Time: 05:00 PM – 08:00 PM**

Student Name:	**ID No.:**

Question No.	Question Full Marks	Question Obtained Marks	Question No.	Question Full Marks	Question Obtained Marks	Question No.	Question Full Marks	Question Obtained Marks
1	4 points		11	4 points		21	4 points	
2	4 points		12	4 points		22	3 points	
3	4 points		13	4 points		23	3 points	
4	4 points		14	4 points		24	3 points	
5	4 points		15	4 points		25	3 points	
6	4 points		16	4 points				
7	4 points		17	4 points				
8	4 points		18	4 points				
9	4 points		19	4 points				
10	4 points		20	4 points				
Obtained Total: / 100								

Exam Instructions

1. Fill in all information required.
2. The exam is composed of **25** questions.
3. Answers without calculations/steps will receive zero marks.
4. Only the following is allowed to be on your desk: SOA approved calculator, pen/pencil, eraser, and sharpener.
5. Calculators cannot be exchanged during the examination.
6. No use of smart devices with communications capabilities (mini laptops, pens, watches, phones, etc.)
7. Cell phones must be turned off and placed under your bench facedown.
8. No questions are allowed during the exam.
9. All material related to the course should be put away
10. Be clean, neat and tidy, else your work may not be marked
11. Students must not communicate with one another in any manner whatsoever during the examination.

GOOD LUCK

Question 1 (4 Points):

John borrows 10,000 for 10 years at an annual effective interest rate of 10%. He can repay this loan using the amortization method with payments of 1,627.45 at the end of each year. Instead, John repays the 10,000 using a sinking fund that pays an annual effective interest rate of 14%. The deposits to the sinking fund are equal to 1,627.45 minus the interest on the loan and are made at the end of each year for 10 years.
Calculate the balance in the sinking fund immediately after repayment of the loan.

(A) 2,130
(B) 2,180
(C) 2,230
(D) 2,300
(E) 2,370

Question 2 (4 Points):

An association had a fund balance of 75 on January 1 and 60 on December 31. At the end of every month during the year, the association deposited 10 from membership fees. There were withdrawals of 5 on February 28, 25 on June 30, 80 on October 15, and 35 on October 31. Calculate the dollar-weighted (money-weighted) rate of return for the year.

(A) 9.0%
(B) 9.5%
(C) 10.0%
(D) 10.5%
(E) 11.0%

Question 3 (4 Points):

An investor deposits 50 in an investment account on January 1.
The following summarizes the activity in the account during the year:

Date	Value Immediately Before Deposit	Deposit
March15	40	20
June 1	80	80
October 1	175	75

On June 30, the value of the account is 157.50. On December 31, the value of the account is X . Using the time–weighted method, the equivalent annual effective yield during the first 6 months is equal to the (time-weighted) annual effective yield during the entire 1-year period. Calculate X.

(A) 234.75
(B) 235.50
(C) 236.25
(D) 237.00
(E) 237.75

Question 4 (4 Points):

A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:
(i) Equal annual payments at an annual effective interest rate of 8.07%.
(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective interest rate of *i*.
The sum of the payments under option (i) equals the sum of the payments under option (ii). Calculate *i*.

(A) 8.75%
(B) 9.00%
(C) 9.25%
(D) 9.50%
(E) 9.75%

Question 5 (4 Points):

A loan is amortized over five years with monthly payments at an annual nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment. Calculate the outstanding loan balance immediately after the 40th payment is made.

(A) 6750
(B) 6890
(C) 6940
(D) 7030
(E) 7340

Question 6 (4 Points):

Project P requires an investment of 4000 today. The investment pays 2000 one year from today and 4000 two years from today.
Project Q requires an investment of *X* two years from today. The investment pays 2000 today and 4000 one year from today.
The net present values of the two projects are equal at an annual effective interest rate of 10%.
Calculate *X*.

(A) 5400
(B) 5420
(C) 5440
(D) 5460
(E) 5480

Question 7 (4 Points):

Seth, Janice, and Lori each borrow 5000 for five years at an annual nominal interest rate of 12%, compounded semi-annually.
Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years.
Janice pays interest at the end of every six-month period as it accrues and the principal at the end of five years.
Lori repays her loan with 10 level payments at the end of every six-month period.
Calculate the total amount of interest paid on all three loans.

(A) 8718
(B) 8728
(C) 8738
(D) 8748
(E) 8758

Question 8 (4 Points):

An investor pays 100,000 today for a 4-year investment that returns cash flows of 60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.
Using an annual effective interest rate of 5.0%, calculate the net present value of this investment today.

(A) -1398
(B) -699
(C) 699
(D) 1398
(E) 2,629

Question 9 (4 Points):

You are given the following information with respect to a bond:
(i) par value: 1000
(ii) term to maturity: 3 years
(iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.
Calculate the value of the bond.

(A) 906
(B) 926
(C) 930
(D) 950
(E) 1000

Question 10 (4 Points):

The current price of an annual coupon bond is 100. The yield to maturity is an annual effective rate of 8%. The derivative of the price of the bond with respect to the yield to maturity is -700.
Using the bond's yield rate, calculate the Macaulay duration of the bond in years.

(A) 7.00
(B) 7.49
(C) 7.56
(D) 7.69
(E) 8.00

Question 11 (4 Points):

Seth borrows *X* for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the third year is 559.12.
Calculate the principal repaid in the first payment.

(A) 444
(B) 454
(C) 464
(D) 474
(E) 484

Question 12 (4 Points):

A firm has proposed the following restructuring for one of its 1000 par value bonds. The bond presently has 10 years remaining until maturity. The coupon rate on the existing bond is 6.75% per annum paid semiannually. The current nominal semiannual yield on the bond is 7.40%. The company proposes suspending coupon payments for four years with the suspended coupon payments being repaid, with accrued interest, when the bond comes due. Accrued interest is calculated using a nominal semiannual rate of 7.40%.

Calculate the market value of the restructured bond.

(A) 755

(B) 805

(C) 855

(D) 905

(E) 955

Question 13 (4 Points):

You have decided to invest in two bonds. Bond X is an n-year bond with semi-annual coupons, while bond Y is an accumulation bond redeemable in $\frac{n}{2}$ years. The desired yield rate is the same for both bonds. You also have the following information:

Bond X
• Par value is 1000.
• The ratio of the semi-annual bond rate to the desired semi-annual yield rate is 1.03125.
• The present value of the redemption value is 381.50.

Bond Y
• Redemption value is the same as the redemption value of bond X.
• Price to yield is 647.80.
What is the price of bond X?

(A) 1019
(B) 1029
(C) 1050
(D) 1055
(E) 1072

Question 14 (4 Points):

A liability consists of a series of 15 annual payments of 35,000 with the first payment to be made one year from now.
The assets available to immunize this liability are five-year and ten-year zero-coupon bonds.
The annual effective interest rate used to value the assets and the liability is 6.2%. The liability has the same present value and duration as the asset portfolio.
Calculate the amount invested in the five-year zero-coupon bonds.

(A) 127,000
(B) 167,800
(C) 208,600
(D) 247,900
(E) 292,800

Question 15 (4 Points):

A 40-year bond is purchased at a discount. The bond pays annual coupons. The amount for accumulation of discount in the 15th coupon is 194.82. The amount for accumulation of discount in the 20th coupon is 306.69.
Calculate the amount of discount in the purchase price of this bond.

(A) 13,635
(B) 13,834
(C) 16,098
(D) 19,301
(E) 21,135

Question 16 (4 Points):

Tanner takes out a loan today and repays the loan with eight level annual payments, with the first payment one year from today. The payments are calculated based on an annual effective interest rate of 4.75%. The principal portion of the fifth payment is 699.68.
Calculate the total amount of interest paid on this loan.

(A) 1239
(B) 1647
(C) 1820
(D) 2319
(E) 2924

Question 17 (4 Points):

Kylie bought a 7-year, 5000 par value bond with an annual coupon rate of 7.6% paid semiannually. She bought the bond with no premium or discount.
Calculate the Macaulay duration of this bond with respect to the yield rate on the bond.

(A) 5.16
(B) 5.35
(C) 5.56
(D) 5.77
(E) 5.99

Question 18 (4 Points):

Trevor has assets at time 2 of *A* and at time 9 of *B*. He has a liability of 95,000 at time 5. Trevor has achieved Redington immunization in his portfolio using an annual effective interest rate of 4%.
Calculate *A/B*

(A) 0.7307
(B) 0.9670
(C) 1.0000
(D) 1.0132
(E) 1.3686

Question 19 (4 Points):

An investor purchased a 25-year bond with semiannual coupons, redeemable at par, for a price of 10,000. The annual effective yield rate is 7.05%, and the annual coupon rate is 7%. Calculate the redemption value of the bond.

(A) 9,918
(B) 9,942
(C) 9,981
(D) 10,059
(E) 10,083

Question 20 (4 Points):

An *n*-year bond with annual coupons has the following characteristics:
i) The redemption value at maturity is 1890;
ii) The annual effective yield rate is 6%;
iii) The book value immediately after the third coupon is 1254.87; and
iv) The book value immediately after the fourth coupon is 1277.38.
Calculate *n*.

(A) 16
(B) 17
(C) 18
(D) 19
(E) 20

Question 21 (4 Points):

Annuity A pays 1 at the beginning of each year for three years. Annuity B pays 1 at the beginning of each year for four years.

The Macaulay duration of Annuity A at the time of purchase is 0.93. Both annuities offer the same yield rate.

Calculate the Macaulay duration of Annuity B at the time of purchase.

(A) 1.240
(B) 1.369
(C) 1.500
(D) 1.930
(E) 1.965

Question 22 (3 Points):

A company has liabilities of 573 due at the end of year 2 and 701 due at the end of year 5.
A portfolio comprises two zero-coupon bonds, Bond A and Bond B.
Determine which portfolio produces a Redington immunization of the liabilities using an annual effective interest rate of 7.0%.

(A) Bond A: 1-year, current price 500; Bond B: 6-years, current price 500
(B) Bond A: 1-year, current price 572; Bond B: 6-years, current price 428
(C) Bond A: 3-years, current price 182; Bond B: 4-years, current price 1092
(D) Bond A: 3-years, current price 637; Bond B: 4-years, current price 637
(E) Bond A: 3.5 years, current price 1000; Bond B: Not used

Question 23 (3 Points):

You are given the following information with respect to a bond:

• Par amount: 1000.

• Term to maturity: 3 years.

• Annual coupon rate: 6% payable annually.

The term structure of interest rates is as follows

Term	Annual Spot Interest Rate
1	7%
2	8%
3	9%

Calculate the annual effective yield rate on this bond for an investor who holds it to maturity, if the bond matures at par.

A. 8.1% B. 8.3% C. 8.5% D. 8.7% E. 8.9%

Question 24 (3 Points):

Toby purchased a 20-year par value bond with semiannual coupons at a nominal annual rate of 8% convertible semiannually at a price of 1722.25. The bond can be called at par value 1100 on any coupon date starting at the end of year 15. What is the minimum yield that Toby could receive, expressed as a nominal annual rate of interest convertible semiannually?

A. 3.2%
B. B. 3.3%
C. C. 3.4%
D. D. 3.5%
E. E. 3.6%

Question 25 (3 Points):

A 1000 par value 5–year bond with 8.0% semiannual coupons was bought to yield 7.5% convertible semiannually. Determine the amount of premium amortized in the 6-th coupon payment.

(A) 2.00
(B) 2.08
(C) 2.15
(D) 2.25
(E) 2.34